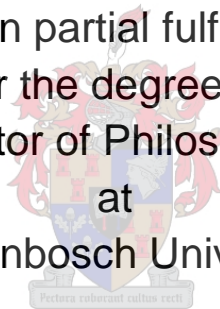


UNPACKING TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE AND SKILLS TO DEVELOP LEARNERS' PROBLEM SOLVING SKILLS IN MATHEMATICS

by

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DECLARATION

By submitting this thesis electronically, I declare that the entirety of the work contained therein is my own, original work, that I am the sole author thereof (save to the extent explicitly otherwise stated), that reproduction and publication thereof by Stellenbosch University will not infringe any third party rights and that I have not previously in its entirety or in part submitted it for obtaining any qualification.

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ABSTRACT

In this study, the pedagogical knowledge of Foundation Phase teachers is explored (and unpacked) in order to obtain insight into their understanding of the teaching and learning of mathematics. The teacher's knowledge is explored, as it is one of the most important variables that impacts on what is done in the classroom. The exploration is undertaken against the background of the very poor overall achievement of learners in the national systemic evaluations and in international assessment studies, which is currently a cause for great concern. This has resulted in different nation-wide intervention programmes that are aimed at improving teacher performance and effectiveness. In this study, the teacher is the focal point of the intervention. Problem-based learning (PBL), which is well-regarded as being one of the best examples of a constructivist learning environment, is introduced to a group of 15 Foundation Phase teachers. The study is an unpacking of the Foundation Phase teachers' pedagogical knowledge and beliefs regarding, and practices in, the teaching and learning of mathematics, as well as in the use of PBL as a vehicle for the teaching and learning of mathematics. The unpacked knowledge can be used to address the challenges that are related to the improvement of the teaching and learning of mathematics in the Foundation Phase.

A combination of qualitative and quantitative research methods, including questionnaires, interviews, lesson observation, and workshops, were used to explore the teachers' current pedagogical knowledge, beliefs and practices with regard to problem-solving. It was also used to expose the teachers to PBL as an alternative approach to teaching and learning mathematics in the Foundation Phase. The study provides a body of knowledge on the Foundation Phase teachers' pedagogical knowledge, practices and beliefs regarding the teaching and learning of mathematics in general, and approaches to problem-solving in particular, thus providing insights into some of the factors that might lie behind learner outcomes.

Study findings indicate that the majority of teachers' daily mathematical teaching culture was deep-rooted in the traditional approach (direct transmission). This approach was characterised by the teachers concerned focusing on the following of rules and procedures, and on doing demonstrations on the chalkboard, whereafter the learners were encouraged to practise what they had learned by asking them to do pen-and-paper calculations. The lessons were generally not structured to develop critical thinking and reasoning skills. In instances where the teachers created learner-centred activities that were conducive to the development of such skills, deep-rooted traditional approaches manifested themselves in the way in which the teachers showed the learners how to solve their given problem at the

earliest signs of any difficulty in doing so was exhibited by the learners. In so doing, the majority of the teachers, despite initially creating learning opportunities by posing problems to their learners, they soon snatched away the selfsame opportunities from them. This was because they did not allow sufficient time for the learners to grapple with a problem, and to engage in critical thinking.

After exposure to PBL, the educators were able to implement PBL so effectively that they could address the problems related to low learner achievement in mathematics, as reflected in the international assessment studies, and in the national systemic evaluations within the current South African context.

OPSOMMING

In hierdie studie is die pedagogiese kennis van Grondslagfase-onderwysers ondersoek ten einde insig te verkry in hulle begrip van die onderrig en leer van wiskunde. Die onderwysers se kennis is ondersoek aangesien dit een van die belangrikste veranderlikes is wat 'n invloed het op dit wat in die klaskamer uitgevoer word. Die ondersoek is onderneem teen die agtergrond dat die algehele prestasie van leerders in die nasionale sistemiese evaluerings en internasionale assesseringstudies uiters swak en 'n bron van groot kommer was. Dit het gelei tot verskillende intervensieprogramme wat gemik is op die verbetering van onderwyserprestasie en -doeltreffendheid. In hierdie studie is die onderwyser die fokuspunt van die intervensie. Probleem-gebaseerde leer (PBL), wat beskou word as een van die beste voorbeelde van 'n konstruktivistiese leeromgewing, is aan 'n groep van 15 onderwysers in die Grondslagfase gebring. Die studie was 'n poging om nuwe kennis te identifiseer ten opsigte van Grondslagfase-onderwysers se pedagogiese geloof en praktyke in die onderrig en leer van wiskunde, en die gebruik van PBL as 'n middel vir die onderrig en leer van wiskunde – kennis wat gebruik kan word om die verwante uitdagings aan te spreek ter verbetering van die onderrig en leer van wiskunde in die Grondslagfase.

'n Kombinasie van kwalitatiewe en kwantitatiewe navorsingsmetodes, wat vraelyste, onderhoude, les-waarneming en werkswinkels ingesluit het, is aangewend om die onderwysers se huidige pedagogiese sienings en praktyke met betrekking tot probleemoplossing grondig te ondersoek en dan voort te gaan om die onderwysers bloot te stel aan PBL as alternatiewe benadering tot onderrig en leer van wiskunde in die Grondslagfase.

Die studie het bevind dat die meerderheid van die onderwysers se huidige onderrigkultuur een was wat diep gewortel is in die tradisionele benadering van onderrig en leer van wiskunde (direkte oordrag): dit is gekenmerk deur die onderwysers se onderrig van wiskunde deur te fokus op reëls en prosedures, demonstrasies aan die klas op die swartbord en leerders dan te laat oefen deur pen- en papierberekeninge te doen. Dié het die meerderheid van die onderwysers daaglik gedoen. Lesse is oor die algemeen nie gestruktureer om kritiese denke en beredenering te ontwikkel nie. In gevalle waar die onderwysers leerder-gesentreerde aktiwiteite geskep het wat weens hulle ontwerp bevorderlik is vir die ontwikkeling van kritiese denke en redenasie, het die diepgewortelde, tradisionele benaderings hulself gemanifesteer in die feit dat die onderwysers, met die eerste aanduiding dat die leerders sukkel, hulle te hulp gesnel het en die leerders gewys het hoe om die probleem op te los. Met dié optrede het die meerderheid van die onderwysers

aanvanklik leergeleenthede geskep (deur probleme aan hulle leerders voor te hou), maar dit spoedig dan weer weggeraap weens die feit dat hulle nie genoegsame tyd toegelaat het vir hulle leerders om met idees te worstel en deel te hê aan kritiese denke nie.

Blootstelling van die opvoeders aan PBL het aan die lig gebring dat opvoeders in die Grondslagfase PBL doeltreffend kan implementeer om probleme rondom lae leerder prestasie in wiskunde aan te spreek wat in internasionale assesseringstudies en in die nasionale sistemiese evaluerings binne die huidige Suid -Afrikaanse konteks weerspieël word.

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DEDICATION

I dedicate this dissertation to my late uncle, Edmond Toga Tambara. I am what you once were.

ACRONYMS

ANA.....	annual national assessment
CAPS.....	Curriculum and Assessment Policy Statement
CDC.....	Centers for Disease Control and Prevention
CLIMMB.....	Curriculum, Literacy, Mathematics and Management Buzz
CORI.....	Concept-Oriented Reading Instruction
DET.....	Department of Education and Training
HOR.....	House of Representatives
IMSTUS.....	Institute of Mathematics and Science Teaching University of Stellenbosch
KCS.....	Knowledge of Content and Students
KCT.....	Knowledge of Content and Teaching
MKT.....	mathematical knowledge for teaching
NCTM.....	The National Council of Teachers of Mathematics
NGO.....	non-governmental organisation
OECD.....	Organization for Economic Co-operation and Development
PBL.....	problem-based learning
PCK.....	pedagogical content knowledge
PCL.....	problem-centred learning
RME.....	Realistic Mathematics Education
SDL.....	self-directed learning
SPSS.....	Statistical Package for the Social Sciences
TALIS.....	Teaching and Learning International Survey
TIMSS.....	Trends in International Mathematics and Science Study
TLLM.....	Teach Less, Learn More
TSLN.....	Thinking School, Learning Nation
WCED.....	Western Cape Education Department

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CHAPTER 1: ORIENTATION, RATIONALISATION AND CONTEXTUALISATION

“The international achievement studies enabled South Africa to benchmark its learner performance, and thus its education system, against those of other countries. Overall, the achievement of learners in the national systemic evaluations and in international assessment studies was very poor and a cause for great concern.”

(WCED, 2008, p. 41)

1.1 INTRODUCTION

1.1.1 Background to the study

This research was carried out when the researcher was a mathematics facilitator in an intervention Project X run by a South African university (which is not named for reasons of confidentiality and protection of the participants). The intervention Project X was involved in the professional development of teachers in the teaching of mathematics at Foundation Phase. It was during the researcher's interaction with the teachers in the project schools, which took the form of lesson observations and feedback discussions, that the researcher was motivated to research the teachers' pedagogical content knowledge (PCK) in general, and that regarding problem-solving, and its use as a vehicle for the teaching and learning of mathematics in particular.

1.1.2 Motivation for the proposed research

Despite the majority of South African learners enjoying above-average levels of public and private education resources, yet their performance in mathematics, on both regional and international tests, has been found to be extremely weak (Fleisch, 2008; Moloi & Chetty, 2011; Moloi & Strauss, 2005; Mullis et al., 1999; Mullis, Martin, Beaton, Gonzalez, Kelly & Smith, 1997; Mullis, Martin, Gonzalez & Chrostowski, 2003; Reddy, 2006; Van der Berg, 2007; Van der Berg, Taylor, Gustafsson, Spaull & Armstrong, 2011; WCED, 2006, 2009). The teacher's role in teaching and learning is identified as a common and necessary factor in addressing this poor performance (Brown, 1992; Even & Tirosh, 2008; Hiebert & Carpenter, 1992; Schoenfeld, 2011; Van der Sandt & Nieuwoudt, 2003), with the teacher's knowledge being one of the most important variables that impact on what is done in the classroom (Fennema & Franke, 1992; Turner-Bisset, 2001).

Similarly, teachers are an important factor in the success of any curriculum, because they filter the curriculum through to the learners (Graham & Fennell, 2001; Swan, 2001; Walsaw,

2010). It has also been observed that it is the teacher who makes the most important contribution to improvements in terms of mathematical achievement (Ball & Forzani, 2009; Ball, Sleep, Boerst & Bass, 2009; Bansilal & Wallace, 2008).

1.1.3 Statement of the problem

The Grade 3 Systemic Assessment indicated that the learners in the schools falling within the parameters of Project X were performing poorest in problem-solving, compared to the other knowledge and skill areas (WCED, 2008). It was, therefore, important to determine the teachers' knowledge of, and skills concerning, problem-solving, and its use as a vehicle for the teaching and learning of mathematics in the project schools.

1.2 THE PURPOSE AND AIMS OF THE RESEARCH

Fleisch (2008) points out that one of the more complex and controversial issues is the average level of teacher competence, with the empirical evidence in this regard being inconclusive. Few studies previously undertaken have provided clues about teachers' pedagogical skills. Currently, pressure exists to establish evidence of teachers' capacity, skills and knowledge regarding learners' learning (Hill, Sleep, Lewis & Ball, 2007, p. 111). The purpose of the present research was to contribute towards the complex issue of understanding teacher competencies in the development of problem-solving skills in learners. The research undertaken in this respect, thus, focused on the teacher as a critical variable in the improvement of the teaching and learning of mathematics in a group of schools that fell within the ambit of Project X.

The aim of this research was:

- to analyse the teachers' PCK in general, and in relation to problem-solving as a vehicle of effective learning;
- to determine what role beliefs play in teachers' use of problem-solving as a vehicle for learning; and
- to facilitate the use of problem-solving as a vehicle for learning.

The research sought to answer the following questions:

- What PCK do teachers have in general, and in relation to problem-solving and its use as a vehicle for learning?
- What beliefs do teachers have about problem-solving in general, and about its use as a vehicle for learning?

- What are the support needs of teachers in using problem-solving as a vehicle for learning?

1.3 RESEARCH DESIGN

The current research adopted a mixed-method research design. The advantage of using such a design is that it combines both qualitative and quantitative methods of research, and it can, hence, both show the results obtained (quantitative), and explain why they were obtained (qualitative) (Denzin & Lincoln, 2000; McMilan & Schumacher, 2006). Three types of mixed-method designs exist: the explanatory design; the exploratory design; and the triangulation design (McMilan & Schumacher, 2006, p. 28). The present researcher used the triangulation design (Bogdan & Biklen, 2006; Cohen, Manion & Morrison, 2007; Kelly, 2007) as use of the design allowed for the simultaneous collection of both qualitative and quantitative data, as well as for the application of “the strengths of each approach to provide not only a more complete result but also one that was more valid” (McMilan & Schumacher, 2006, p. 28). Details of the research design are shown in Chapter 4.

1.4 THE FOCUS ON PROBLEM-SOLVING

Problem-solving skills, in the current context, refer to the learner's ability to perform calculations; organise data; recognise patterns; make models; write equations; guess; eliminate possibilities; and check results (Reiss & Törner, 2007; Salmon & Grace, 1984; Voyer, 2011).

The development of problem-solving skills is important for a number of reasons. Firstly, problem-solving is a skill that is central to mathematics learning (Cooper, 2010; Dhlamini, 2009; Govender, 2010; Janassen, 2000; Sweller, Clark & Kirschner, 2010). It is one of the critical outcomes of the National Curriculum Statement, and of the Curriculum and Assessment Policy Statement (CAPS) (Republic of South Africa. Department of Education, 2011). Problem-solving has, in fact, been allocated time daily in mathematics lessons in Grade 1 to Grade 3 (Republic of South Africa. Department of Education, 2011).

Secondly, problem-solving is the cognitive domain to which 40% of the time is devoted during learner assessment in the mathematics assessment framework for the Trends in International Mathematics and Science Study (TIMSS) (Heugh, Diedericks, Prinsloo, Herbst & Winnaar, 2007). TIMSS is an international comparative study that primarily measures learner achievement in mathematics and science. The TIMSS assessment framework has been designed to ensure comparability, and so as to benchmark performance (Reddy, 2006). The same framework is used by the Department of Education in the design of its systemic tests (Heugh, Diedericks, Prinsloo, Herbst & Winnaar, 2007).

Thirdly, the learners who are assessed at the end of the Foundation Phase (Grade 3) are performing poorly in terms of problem-solving. Observations are that the poor performance of South African learners in international tests (namely, TIMSS), and in schools in general, requires that an emphasis should be placed on problem-solving in the teaching and learning of mathematics (Cooper, 2010; Mwakapenda & Dhlamini, 2010; Van der Berg, Taylor, Gustafsson, Spaul & Armstrong, 2011). This observation was confirmed in the five schools involved in Project X, according to their 2008 Grade 3 assessment results, which showed that the learners were performing poorest in problem-solving, and thus affecting the school's overall score negatively (see Table 1.1 below).

	*School A	School B	School C	School D	School E
	2008	2008	2008	2008	2008
Counting, ordering and representing numbers	N/A	42.1%	84.1%	52.7%	36.2%
Calculations	N/A	38.3%	79.8%	39.0%	32.8%
Problem-solving	N/A	11.4%	49.5%	12.3%	3.3%
Overall score	N/A	29.6%	81.8%	30.1%	14.4%

*In 2008, School A was a new school, and an offspring of School B.

Table 1.1: Percentage of learners performing well in the knowledge and skills tested (WCED, 2008)

The poor performance in problem-solving illustrated above was a compelling reason for examining the teachers' mathematical knowledge for teaching (MKT). MKT includes mathematical reasoning, insight, understanding, and the skills needed to teach mathematics, as well as the skills needed to develop learners' problem-solving abilities (Hill, Rowan & Ball, 2005; Silverman & Thompson, 2008). Finally, there has been a general shift towards problem-solving, rather than towards the mastering and applying of skills (Hiebert et al., 1996). As South African learners continue to perform poorly in the TIMSS, there is a need to learn from such countries as Singapore that have consistently performed well in the TIMSS. Singapore was first in both the fourth and the eighth grades in the TIMSS comparison assessments that were conducted in 1995, 1999 and 2003, and it ranked among the top three in 2007 (Clark, 2009). This success has been attributed to various factors, including a coherent national curriculum, teacher training, a public belief in the importance of mathematics, and, notably, an emphasis on the importance of problem-solving. In 1992, Singapore began to emphasise the latter in its curriculum (Clark, 2009).

Clark (2009) points out that, in addition to paying careful attention to the teaching and cultivating of problem-solving skills, the belief prevails in Singapore that problem-solving is a requirement for the twenty-first century. This outlook, the researcher believes, is worth

considering in the efforts that are exerted to making one's country's learners competitive on an international scale.

In South Africa, the current curriculum reforms' emphasis on problem-solving is a desirable shift towards good practice, as an emphasis on problem-solving has been found to prevail in countries that continue to do well in the TIMSS. Most South African teachers have only experienced a very traditional, whole-class approach, in which the teacher is the expert conveyor of knowledge, and in which the pupils are encouraged to be passive recipients. The selfsame teachers are being asked to adopt changes that are very different from their current understanding of the teaching and learning of mathematics. According to Stigler and Hiebert (1998, p. 6), there is a need to understand that teaching is a cultural activity, resulting in the need for growing awareness of the cultural scripts that teachers use. As the South African current curriculum emphasises problem-solving as a way of learning and practising mathematics, it has become equally important to understand their present teaching culture (Stigler & Hiebert, 1998).

1.5 DEFINITION OF KEY TERMS

The key terms that are used in this research are explained below:

A **teacher's knowledge** refers to the teacher's awareness, or understanding, of a circumstance or fact, which is gained through association or experience, books, media, encyclopaedias, academic institutions, and other sources.

A **teachers' pedagogical knowledge** refers to the specialised knowledge of teachers that is aimed at creating effective teaching and learning environments for all the students that they teach.

A **teacher's knowledge and skills** refers to the teacher's awareness, or understanding, of a circumstance or fact, and their ability to use and/or to apply that awareness, or understanding, of a circumstance, or fact, in context.

A **teacher's pedagogical skills** refers to the teacher's ability to plan, and to provide, a set of learning opportunities that offer access to crucial concepts and skills for all learners, as well as to the teacher's ability to assess student learning.

Mathematical knowledge for teaching (MKT): MKT refers to the mathematical knowledge that is used in the classroom. Exceeding the knowledge of formal mathematics, it is the mathematical knowledge that one needs for carrying out one's work as a teacher of mathematics (Hill et al., 2008).

Pedagogical content knowledge (PCK): PCK refers to the teacher's knowledge of the nature of the matter to be taught, of how the learners learn the matter, of how best to teach the matter, of the materials that are suitable for teaching the matter, and of how the matter fits into the curricula.

Common content knowledge: Common content knowledge is the mathematical knowledge and skills that are used in settings other than teaching, with an example of such knowledge being that of the algorithm required for multiplying two numbers together (Ball, Thames & Phelps, 2008). Knowledge of this nature informs such teaching tasks as knowing whether a learner's answer is correct, knowing the definition of a concept or object, and knowing how to carry out a procedure (Hill & Ball, 2009; Sullivan, 2008).

Specialised mathematical knowledge: This form of knowledge includes being able to model integer arithmetic, using different representations (Hill & Ball, 2009; Sullivan, 2008).

Horizon knowledge: Horizon knowledge is a kind of mathematical 'peripheral vision' that is required in teaching. It encompasses having a view of the larger mathematical landscape that teaching requires (Ball, Thames & Phelps, 2008, p. 69).

Problem-solving: Problem-solving refers to the learners' ability to use, and to apply, mathematics in relation to practical tasks, real-life problems, and mathematics itself (Hiebert et al., 1996; Libeskind, 1977; Schoenfeld, 1992). It applies to a wide variety of situations, ranging from routine mathematical problems occurring in an unfamiliar context, to open-ended investigations that make use of the relevant mathematics and thinking processes (Fai, 2006; Hino, 2007). Problem-solving is a process wherein the learner encounters a question for which he/she has no immediately apparent solution or algorithm available to directly apply so as to obtain the appropriate answer (Tripathi, 2009). It is an important way of doing, learning and teaching mathematics (Chapman, 2005).

Problem-based learning (PBL): PBL is a learner-centred instructional approach in which learners solve problems collaboratively, and then reflect on their experiences. It is a learning environment in which problems drive the learning involved (Kyeong Ha, 2003; Schroeder & Lester, 1989).

Unpacking: Unpacking refers to revealing, deciphering, encoding, shedding light on, or exploring the teachers' PCK, practices and beliefs by means of lesson observation, interviews and questionnaires.

1.6 ETHICAL CONSIDERATIONS

The ethical issues of informed consent, deception, confidentiality, anonymity, privacy, and caring (Christians, 2000; McMillan & Schumacher, 2006), were addressed according to the ethical code of Stellenbosch University, hence the following ethical conditions and considerations were fulfilled during this research.

1.6.1 Ethical considerations regarding research participants

Participant's consent: The participants were provided with an opportunity to give their consent to freely participate in the research. They were, therefore, not involved in the research without their knowledge or consent, and neither were they coerced into participating in it, nor were they induced into committing acts that diminished their self-esteem.

Participant's anonymity: The participants' right to remain anonymous was respected throughout the research.

Transparency: Transparency was ensured by openly discussing the purpose, the objectives, and the goals of the research with everyone who was involved in the research. At no time did the researcher withhold information about the true nature of the research.

Honesty and uprightness: The evaluation of the research was carried out with the honesty and uprightness expected in research. There was no fabrication, falsification or misrepresentation of evidence, data, findings, and conclusions. There was no tampering with the evidence. The researcher communicated the findings in clear, straightforward and appropriate language to all stakeholders.

Participant's respect, safety, dignity and self-worth: The researcher respected and safeguarded the safety, dignity and self-worth of all the research participants by being mindful of the cultural, religious, gender and other significant differences present in the research population.

1.6.2 Ethical considerations regarding research data

Criteria for generation of data: The data generated in this research were scrutinised according to the criteria for data generation, so as to ensure authenticity, believability, validity and reliability (Janesick, 2000; Lincoln & Guba, 2000).

Authenticity and believability of data: Authenticity of data refers to the genuineness of data, while the believability of data is the extent to which data are regarded as true and credible, with both aspects being major reflections of quality (Marco & Larkin, 2000, p. 693).

The believability of data depends on their *origin* or source, and on their subsequent *processing history* (Prat & Madnick, 2008, p. 7). The two informal ways by means of which data can be assessed for authenticity and believability are external criticism and internal criticism.

External criticism: In this research study, external criticism was used to verify whether the data concerned were collected from legitimate sources. The data in this research came from real teachers in the Foundation Phase, resulting in their accuracy, as far as they comprised real responses from teachers (*origin*). The data, however, varied among the teachers, as they took the form of spontaneous responses to open-ended questions.

Internal criticism: Internal criticism is concerned with data accuracy and bias (Marco & Larkin, 2000; Prat & Madnick, 2008). As a result, the data generation and processing methods undertaken in the research were discussed with various researchers in the field of educational research, in order to substantiate the legitimacy of the methodology used (*processing history*).

1.6.3 Validity and reliability of the data

This section briefly introduces the concepts of validity and reliability, which are examined in greater detail in Chapter 6. The validity of a research measuring instrument is the extent to which it measures what it is intended to measure, whereas reliability is the consistency with which the research instrument yields a certain result when the sample being measured has not changed (McMillan & Schumacher, 2006, p. 134; Shank, 2006). The validity and reliability of a research instrument influence the extent to which one is able to learn something about the subject under research (Golafshani, 2003; Lincoln & Guba, 2000, p. 178). The probability that one will obtain statistical significance in one's data analysis, and of being able to draw meaningful conclusions from one's data, is related to the data validity and reliability (Howe & Lewis, 1994).

On completion of the draft questionnaire, its validity was established by means of consulting the necessary experts, and by pilot testing it. The questionnaire was pilot tested with teachers teaching in the Foundation Phase in five schools with the same demographics as the research sample schools. The data collected from the pilot test were analysed using the Statistical Package for the Social Sciences (SPSS). The test provided two key pieces of information, namely a 'correlation matrix' and a 'view alpha if item deleted' column (Radhakrishna, 2007). Changes were made based on both a field test, and on expert opinion.

1.7 SIGNIFICANCE OF THE STUDY

At the time at which the present study was undertaken, little information existed about the South African Foundation Phase teachers' pedagogical knowledge, beliefs and practices related to problem-solving in the teaching and learning mathematics. Such information is important for understanding and improving the teaching and learning of mathematics in the Foundation Phase. The teachers' pedagogical knowledge, beliefs and practices are closely linked to the teachers' strategies for coping with the day-to-day challenges of shaping the learners' mathematics learning environment, as well as the learners' motivation and performance. Very little research has been done on profiling teachers' MKT related to problem-solving, and its use as a vehicle for the teaching and learning of mathematics. To the best of the researcher's knowledge, this study is the first full-fledged research of its nature to have, as yet, been undertaken at Foundation Phase level, and to have focused on the teacher's pedagogical knowledge of, and skills for, mathematics problem-solving, and the facilitation thereof as a vehicle for learning mathematics in South Africa. The overall significance of the study, therefore, is that it provides a robust relevant analysis of the teacher's pedagogical knowledge, beliefs and practices. It was set to provide information that would be useful in the review and development of teacher professional development that would be conducive to effective learning.

The following benefits were intended to result from the study:

1. The analysed data would provide an overall picture of the current situation regarding teachers' pedagogical knowledge in general, and regarding problem-solving, and its use as a vehicle for the teaching and learning of mathematics.
2. Professional development needs for the intervention programme would be identified.
3. Recommendations would be made about how to improve the teaching of mathematics through PBL, which is an approach that uses problem-solving as a vehicle for the teaching and learning of mathematics.

1.8 CONTEXT OF THE STUDY

This study was carried out while the researcher was a mathematics facilitator involved in an intervention Projected X run by the Institute for Mathematics and Science Teaching. The three-year intervention project involved the professional development of teachers in the teaching of mathematics at Foundation Phase. In my brief discussion with these teachers, I had observed that the teachers seemed to possess scant knowledge of how to use a problem-based approach in the teaching of mathematics. I was curious both about their beliefs, and about their PCK, regarding the use of problem-solving as a vehicle for learning. As a paucity of recorded evidence existed regarding the use of PBL in the Foundation Phase

in South Africa, I decided to address this gap in my study. I then decided to investigate the beliefs and PCK regarding problem-solving of the teachers in the 5 schools involved in the project. I also wanted to investigate ways in which to support the teachers in their use of problem-solving as a vehicle for learning mathematics. To contextualise the above-mentioned research, this section discusses the following:

- the role of the mathematics facilitator;
- the project schools;
- the nature of the Institute;
- an overview of Project X; and
- The Foundation for Learning Campaign.

1.8.1 The role of the mathematics facilitator

In addition to discussing the role of a facilitator, the section draws a link with the research. Most professional development initiatives that have been undertaken in South Africa have been prompted by the TIMSS results, and by the subsequent desire to improve learner achievement. In the past (Higgins, 2005), the main focus of such professional development has been the teacher's content knowledge. However, lately the shift has been towards helping the teacher implement the new practices in their everyday teaching that is aimed at improving learner achievement.

The primary responsibility of the mathematics facilitator in Project X was to increase teacher capacity in terms of mathematics content skill, knowledge and pedagogy, so as to improve the current low learner performance in mathematics. In view of the renewed emphasis on problem-solving, it was the hope of the present facilitator, therefore, that the research undertaken would provide data that would contribute towards the understanding of the teacher's mathematics content skill, knowledge and pedagogy. The professional growth of the educators concerned in the new teaching practice, in which problem-solving is both a primary goal and a vehicle for teaching mathematics, was anticipated. Such growth was intended to promote the understanding that was to form the basis for professional development in relation to problem-solving.

In general, a mathematics facilitator (Carlson, Moore, Bowling & Ortiz, 2007) must be able to demonstrate exemplary classroom mathematics practice. In addition, they must be able to observe and coach classroom teachers, to plan and conduct mathematics professional development, and to collect, analyse and report school and district data to administrators and others, as needed or requested.

The core functions of the mathematics facilitator in Project X were in many ways similar to those described above. They were to:

- provide expertise in mathematics content and mathematics pedagogy;
- present demonstration lessons to the educators;
- work collaboratively to improve the instruction of mathematics, using research-based approaches and intervention;
- demonstrate and share exemplary classroom mathematics practices;
- plan and conduct mathematics professional development for the mathematics educators; and
- apply a variety of instructional theories and models, incorporating best practices, child-centred learning, and differentiated instruction approaches aimed at achieving the success of all learners concerned.

The observation of poor performance in the key area of problem-solving by learners in the schools in Project X motivated the facilitator to carry out the research in line with the third point bulleted above.

1.8.2 Description of the project schools

For one to have an appreciation of the environment within which the research was carried out, the following brief description of each of the schools in the project is given. This description is based on the data that were collected during the initial phase of Project X.

1.8.2.1 School A

Demographic data

The school had 5 educators and 250 learners in the Foundation Phase at the dual-medium school (isiXhosa and English) concerned. IsiXhosa was the medium of instruction in the Foundation Phase. The Foundation Phase had an average of 45 to 50 learners per class. This comparatively new school was established in 2006 as a spring-off from school B, which is described below.

Resources

Being a new, totally prefabricated township school, with overcrowded classrooms and an inadequate number of up-to-date textbooks, school A lacked both a library and a laboratory.

Little, if any, provision of reading materials made it difficult for the learners to develop the grade-appropriate reading skills required.

Management and staff

Good management and coordination were in place, with both the principal and the educators being keen to see the performance of learners at their new school improve. The adoption of a team approach was evident in the school.

Learners

Most of the learners came from poor families and social backgrounds, with some of the learners being said to come to school without breakfast. Over half of the learners were on the school's feeding scheme. A growing number of the learners were said to be in the high-risk category. The school did not experience any serious cases of absenteeism from either the educators, or the learners. No serious disciplinary problems were present among the learners in the Foundation Phase.

Community/Parental involvement

The school shared the same community with school B described below. The educators had observed that a significant number of their learners came from single-parent households. The level of poverty was high in the community, due largely to the prevailing levels of unemployment. The educators also observed that the home supervision of learners was lacking.

1.8.2.2 School B

Demographic data

The school had 15 teachers and 675 learners in the Foundation Phase. It was a dual-medium school (isiXhosa and English), with isiXhosa being the medium of instruction in the Foundation Phase. The Foundation Phase had 40 to 45 learners per class.

Resources

The school was a poorly resourced previous Department of Education and Training (i.e. ex-DET) (African) township school, with overcrowded classrooms, and an inadequate number of up-to-date textbooks. Some of the school buildings were in disrepair, probably as a result of the neglect of maintenance work.

Management and staff

Good coordination was in place in the Foundation Phase, with both the deputy principal and the teacher in charge showing efficient control of the functioning of the phase. The latter member of staff was both very energetic and keen to have the school's Foundation Phase performance improve.

Learners

The learners generally experienced numeracy problems. Both the levels of literacy and of numeracy were low, as indicated by the low annual national assessment (ANA) and systemic test scores. The learners experienced transition problems from Grade 3 to Grade 4. The educators and the learners at the school did not exhibit any absenteeism. The school had no serious problems with the discipline of the learners in the Foundation Phase.

Community/Parental involvement

Some parents were seasonal workers. Unemployment severely impacted on the community. The school management was not satisfied with the amount of parental support that they received.

1.8.2.3 School C**Demographic data**

The school had 9 educators and 35 to 37 learners per class in the Foundation Phase. The school was a previous House of Representatives (i.e. ex-HOR) (coloured) school. Despite the school being dual-medium (English/Afrikaans), in addition some isiXhosa-speaking learners received their instruction in either English or Afrikaans.

Resources

The school was well resourced, with a science laboratory, a computer laboratory, and a library.

Management and staff

The school had an inspiring principal, who was very involved in, and enthusiastic about, the project. Both the principal and the educators were very enthusiastic about the project. Good management and coordination was in place. A motivated team approach was evident, and serious commitment to learner success was observed.

Learners

The learners were from mixed social backgrounds, among whom there were a number of unemployed and poor parents. The language of instruction was an issue for the isiXhosa learners. A number of the learners were in the high-risk category. The learners experienced problems in transitioning from Grade 3 to Grade 4. The prevailing literacy and numeracy levels were said to be improving at the time of the study. Neither absenteeism nor discipline was a problem, with measures to curb any arising already being in place.

Community/Parental involvement

The school is based in a community of parents of mixed backgrounds. Although unemployment was a challenge of some significance, the parents assisted with fundraising.

1.8.2.4 School D**Demographic data**

The school had 12 educators and 503 learners in the Foundation Phase. It was a dual-medium school (English and Afrikaans), although 10% of the learners were isiXhosa speakers. Per class in the Foundation Phase there were 40 to 42 learners.

Resources

This ex-HOR (Coloured) school was well resourced, with a Khanya Computer laboratory, plus a second computer laboratory.

Management and staff

The principal and educators were very enthusiastic about the project. They expressed a need for new ideas and assistance. Good management and coordination was in place. A team approach was evident. The school was striving to improve its systemic test results.

Learners

The learners came from poor social backgrounds. Levels of unemployment were high among the parents. Language was an issue for the isiXhosa-speaking learners, as the language of instruction (English or Afrikaans) was not their mother tongue. Some of the Afrikaans-speaking learners preferred to be taught in the English medium. A number of learners were in the high-risk category. Literacy and numeracy levels were low. Absenteeism and discipline were not problematic at the school, as measures had been put in place to minimise the existence of either. Staff turnover figures were stable.

Community/Parental involvement

Single parenting and unemployment were factors of great significance in the community, which was generally poor. Despite the existence of such challenges, the parents still found energy and time enough to assist with sport coaching.

1.8.2.5 School E**Demographic data**

The dual-medium (English/isiXhosa) school had 14 educators and 754 learners in the Foundation Phase. Per class in the Foundation Phase there were 40 to 55 learners.

Resources

The school was a poorly resourced ex-DET (African) township school, with overcrowded classrooms, and inadequate up-to-date textbooks. Over a quarter of its classes were accommodated in prefabricated structures. The school was equipped with a computer laboratory, a library, and an empty science laboratory.

Management and staff

Both the principal and the educators expressed the need for the project. The staff showed a positive attitude towards the project, and they expressed the need for new ideas and assistance. The principal expressed the need for the school to improve its learner performance, which was, at that stage, very poor.

Learners

A number of the learners, who were taught in isiXhosa up to Grade 3, were in the high-risk category. The prevailing literacy and numeracy levels were low. The learners experienced problems in transitioning from Grade 3 to Grade 4. Absenteeism and discipline were not problematic at the school, as measures were in place to minimise such challenges. The staff turnover was stable.

Community/Parental involvement

The levels of parental involvement were very low. The school community was generally poor, with it being beset by unemployment.

1.8.3 The Institute

The Institute, which is part of the Faculty of Education, is found in the Department of Curriculum Studies. Its main focus is the improvement of the teaching and learning of Mathematics and Science in schools. This is achieved through the empowerment of the teachers concerned, and through the improved instruction of the learners. The Institute targets mostly the disadvantaged communities, both in the rural and in the urban areas, of the province in which it is located.

The Institute's initiatives focus on rural and urban areas in the province through the implementation and maintenance of systemic, long-term intervention programmes whose primary aims are to:

- equip and support teachers academically and didactically to effectively implement strategies for classroom and curriculum management; and
- widen the base of learners who take Mathematics and Science, preparing them with the necessary knowledge, life skills and motivation to enrol for tertiary studies in the fields of the Natural Sciences, Mathematics, and related applied fields

The Institute's intervention programmes (of which Project X was one), among other aspects, include the provision of:

- in-depth, interactive, hands-on workshops for teachers; and
- facilitators who visit the relevant schools to monitor the progress of teachers and learners, and to offer them support.

1.8.4 Overview of Project X

In May 2009, Project X, whose objective was to address literacy, numeracy and school management in a group of poor performing schools, was implemented. The focus of the intervention was Numeracy and Literacy in the Foundation Phase. The researcher was a mathematics facilitator in the numeracy intervention arm of the project, with the research being carried out in the 5 schools in which he was a mathematics facilitator.

- **The nature of the needs prompting the implementation of Project X**

At the time of the conceptualisation of Project X, the Institute was part of a consortium of nine service providers running a collaborative project. For the project, the Institute selected five secondary schools, and ten feeder primary schools. The project focused on the teaching and learning of Mathematics, Mathematics Literacy, Natural Science, the Life Sciences and the Physical Sciences in the Intermediate and Senior Phases, or from grades 4 to 12.

In the course of implementation of this project, the schools concerned indicated that the problems being experienced in the Intermediate Phase were being inherited from the Foundation Phase. Hence, the request was made that an intervention programme be conducted in the Foundation Phase, too. Project X was, thus, launched in response to the expressed demand, in acknowledgement of the need for intervention at the phase in question.

- **The objective of Project X**

The objective of Project X was to assist, motivate and equip the Foundation Phase teachers concerned with the necessary subject knowledge, didactical skills and sound classroom management skills to enable them to effectively improve the numeracy skills and knowledge of their learners. It was also aimed at creating a stable and organised environment in which learning could be facilitated.

1.8.5 The Foundation for Learning Campaign

Project X was implemented during the era of the Foundation for Learning Campaign. This was a four-year campaign that was run by the Department of Education to create a national focus on the improvement of numeracy abilities in South Africa. The initial focus of the campaign was on primary schools, starting with the Foundation and Intermediate Phases. The campaign, in its provision of guidelines for daily teacher activities during numeracy instruction, provided for 15 minutes for grades 1 and 2, and for 20 minutes for Grade 3, as the amount of time that was stipulated for problem-solving. The campaign was a national response to national, regional and international studies that had shown, over a number of years, that South African children were not able to count at the expected levels, and that they were unable to execute tasks that demonstrated key skills associated with numeracy. All primary schools were expected to increase their average learners' performance in numeracy to not less than 50%. Project X had, thus, come at the right time for the schools concerned, as it provided the help that was needed to achieve the identified target.

1.9 LIMITATIONS

Limitations are the potential research weaknesses that are related to the research design, or to the research methodology concerned (Pajares, 2007). If they are not adequately controlled, the limitations of a study can restrict the application and interpretation of the research findings involved. The notable limitation of the current research lay in its purposive sampling (Devers & Frankel, 2000, p. 265; Oliver, 2006, p. 245). The researcher's lack of capacity to take the optimum number of samples was the major limitation. This weighed

heavily against the generalisability of findings to situations outside the geographical area of research. The purposive sample of 48 teachers taken did not allow for generalisation about all teachers' PCK regarding problem-solving, and its use as a vehicle for the teaching and learning of mathematics. Any inferences made were, therefore, speculative.

As the research was carried out in government schools, there was the possibility of time delays in obtaining clearance from the province's Education Department. It was anticipated that there would be bureaucratic delays in granting permission and access to the schools, with the effects of such only being able to be minimised by allowing sufficient time for the planning stage of the research.

1.10 DELIMITATIONS

The delimitations of a research study are those characteristics of the research that limit its scope, and which are determined by the conscious decisions that are made in relation to exclusion or inclusion during the development of the research proposal concerned (Cline, 2011). This research could have focused on a number of learner-related research questions that were, however, not pursued. These questions were:

- What knowledge and skills do learners have to solve problems?
- How do learners use such knowledge and skills to solve problems?

The above-mentioned questions were not pursued in this research due to the focus of the research being on the development of an understanding of what knowledge teachers had about problem-solving, and of its use as a vehicle for teaching and learning mathematics. Therefore, the questions mentioned would not have been of direct relevance to the research. The inclusion of the above-mentioned questions, while interesting, would have been beyond the scope of this research in view of the limited time and budget available.

In spite of the limitations and delimitations pointed out above, the results and findings of this research can, nevertheless, be seen as valid and useful within the context of the project. Though the generalisations can be seen to be of limited import, useful inferences have still been made possible for those who are concerned in the area of improving the teaching and learning of mathematics in the Foundation Phase.

1.11 OVERVIEW OF THE RESEARCH

Chapter 1 has been devoted to discussing the motivation for this research. It has also discussed why problem-solving is increasingly becoming crucial in teaching mathematics. Figure 1.1 below summarises the research structure, and the aspects that are dealt with in each chapter.

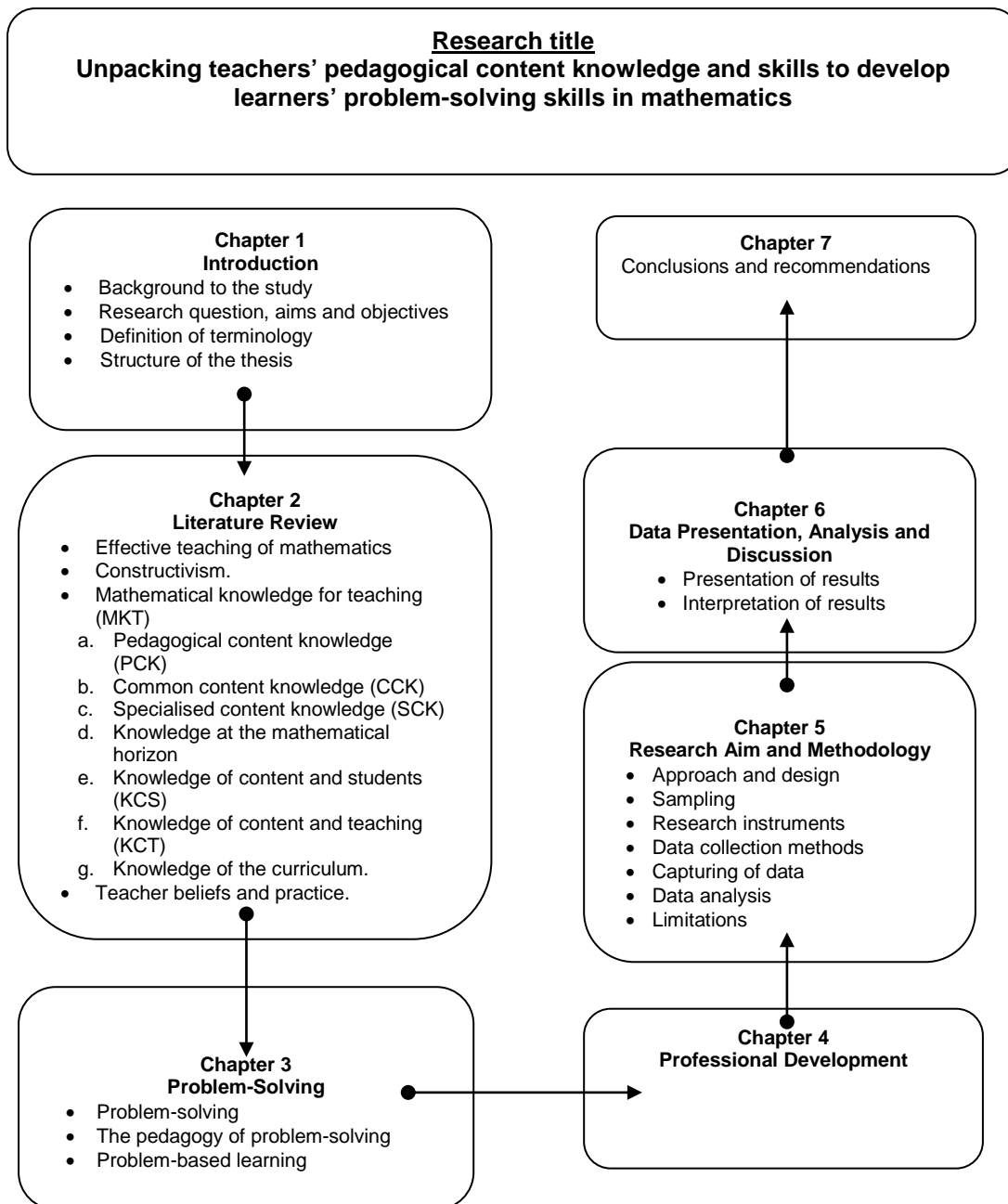


Figure 1.1: Summary of the research structure

Chapter 2 reviews the literature that forms the theoretical basis of the research. It focuses on the issues relating to effective teaching. The central theoretical framework employed for the discussion was constructivism. The theory of MKT and its components was a key issue of the discussion. The theories and approach were central to the formulating of the key issues covered in this research.

In **Chapter 3**, problem-solving is defined, and the pedagogy of problem-solving is discussed. Different approaches to problem-solving, such as teaching **about** problem-solving, teaching **for** problem-solving, and teaching **through** problem-solving are discussed (Schroeder &

Lester, 1989). The chapter discusses why problem-solving and teaching through problem-solving have proved to be so successful in countries that have scored high in terms of the TIMSS.

Professional development is the focus of **Chapter 4**, in which the various professional development models are discussed. The chapter also focuses on how professional development can change the teacher's classroom practices, beliefs and attitudes.

Chapter 5 focuses on the empirical study, describing and justifying the approach adopted, and the design used in the research. The sampling procedure, the research instruments, and the data collection methods are described, with the rationale for the selection of each being given. The chapter also describes how the data were captured and analysed, and how the results were presented. The chapter concludes by discussing the research limitations, and the sources of error.

In **Chapter 6**, the research results are presented through histograms, tables and ATLAS.ti analysis outputs. The chapter presents the information that was gathered by means of the administration of the questionnaire, teacher interviews, and lesson observations. The chapter summarises the research findings, and it discusses the interpretation of results, focusing on how they provide answers to the research questions asked.

Based on the findings presented in Chapter 6, in **Chapter 7**, conclusions are drawn to form an overall picture of the current situation regarding the teachers' PCK regarding problem-solving and its use as a vehicle for the teaching and learning of mathematics at Foundation Phase level. The chapter includes recommendations, and a discussion of the implications for the effective teaching and learning of mathematics, as well as for the ways in which teachers can successfully facilitate the use of problem-solving as a vehicle for learning.

CHAPTER 2: THE EFFECTIVE TEACHING AND LEARNING OF MATHEMATICS

“Effective mathematics teaching requires understanding what students know and need to learn and then challenging and supporting them to learn it well.”

(Wilson, Cooney & Stinson, 2005, p. 84)

2.1 INTRODUCTION

Given the background of poor learner performance in numeracy in South Africa (Atagana et al., 2010; Howie, 2001, 2003, 2006; Reddy, 2006), and in the project's schools in particular, the issue of effective teaching and learning is very pertinent. This chapter discusses a range of factors that are theoretically associated with the effective teaching and learning of mathematics. These factors include teacher knowledge, teacher understanding and practices, teacher beliefs, and the theories underlying effective teaching and learning.

2.2 EFFECTIVE TEACHING OF MATHEMATICS

Teaching of mathematics takes place so that the learners *can learn* the mathematics involved (Fosnot & Dolk, 2005, p. 175). Hence, the effective teaching of mathematics is likely to have a positive effect on a range of learner outcomes (Hopikins, Hollingsworth & Loudon, 2009; Ingaverson, Beavis, Bishop, Peck & Elsworth, 2004). Traditionally, “the teaching of mathematics has relied heavily on exposition by the teacher together with consolidation and practise of fundamental skills and routines by the learner” (Orton & Forbisher, 1996, p. 11). The fault of this approach to the teaching of mathematics is that, for some children, exposition never leads to mastery; many of the procedures, or routines, are not remembered correctly, or they are not remembered at all, and sometimes the procedures, or routines, are confused (Mji & Makgato, 2006; Muijs & Reynolds, 2000; Nkhoma, 2002; Orton & Forbisher, 1996).

There is, therefore, general agreement among educationists, learning theorists and psychologists that if we are to produce learners who develop into thinking and problem-solving adults, we need to use teaching methods that foster these competencies (Anthony & Walshaw, 2009; Orton & Forbisher, 1996; Schifter, 2005). It is, therefore, important that, for the purposes of the current research, that the nature of the effective teaching of mathematics be defined.

The effective teaching of mathematics is teaching that enables the learners to acquire specific skills and knowledge, and which promotes the obtaining of better learner outcomes than might otherwise have been possible (Dick & Reiser, 1989; Leinhardt, 1988; Russ, Sherin & Sherin, 2011). The evidence of effective teaching should be based on the observation of the quality of opportunities that the teacher provides for learner learning in the classroom, in relation to the teaching standard. Therefore, one could say that the effective teaching of mathematics revolves around the learner being involved in constructing mathematical understanding through exploration, problem-solving, discussion, and practical experience (Haylock, 2010; Hiebert, Morris & Glass, 2003; Ingavarsen & Rowe, 2008).

From the above, one has the impression that effective teaching should, thus, be conceptualised in terms of its impact on the learner. The true measure of effective teaching/learning is, therefore, what the learner knows, understands, and/or is able to demonstrate after completing a process of learning, measured against what the learner is expected to know, understand, and/or be able to demonstrate, after the completion of a process of learning.

The effective pedagogy of mathematics is “aligned with the shifting away from the traditional emphasis of learning rules for manipulating symbols to practises in which learners are actively engaged with the mathematics” (Anthony & Walshaw, 2009, p. 148).

The need for producing learners who are suitably equipped to develop into thinking and problem-solving individuals has resulted in a paradigm shift in the teaching of mathematics, from behaviourist knowledge transmission to the constructivist paradigm. This shift is “emphasizing cognitive processes whilst the socio-constructivist (social-cultural) paradigm emphasizes social factors in constructing shared knowledge” (Silfverberg & Haapasalo, 2010, p. 732), as is shown in Figure 2.1 below.

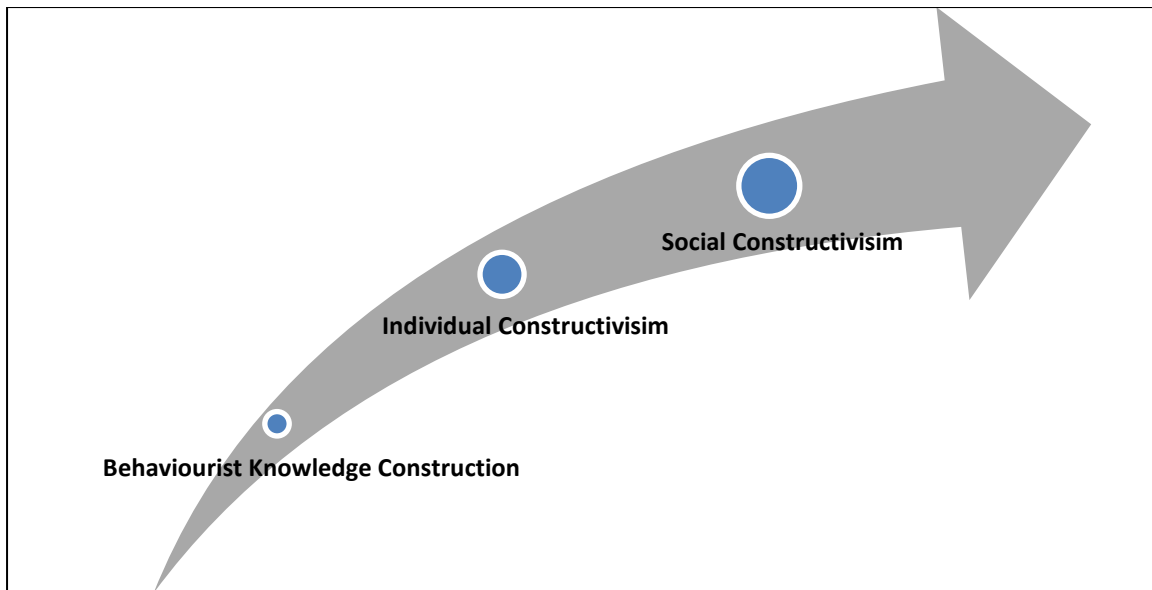


Figure 2.1: Paradigm shifts affecting the teaching of mathematics

The paradigm shifts in the teaching and learning of mathematics, as are illustrated in Figure 2.1 above, have been necessitated by the need for learning mathematics with **understanding**, rather than by the need for being able to repeat remembered routines, and to demonstrate particular basic skills. However, it is important to note that the “paradigm shifts concerning teaching and learning do not necessarily lead to changes in the practice of teaching mathematics” (Silfverberg & Haapasalo, 2010, p. 735). This is demonstrated in the Ravitz Report (Ravitz, Becker & Wong, 2000, p. 11), which shows that, although the mathematics community in the United States is “officially committed to constructivist ideas, the culture of traditional direct teaching is still strong, especially in the teaching of mathematics”.

In the case of South Africa, learners tend to perform poorly in mathematics in both local and international standardised tests, in spite of the reforms that have been put in place, and the efforts that have been made, to try and improve performance. This raises a question about whether the adoption of different teaching methods (for example, those used by countries leading in TIMSS) would be likely to lead to an improved level of success.

It might be the case in South Africa that, although the mathematics authority has introduced many reforms in the teaching of mathematics since 1994 that are aimed at improving learners’ performance, the culture of direct teaching (which has proved ineffective) is still very strong among those who are supposed to effect change, namely the teachers.

The need for learning mathematics with **understanding**, rather than for being able to repeat remembered routines, and to demonstrate the particular basic skills that are referred to above, has become the measure for the effective teaching of mathematics. In the next section, such understanding is discussed, as it is a critical component of the effective teaching of mathematics.

2.3 DEFINING UNDERSTANDING

Haylock (2010, p. 3) defines the understanding of mathematics as “learning in which the learner is involved in constructing understanding through exploration, problem-solving, discussion and practical experience”. Hiebert and Carpenter (1992, p. 67) define the degree of understanding as being:

...determined by the number and the strength of connections. A mathematical idea, procedure, or fact is understood thoroughly if it is linked to existing networks with stronger or more numerous connections.

Skemp (1976) identifies two types of understanding: **relational** and **instrumental**. He describes relational understanding as knowing both what to do and why and the process of learning relational mathematics as being one of building up a conceptual structure. He describes instrumental understanding, in contrast, as simply understanding rules without reasons. Both forms of understanding are discussed separately, and in detail, below.

$$\begin{array}{r}
 134 \\
 24 \overline{) 3216} \\
 \underline{-24} \\
 81 \\
 \underline{-72} \\
 96 \\
 \underline{-96} \\
 0
 \end{array}$$

Figure 2.2: Using the long-division algorithm

2.3.1 Instrumental understanding

Instrumental understanding is understanding in which all the learner knows is what to do. Being dependant on memory, the learner does not necessarily know why they are doing what they are doing, or why what they doing produces the correct answer. In Figure 2.2 below, the individual might know how to carry out long division using a particular algorithm, but they might not know why they bring down the digits during the procedure. Such understanding is, thus, rather a product of memory than knowing what to do (Skemp, 1976).

The teaching characteristics that produce instrumental understanding are demonstrating procedures, which emphasise memorisation and drill, pen and computations, and teaching by rules (Orton & Forbisher, 1996; Romberg & Kaput, 1999). The short-term effects of this understanding (in terms of which the learner knows what to do) are great, and such understanding is what is often rewarded (Lindquist, 1989, p. 10). The long-term effects of this form of understanding are negative, as is shown by the present state of mathematical learning, in terms of which the learners remember little of what they have been taught (Barmby, Harries, Higgins & Suggate, 2007; Lindquist, 1989).

2.3.2 Relational understanding

The second form of understanding, relational understanding, is less dependent on memory, as the learner has the ability to know what to do, and why (Skemp, 1976). The kind of teaching characteristics that produce relational understanding emphasises carrying out mathematical procedures, making connections, constructing one's own mathematical concepts, and problem-solving (Clarke, 2002; Haylock, 2010; Orton & Forbisher, 1996). A learner who, when given the fraction $\frac{6}{8}$, can illustrate the fraction through diagrams, give examples of it, calculate equivalencies, and approximate the size of the fraction, is an example of a learner with relational understanding. In contrast, a learner who only knows the name of, and the procedure for, simplifying the fraction to $\frac{3}{4}$, possesses instrumental understanding. The effective teaching of mathematics, therefore, entails the learner being provided with opportunities to construct understanding through exploration, problem-solving, discussion, and practical experience.

It is important to reiterate that the paradigm shift that was illustrated in Section 2.2 (see Figure 2.1) has, in many cases, not led to similar changes in the teaching of mathematics among the teachers (Senger, 1999; Silfverberg & Haapasalo, 2010). Kleve (2010, p. 158) also observes that “traditional beliefs and practice regarding school mathematics are challenged by reform-oriented curricula and the teachers’ deeply held beliefs can serve as an obstacle in implementing the new reforms”.

South African teachers have experienced, and continue to experience, several different curriculum reforms. Recent examples of such are outcomes-based education (OBE), the Revised National Curriculum Statement (Department of Education, 2005), the Foundations for Learning Campaign, and, most recently, CAPS. These curriculum changes are driven by the desire to “improve learner numeracy and literacy performance in view of very poor achievement by learners in the national systemic evaluations and in international assessment” (Republic of South Africa. Department of Education, 2008, p. 41). The lack of

significant change in learner performance might, in part, be due to the teachers' deeply held traditional beliefs about the teaching and learning of mathematics serving as an obstacle to the implementing of the reforms (Kleve, 2010, p. 159). Assessment in TIMSS, and other external assessments that are constructed around the TIMSS assessment framework, tend to emphasise high-order thinking and analytics skills (Education Alliance, 2006, p. 11). The challenge of teaching mathematics in the current decade is that it requires teachers to adopt teaching strategies that develop the relevant skills (Hiebert & Grouws, 2007, p. 390; Peterson, 1989, p. 6). If teachers continue to use the traditional approaches that yield only instrumental understanding, then the learners will continue to perform badly in tasks that focus on higher order thinking, and on the acquisition of critical analytic skills. The following section briefly discusses the changes that have taken place in the teaching and learning of mathematics during the last century.

2.4 EDUCATIONAL CHANGES OVER THE YEARS

This section discusses the five different phases (which are shown in Figure 2.4 below) that the teaching of mathematics has gone through internationally. Such an overview is critical, as it provides a foundation for understanding how learners learn. Understanding the different phases should serve to provide a valuable background to comprehending the current issues in mathematics education, and it should help to avoid tunnel vision regarding the present educational problems.

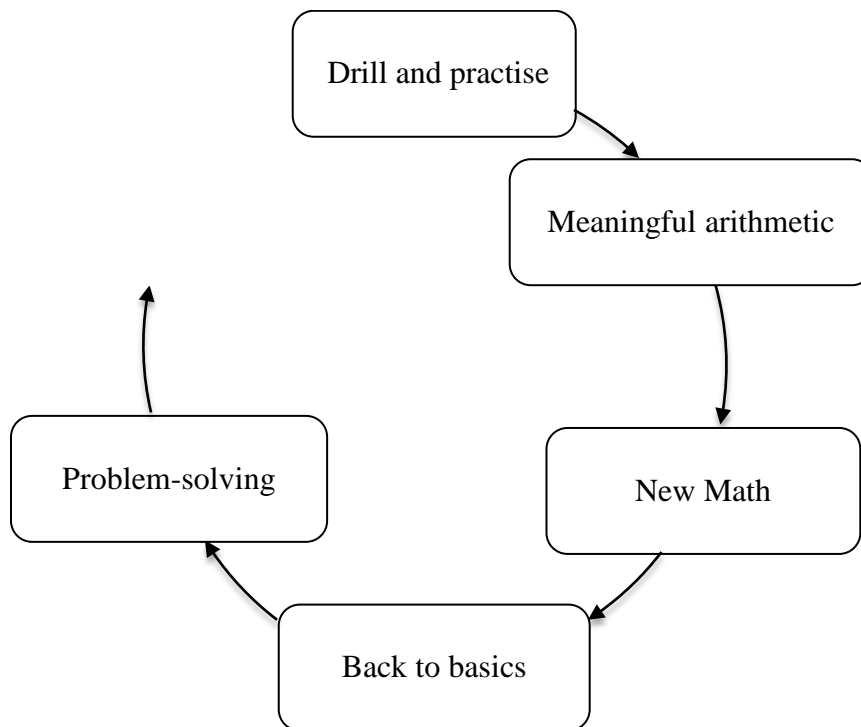


Figure 2.3: Different phases in mathematics education (adapted from Lambdin & Walcott, 2007, p. 5)

2.4.1 The drill and practice phase

The period of drill and practice stretched from around 1920 to 1930. The main theories were connectionism, or associationism, and the main focus was computation. This was to be achieved by means of the rote memorisation of facts and algorithms. The work required to be mastered was broken down into small learning steps (Lambdin & Walcott, 2007). However, people began to query whether the mathematics learnt in school was of any use in everyday life. This ushered in a new era, which focused on making sure that the skills learnt comprised meaningful arithmetic (Lambdin & Walcott, 2007).

2.4.2 The meaningful arithmetic phase

The meaningful arithmetic phase's focus was on understanding arithmetic ideas and skills, and on being able to apply these in everyday problems. This was to be achieved through an activity-based approach. The learning approach entails learners being active participants in the learning process, through doing arithmetic activities, and through critically reflecting on the activities involved, as opposed to the conventional, informative learning process, which pivots around the deskbound act of knowing (Akerson, Abd-El-Khalick & Lederman, 2000; Fullon, Walsh & Prendergast, 2013; Lakshmi & Cheng Hee, 2005). This phase stretched from the 1940s to the 1950s.

2.4.3 The new maths phase

The new maths phase, which stretched from the 1960s to the 1970s, was based on developmental psychology's sociocultural theory, as propounded by such theorists as Bruner, Piaget and Dienes. The focus was on understanding the structure of mathematics, and on introducing abstract mathematics early on in the school curriculum, on the assumption that both the learners' skills and their understanding would improve (Herrera & Owens, 2001). The new maths era fine-tuned the use of meaningful arithmetic.

The ideas pertaining to this phase were grounded on the theory that children move through three levels of development during learning. First comes the *enactive level*, at which level the child tends directly to manipulate objects, followed by the *iconic level*, during which the child manipulates mental images of the objects, rather than directly manipulating the objects themselves. Thirdly, and finally, comes the *symbolic level*, in which symbols, rather than objects or mental images of objects, are manipulated (Herrera & Owens, 2001; Lambdin & Walcott, 2007).

Many of the supporters of the new maths phase designed curricula based on the discovery of mathematics through the manipulation of such objects as blocks, sticks, chips, and other

materials. The concepts were then represented pictorially, and, finally, the appropriate mathematical symbols were introduced. Discovery learning was also promoted during this phase. While the authors of the new maths materials agreed that it was too much to expect learners to rediscover each element of the curriculum, “they wrote text books that adopted a guided discovery approach” (Lambdin & Walcott, 2007, p. 11). Schools were to produce graduates who were capable of understanding the mathematics and science that were essential to grasp, so as to be able to compete in the new technologically driven world.

2.4.4 The back to basics phase

Many parents, politicians and even teachers found it difficult to appreciate the new maths phase, because they found it was so different in its emphasis to what they had been taught. Misgivings were raised regarding the usefulness of the new maths, especially with regard to whether it was capable of sufficiently preparing the learners to comprehend the mathematics that was needed for life, including for the workplace. This apprehension resulted in a rapid return to basics in the 1970s, with the schools concerned returning to teaching facts and skills through demonstration, drill and practice (Cheek & Castle, 1981; Jardine, Clifford & Friesen, 2003; Lambdin & Walcott, 2007). It appears that the back-to-basics drive was not founded on any learning theories, although it was a return to connectionism (Cheek & Castle, 1981; Fey, 1979; Lambdin & Walcott, 2007).

2.4.5 The problem-based phase

Ten years after returning to basics, there was growing concern that focusing mathematics teaching on the acquisition of basic facts and skills did not sufficiently prepare learners for their future life and career (Lambdin & Walcott, 2007; Pressley, 1986; Schoenfeld, 2004). Accordingly, the 1980s marshalled in a new phase, during which the presiding politicians expressed their concerns about the levels of competitiveness present in commerce and finance (Lambdin & Walcott, 2007), and about the need for the nation’s overall economic and technological progress. Of key importance was that all the learners were able to use mathematics to solve problems (Lambdin & Walcott, 2007, p. 15).

Piaget’s developmental psychology theory and Vygotsky’s sociocultural theory dominated the learning theories that were prevalent during this phase. The developmental psychology and sociocultural theories developed into the modern-day constructivism learning theory (which is discussed at length in Subsections 2.6.3 to 2.6.5). The latter theory’s view of mathematics learning is that the learner must invent their own methods of counting, adding, and other mathematical procedures (Von Glasersfeld, 2005). Learning is an “active process where learners should learn to discover principles, concepts, and facts for themselves, hence the importance of encouraging intuitive thinking in learners” (Lutta, 2008, p. 4).

This is in contrast to the traditional view where the teacher teaches and the child supposedly understands and assimilates more or less than what has been taught. PBL was plausibly aligned to constructivism and is centred on the belief that learners best learn when they themselves are engaged in figuring out things and that working out their own approaches to problems is the best way learners can become mathematically literate and proficient (Lambdin & Walcott, 2007, p. 16).

During the problem-solving phase, considerable attention was paid to learners working in cooperative groups. During the later period of the problem-solving phase, problem-solving was further refined into teaching *about* problem-solving, teaching *for* problem-solving and teaching *via* problem-solving, all three of which are discussed in detail in Subsections 3.3.1 to 3.3.3.

2.5 TAKING COGNISANCE OF THE PAST, WHILE FOCUSING ON THE FUTURE FOR SOUTH AFRICA

The focus on the past empowers one to make informed decisions about contemporary educational practices. When one knows the origins of the numerous psychological theories, and how they influenced twentieth century mathematics teaching and learning, one can avoid the pitfalls of the past. When one analyses the phases through which mathematics teaching has gone, one observes that constructivism and problem-solving are at the root of recent curriculum reforms, and that they continue to thread their way through contemporary mathematics.

As South Africa grapples with solutions to improving the teaching of mathematics, the nation as a whole cannot be blind to the events of the past with regard to the teaching of mathematics. It would, therefore, be short-sighted for people in South Africa to call for a return to basics. Focusing mathematics teaching on the acquisition of basic facts and skills does not adequately prepare learners for their future life and career (as has been observed in countries that have previously followed this route).

2.6 OVERARCHING THEORY UNDERLYING EFFECTIVE TEACHING AND LEARNING

The job of teaching mathematics cannot be effectively done if it is not grounded in theory, however limited or small-scale such theory is (Fosnot, 2005; Orton, 2004). Accordingly, this section of the chapter considers the theoretical views that are related to the effective teaching and learning of mathematics, as they are needed as the basis of the day-to-day teaching and learning processes in the classroom. The three main theories of learning are: behaviourism; maturationism; and constructivism (Fosnot & Perry, 2005, p. 8). In this

chapter, the first two theories noted will be briefly discussed, whereas extensive discussion will be undertaken regarding constructivism, as it is the foundation of the current reform movement in the teaching and learning of mathematics (Boghossian, 2006, p. 713; Fosnot & Perry, 2005, p. 8). Also noted by Melton et al. (2003, p. 177) is the fact that in “recent years there have been a visible shift in educational practice toward social constructivism as the dominant learning theory”.

2.6.1 Behaviourism

Although behaviourism formed the traditional basis for schooling for most of the past century, in recent years there has been a noticeable shift towards constructivism as the dominant learning theory (Doolittle & Camp, 1999; Duit & Treagust, 1998). Behaviourism explains learning as a system of behavioural responses to physical stimuli. In the basic system of belief in behaviourism the following tenets hold sway:

- teacher-centredness;
- the teacher as expert;
- the teacher as transmitter of information;
- learning as a solitary activity;
- assessment primarily through testing;
- an emphasis on ‘covering’ the material;
- an emphasis on short-term memorisation; and
- strict adherence to a fixed curriculum (Forcier & Descy, 2007).

The behaviourism view holds that a learning result is indicated by a change in the behaviour of a learner.

2.6.2 Maturationism

The maturationism theory defines conceptual knowledge as being dependent on the developmental stage of the learner concerned (Fosnot & Perry, 2005, p. 9; Wyeth & Purchase, 2003, p. 94). The learner is viewed as an active ‘meaning maker’, who interprets experiences with cognitive structures that are a result of maturation. Hence, the model of teaching evolving from this theory is known as “developmental appropriate practice” (Fosnot & Perry, 2005, p. 10), which is an approach to teaching that is grounded in the research on maturationism. The teacher in a developmentally appropriate classroom often acts as a facilitator of learning, and makes teaching/learning decisions based on research into child development and learning (Maxwell, McWilliam, Hemmeter, Ault & Schuster, 2001; Stahl & Kuhn, 2003). Developmentally appropriate practice supports “child-initiated and hands-on activities, with teacher-directed instruction primarily occurring in response to individual children's needs; either in short interactions between teachers and individual children or in

very brief interactive whole group activities” (Buchanan, Burts, Bidner & White, 1998, p. 460).

2.6.3 Constructivism

Constructivism, which is the most current psychology of learning, is discussed in this section, because it is the “theory of learning and development that is the basis of the current reform movement in teaching and learning” (Fosnot & Perry, 2005, p. 8). It is a theory of knowledge that states that individuals generate knowledge and meaning from the interface between the individual’s experiences and his/her ideas (Von Glasersfeld, 2005). The theory of constructivist learning, which has had wide-ranging impact on teaching methods in education, underlies many education reform movements, such as South African outcome-based education (Du Plessis, Conley & Du Plessis, 2007). Constructivism is driven by the idea that learners must construct knowledge in their own minds. Rather than simply imparting knowledge to learners, teachers tend to facilitate the process of knowledge acquisition (Booker, Bond, Briggs & Davey, 1997; Du Plessis et al., 2007; Geelan, 1997; Matthews, 2000; Venter, 2001). The teacher “facilitates this process of knowledge construction” (Du Plessis et al., 2007, p. 4). It is important to note that constructivism, instead of being a particular pedagogy (Cobb, 1990; Fosnot, 2005; Noddings, 1990), is a theory defining how learning takes place, irrespective of whether the learners concerned utilise their experiences to understand a speech that they hear, or to follow the directions for constructing a model aircraft. In both cases, the concept of constructivism proposes that learners construct knowledge out of their own experiences. The basic tenets of belief in constructivism are:

- learner-centredness;
- the teacher as a member of the learning community;
- the teacher as coach, mentor, and facilitator;
- learning as a social, collaborative endeavour;
- assessment as being interwoven with teaching;
- an emphasis on discovering and constructing knowledge;
- an emphasis on application and understanding; and
- the pursuit of student questions, which are highly valued (Forcier & Descy, 2007).

Table 2.1 below shows comparisons of the various systems of belief in constructivism and behaviourism.

Constructivism	Behaviourism
<ul style="list-style-type: none"> • Problem-oriented 	<ul style="list-style-type: none"> • Replication-oriented
<ul style="list-style-type: none"> • Teacher as coach 	<ul style="list-style-type: none"> • Teacher as fountain of knowledge
<ul style="list-style-type: none"> • Multiple perspectives/truths 	<ul style="list-style-type: none"> • Single truth
<ul style="list-style-type: none"> • The necessity for instructional goals and objectives to be negotiated, and not imposed 	<ul style="list-style-type: none"> • Non-negotiable
<ul style="list-style-type: none"> • Learners' interpretations of multiple perspectives of the world 	<ul style="list-style-type: none"> • One perspective
<ul style="list-style-type: none"> • Reflective practice fostered 	<ul style="list-style-type: none"> • Reflection irrelevant and unnecessary
<ul style="list-style-type: none"> • Context- and content-dependent; dependent on knowledge construction 	<ul style="list-style-type: none"> • Context-independent
<ul style="list-style-type: none"> • Use of errors as a mechanism to provide feedback on learners' understanding 	<ul style="list-style-type: none"> • Use of errors to reinforce behaviour
<ul style="list-style-type: none"> • Sensitivity toward, and attentiveness to, the learner's previous constructions 	<ul style="list-style-type: none"> • Learners' previous 'constructions' true if they accord with the teacher's
<ul style="list-style-type: none"> • Encouragement of ownership and voice in the learning process 	<ul style="list-style-type: none"> • Learner participation regarded as unnecessary
<ul style="list-style-type: none"> • Knowledge construction emphasised 	<ul style="list-style-type: none"> • Knowledge reproduction emphasised
<ul style="list-style-type: none"> • Learner exploration encouraged in order to seek knowledge 	<ul style="list-style-type: none"> • Learner exploration neither encouraged, nor discouraged
<ul style="list-style-type: none"> • Collaboration and cooperative learning favoured 	<ul style="list-style-type: none"> • Collaboration and cooperative learning discouraged

Table 2.1: An overview of constructivism and behaviourism (adapted from Boghossian, 2006, p. 722)

From the preceding discussion, constructivism can be seen to replace the teacher (as the centre of knowledge) at the centre of the learning and teaching process with the learner (as the constructor of own knowledge). At this point, it is necessary to point out that “neither of these theories of learning can be regarded as exclusively right or wrong” (Brown, 2006, p. 109). It is, however, necessary to point out that constructivism is presently accepted as the most relevant of the three theories, and that most education policies, education models and education practices focus on constructivism (Boghossian, 2006; Brown, 2006; Fosnot & Perry, 2005). It is for this reason that constructivism is discussed at length in this chapter.

Most teachers' planning models are based on their verbal clarifications, or on their visual demonstrations, of a procedure or skill that are then followed up on by the learners practising the procedure or skill taught. The traditional lesson plan focuses on what the teacher intends to do in the lesson, and learning is teacher-focused; hence, the lesson plan states what the teacher will do during the lesson (Gagnon & Collay, 2010).

The constructivist theory implies that learners tend to play a more active role, instead of the teacher being the major role-player involved. Hence, in terms of constructivist approaches, the teacher plans for learning, rather than for teaching. In constructivism, the focus is on the learners' learning experience. Thus, when planning a learning experience for the learners, the teacher focuses on what experience the learners will have. The teacher's attention is on figuring out how to organise what the learners will do, instead of focusing on his/her own teaching actions (Gagnon & Collay, 2010; Schifter, 2005).

2.6.4 Constructivist learning

Gagnon and Collay (2010) point out that constructivist learning has emerged during the first decade of the twenty-first century as a prominent approach to teaching. Such learning represents a paradigm shift from a form of education that is based on behaviourism to a form of education that is based on cognitive theory. In this respect, the following applies:

- The behaviourist epistemology focuses on intelligence, domains of objectives, levels of knowledge, and reinforcement.
- The constructivist epistemology assumes that learners construct their own knowledge on the basis of their interaction with the environment.

The four epistemological assumptions that lie at the heart of constructivist learning (Gagnon & Collay, 2010) are as follow:

- Knowledge is *physically* constructed by learners who are involved in active learning.
- Knowledge is *symbolically* constructed by learners who are making their own representations of action.
- Knowledge is *socially* constructed by learners who convey their meaning making to others.
- Knowledge is *theoretically* constructed by learners who try to explain things that they do not completely understand. (As the learners grapple to understand the things that they do not understand, they actually build up their understanding of such things).

Therefore, learners are active participants in learning, rather than the inert receivers of information. This is the fundamental message of constructivism. Learners who are involved in lively learning construct their own understanding, and build their own knowledge in the process. Such an understanding ties in well with mathematics, which is a cumulative, vertically structured discipline. One learns mathematics by building on the mathematics that one has previously learned. The different forms of constructivism that are identified in the literature are summarised in Table 2.2 below.

Type of constructivism	Originator(s)	Description
1. Personal constructivism	Kelly and Piaget	<ul style="list-style-type: none"> Emphasises the idea that individuals construct knowledge for themselves through construing the repetition of events. Emphasises that knowledge is individual and adaptive, rather than objective.
2. Radical constructivism	Von Glasersfeld	<ul style="list-style-type: none"> Claims that knowledge is not "transferred directly from the environment or other persons into the learner, but has to be actively constructed within the individual mind for the purpose of survival.
3. Social constructivism	Solomon	<ul style="list-style-type: none"> Believes that ideas are held by individuals. Emphasises the social effects of the desire for consensus and peer approval. Describes two separate 'domains of knowledge' that are difficult to bring together, namely socially acquired life-world knowledge, and symbolic school knowledge.
4. Critical constructivism	Taylor	<ul style="list-style-type: none"> Suggests that the processes of teaching and learning are socially constructed, and that certain socially 'repressive myths', such as 'cold reason' and 'hard control', can lead to the failure of constructivist reforms in classrooms. Suggests that constructivism can most fully fulfil its potential through social reconstruction, and that emancipatory interests must overcome the existing technical/rational status quo
5. Contextual constructivism	Cobern	<ul style="list-style-type: none"> Agrees with Solomon on the importance of social interactions for human cognition, adding <i>culture</i> as a central force in the development of <i>ideas</i>

Table 2.2: Forms of constructivism (adapted from Venter, 2001, p. 88)

One observes a common thread running through the five above-mentioned forms of constructivism. The main point of departure is how the knowledge is constructed, and who is involved in the process of its construction. In spite of the many types of constructivism (see Table 2.2 above), the central principle remains *that knowledge is constructed*, and that it is not transferred directly from the teacher, the environment, or other persons to the learner.

Constructivist learning suggests that the starting point of concern is what knowledge is, and how it is constructed by the learner. The advocates of constructivism concur that learning is a dynamic process of constructing understanding, and that it is not a passive process of receiving information. The constructivist methodology can be adapted to any subject or educational programme by means of involving the learner, and by allowing them to construct their own knowledge, instead of them being inactive beneficiaries of the knowledge that is transmitted to them by the teacher. Use of the constructivist approach can be incorporated into 45- or 50-minute class periods, so as to teach a particular concept, skill, or attitude (Gagnon & Collay, 2010).

2.6.5 Criticism of educational constructivism

The TIMSS study has shown that the problem of relatively poor national performance in mathematics is not unique to South Africa. This concern has been also raised in the United States, Australia (Schollar, 2008), and Canada (Kershaw, 2010). Much of the recent research and policy literature flowing from these countries has one thing in common: an increasing focus on the nature of the curriculum, *the learning theory upon which it is based*, and the teaching practices that it encourages. In short, constructivism is under increasing pressure to offer observationally solid confirmation that it is a compelling hypothetical premise for a national curriculum, and particularly for the teaching and learning of the basics for mathematics to young learners in primary schools (Schollar, 2008).

Evaluations of many different relatively small-scale interventions by non-governmental organisations (NGOs) mainly funded by the private sector and independent development trusts have recently been rescrutinised in an effort to distil their findings about what is going on in mathematics education in South Africa (Schollar, 2008). In the majority of the studies concerned, the reported achievements in changing teaching practices that have been encouraged by the adoption of constructivist approaches have not resulted in corresponding improvements in learner performance levels. As was shown in the following section, research support for constructivist teaching techniques has been mixed, with some research supporting the techniques, and other research contradicting the aforementioned results.

2.6.6 The effectiveness of constructivism in relation to instructional design

Several teachers (Kirschner, Sweller & Clark, 2006; Mayer, 2004) have questioned the effectiveness of constructivism in relation to instructional design, as it applies to the development of instruction for beginners. While some constructivists maintain that 'learning by doing' improves learning, opponents of constructivists claim that only slight experimental proof exists to back this declaration in terms of the context in which young learners are meant to learn (Kirschner et al., 2006; Mayer, 2004). Sweller and colleagues maintain that young learners lack the fundamental mental simulations, or schemas, that are necessary for learning by doing.

In response to the above criticism, Clark, Nguyen and Sweller (2006) describe worked examples advocating constructivist theory as serving as an instructional design solution for procedural learning. They describe the solution as a very effective, empirically validated method of teaching learners procedural skill acquisition. The evidence for the effectiveness of learning by means of studying previously worked out examples, which is known as the

worked example effect, has been found to be useful in many domains, including music, chess, athletics, physics, mathematics, and programming (Hmelo-Silver & Duncan, 2007).

2.6.7 The effectiveness of using constructivist techniques for all learners

Mayer (2004) argues that not all the teaching techniques that are based on constructivism are efficient, or effective, for all learners. Abadzi (2006, p. 76) points out that “while there are many enthusiastic articles about the constructivist philosophy, there is little hard evidence regarding its benefits for poor students”. Mayer (2004, p. 15) also suggests that many teachers misapply constructivism as a means of employing teaching techniques that require learners to be behaviourally active. He describes this inappropriate use as the constructivist teaching fallacy, “because it equates active learning with active teaching”. Therefore, Mayer proposes that learners should be ‘cognitively active’ during learning, and that the teachers concerned should use ‘guided practice’.

Hmelo-Silver, Duncan and Chinn (2007) respond to the above criticism by arguing that such critics as Kirschner et al. (2006), Meyer (2008), and Abadzi (2006) have overlooked research favourable to PBL. The former researchers include in their response a 2003 meta-analysis showing that PBL has benefits for knowledge application over those of the traditional curriculum.

2.6.8 Unguided methods of instruction

Kirschner et al. (2006) describe constructivist teaching methods as being unguided methods of instruction. They suggest, in preference, more structured learning activities for learners with little or no prior knowledge.

In response to the above criticism, Hmelo-Silver and Duncan (2007) point out that many constructivist techniques incorporate large amounts of guidance in the form of scaffolding, which is a fact that Kirschner et al. (2006) overlooked. Hmelo-Silver and Duncan (2007) state that highly scaffold constructivist teaching methods, like PBL and inquiry learning, are effective, therefore the above-mentioned evidence does not support the conclusion of Kirschner et al. (2006).

Hmelo-Silver and Duncan (2007) and Downes (2007) argue that the critics of constructivism create a false dilemma between ‘guided’ and ‘unguided’ instruction, without recognising the continuum of guidance and structure that is possible with the use of constructivism, PBL, and other methods.

2.6.9 Constructivism as a thoroughly problematic doctrine

Meyer (2008, p. 339) states that constructivism "is an example of fashionable but thoroughly problematic doctrine that can have little benefit for practical pedagogy or teacher education".

2.6.10 Response to criticism

The following studies provide supporting evidence of the success of the constructivist PBL and inquiry learning methods:

- 1) **The GenScope Project:** Learners using the GenScope software, which is an inquiry-based science software application, have shown significant gains over the control groups, with the largest gains being shown in learners doing the basic courses (Hmelo-Silver & Duncan, 2007).
- 2) **Study by Geier:** Geier's study focused on the effectiveness of inquiry-based science instruction of middle school learners, as demonstrated by their performance on high-stakes standardised tests. The improvement involved was 14% for the first cohort of learners, and 13% for the second cohort. The study also found that inquiry-based teaching methods greatly reduced the achievement gap experienced by African-American learners (Hmelo-Silver & Duncan, 2007).
- 3) **Concept-Oriented Reading Instruction (CORI):** This comparative study compared three instructional methods for 3rd-grade reading: a traditional reading approach; a strategies instruction-only reading approach; and a reading approach with strategies instruction, and with constructivist motivation techniques including learner choices, collaboration, and hands-on activities. CORI resulted in the improvement of learners' reading comprehension, cognitive procedures, and motivation (Guthrie, 2004).
- 4) **Jong Suk Kim study:** This study found that the utilisation of constructivist teaching methods with 6th graders enhanced learners' achievement rates more than did the utilisation of traditional teaching methods. In addition, the study also found that learners preferred constructivist methods over traditional methods. However, Kim found no difference in learner self-concept, or in learning strategies, between those who were taught by constructivist methods, and those who were taught using traditional methods (Kim, 2005).
- 5) **Doğru and Kalender study:** This study compared science classrooms using traditional teacher-centred methods in contrast to those using constructivist learner-centred methods. In their initial test of learner performance, immediately following the lessons, they found no significant differences between classes that were taught using traditional methods, and those using constructivist methods. However, in the follow-up evaluation, which took place after 15 days, learners who had learnt through constructivist methods indicated better knowledge retention than did the learners who

had learnt the same subject matter through conventional methods (Doğru & Kalender, 2007).

The fundamental principle that emerges from constructivism is that learning is a constructive activity that the learners themselves have to carry out. The task of the teacher is not to dispense knowledge, but it is, rather, to provide the learners with opportunities and incentives to build up their own knowledge base (Von Glasersfeld, 2005, p. 7). From the above studies, one observes that constructivist approaches have the following edge over traditional approaches: better knowledge retention; preferred by the learners; improved reading comprehension; enhanced cognitive strategies; increased motivation; and significant gains in the self-confidence of the learners.

This section has discussed a paradigm that is intended to transform the culture of teaching in the long run. However, other issues, such as the renewal of the curriculum, textbooks taking into account the paradigm shift, and the raising of teachers' competence all also need to take place.

The following section focuses on the teacher's professional competencies, through examining the teacher's knowledge for teaching. What the teacher knows inevitably has great bearing on what happens in the classroom, and, ultimately, on how, and what, the learners learn (Van der Sandt & Nieuwoudt, 2003). Generally, an agreement exists that the possession of a body of knowledge is critical for a teacher to ensure that the learners whom they teach are able effectively to learn mathematics (Fennema & Franke, 1992; Silversman & Thompson, 2008; Walshaw, 2010). This body of knowledge is referred to as *mathematical knowledge for teaching (MKT)*.

2.7 MATHEMATICAL KNOWLEDGE FOR TEACHING (MKT)

Hill et al. (2008, p. 431) define MKT as:

....not only mathematical knowledge common to individuals working in diverse professions, but also the subject matter knowledge that supports that teaching, for example why and how specific mathematical procedures work, how best to define a mathematical term to a particular grade level, and the types of errors learners are likely to make with a particular content.

The above definition implies that, for one to be an effective teacher of mathematics, one needs to possess various aspects of knowledge. This body of knowledge, which is referred

to as MKT, is critical for a teacher to have to ensure that the learners involved effectively learn mathematics. The teachers' knowledge for teaching mathematics is "multifaceted and topic specific" (Pournara, 2013, p. 1). Hill and Ball (2009, p. 70) illustrate this body of knowledge (MKT) through the domain map in Figure 2.5 below.

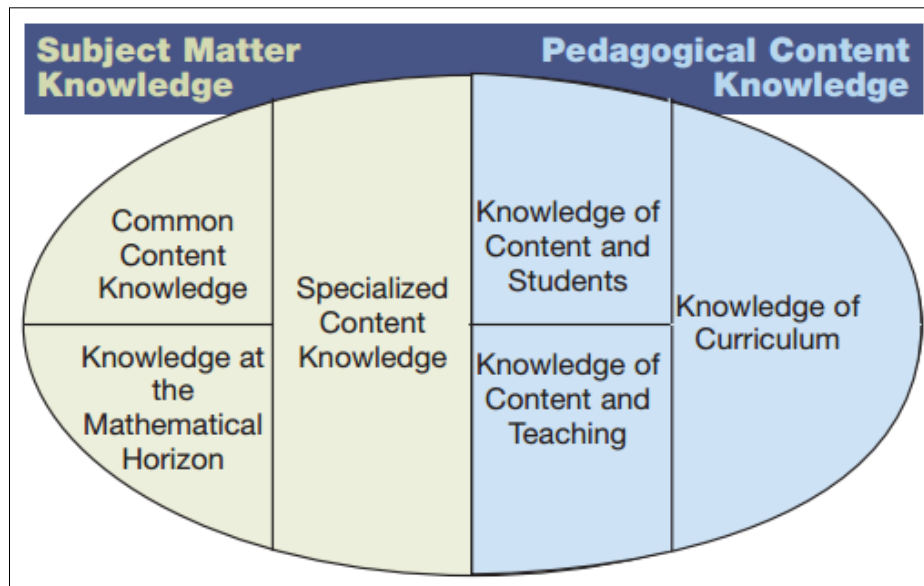


Figure 2.4: Domain map for the mathematical knowledge of teaching (Hill & Ball, 2009, p. 70).

Figure 2.5 above shows that a teacher of mathematics not only needs to be able to compute properly (i.e. to possess *subject matter knowledge*), but that they also need to know how to utilise pictures, or outlines, to speak about mathematical concepts and processes to the learners, to offer the learners clarification of common rules and mathematical procedures, and to examine learners' answers and descriptions (PCK) (Hill et al., 2005).

Hill and Ball (2009) similarly observed that some MKT is primarily a combination of mathematics knowledge with other types of knowledge, such as the knowledge of learners, or the knowledge of teaching, or of the curriculum. Such blended types of content knowledge are knowledge of the content and learners, knowledge of the content and teaching, and knowledge of the content and curriculum.

A teacher requires to have possession of all of these types of knowledge. The twenty-first century has moved beyond the previous focus on the teachers' professional characteristics. The focus has, instead of being on the number of mathematics professional development

courses that teachers have attended, and/or on the mathematical content that they know, come to lie increasingly on the mathematical knowledge that they must have and use when teaching mathematics for understanding (Cavey, Whitenack & Lovin, 2006; Sullivan, 2008).

Hill and Ball (2009) point out that teaching mathematics requires specialised knowledge about the subject. They argue that knowing the mathematical rules is not enough. Instead, mathematical understanding requires teaching. MKT consists of knowing how learners think, how to prepare instructional opportunities, and how to master various modes of delivering instruction. MKT, therefore, goes beyond the possession of mathematical content knowledge; it is the mathematical knowledge that is used in the classroom that exceeds the knowledge of formal mathematics (Shuhua, Kulm & Zhonghe, 2004, p. 147). It is the mathematical knowledge that one needs for carrying out one's work as a teacher of mathematics (Adler & Davis, 2006; Hill et al., 2008).

Secondly, though it would be unwise to say that mathematical knowledge is not important to the teaching of mathematics, conventional content knowledge seems to be insufficient for skilfully handling the mathematical tasks that are involved in teaching. The teachers' MKT is useful for, and usable in, the work that teachers do as they teach mathematics to their learners (Ball & Stylianides, 2008). In addition to mathematical knowledge, effective teaching demands that teachers know other aspects of teaching, as well as about their learners, and about the cultural, political, and social context within which they work (Isiksal & Cakiroglu, 2010).

Since the focus of this chapter is on the effective teaching of mathematics, the following aspects of the situation are discussed in detail in the remainder of the chapter: the different domains of MKT; the relationship between teachers' MKT and the mathematical quality of their teaching; how MKT affords for the effective teaching of mathematics; and how the lack of MKT constrains the teaching of mathematics.

2.8 SUBJECT MATTER KNOWLEDGE

As observed above, it would be unwise to say that mathematical *subject matter knowledge* is not important to the teaching of mathematics, although conventional content knowledge seems to be insufficient for the skilful handling of the mathematical tasks of teaching. In the current study, subject matter knowledge is not discussed in as much detail as is PCK, because the focus of the study is on Foundation Phase teachers, who, in this case, had a matric qualification, and who, in general, had mastery of adequate mathematical content to be able to teach in this phase. The teachers concerned had a greater need for PCK than they did for subject matter knowledge. The domain of subject matter knowledge has three components: common content knowledge; specialised content knowledge; and knowledge at the mathematical horizon (Hill & Ball, 2009).

Common content knowledge: This form of knowledge refers to the mathematical knowledge and skills that is used in settings other than teaching, such as the knowledge of the algorithm that enables the multiplying together of two numbers (Ball et al., 2008).

Specialised content knowledge: This form of knowledge refers to the mathematical knowledge and skills that are unique to teaching, such as knowing how the algorithm to multiply two numbers together relates to place value and the distributive property (Ball et al., 2008).

Knowledge at the mathematical horizon: This form of knowledge refers to an awareness of how mathematical topics are related over a span of mathematics included in the curriculum, for example knowing how the algorithm to multiply two numbers together is related to multiplying two polynomials together (Ball et al., 2008).

The implication of the above is that teachers need to know more than merely the common content of mathematics.

2.9 PEDAGOGICAL CONTENT KNOWLEDGE

The following section discusses the components of MKT that have been identified under PCK. Each component (see Figure 2.6 below) will now be discussed further, in terms of definition and linkages with the other components.

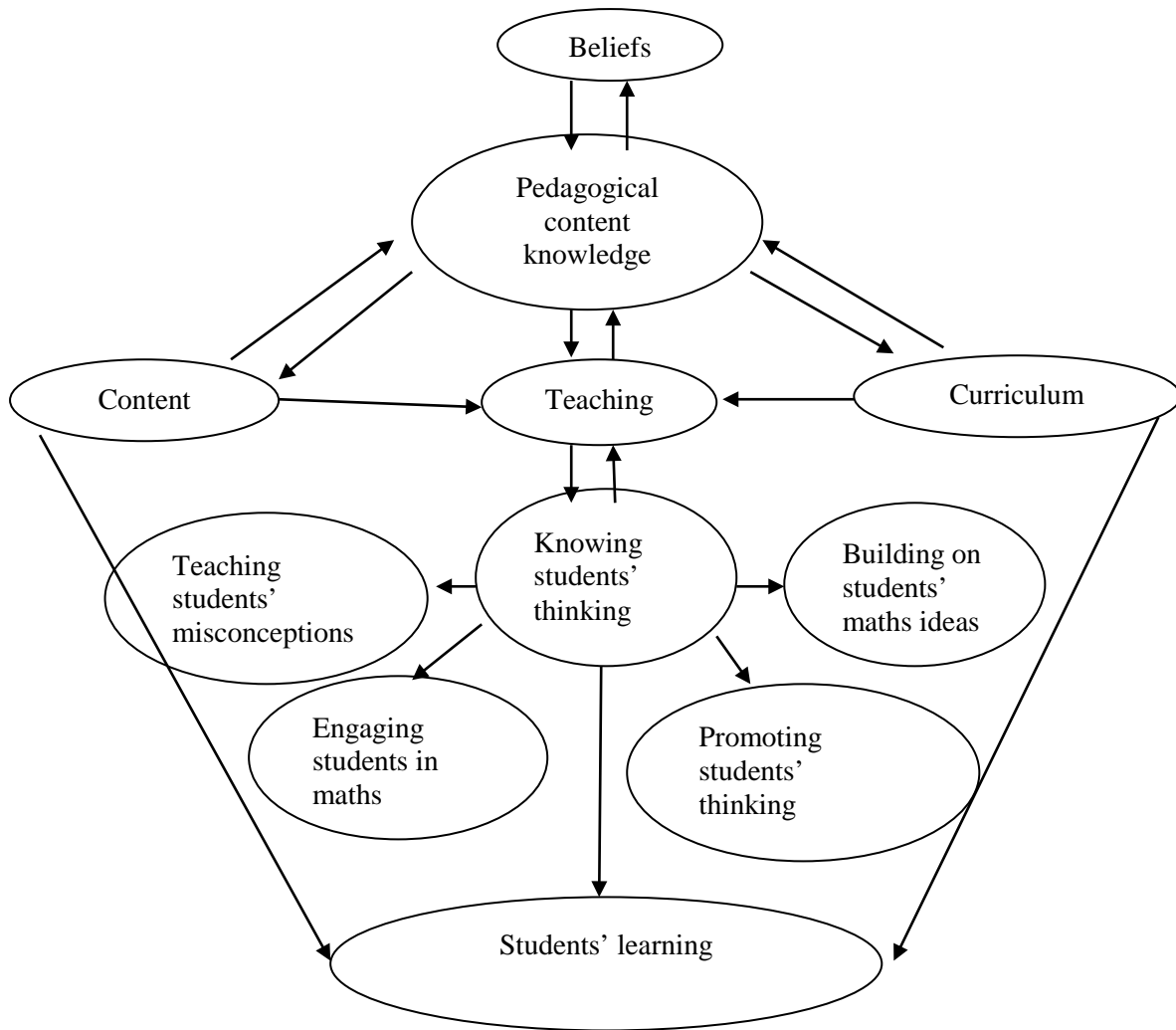


Figure 2.5: The network of pedagogical content knowledge (Shuhua, Kulm & Zhonghe, 2004)

While all three components of PCK are central to effective teaching, the main element of PCK is the knowledge of teaching. Figure 2.5 shows “the interactive relationship among the three components and shows that knowledge of teaching can be enhanced by content and curriculum knowledge” (Shuhua et al., 2004, p. 147). One notes, from the above discussion, that effective teaching is a product of a teaching process that is dependent on the teacher’s PCK, beliefs and knowledge of the learner. The rich interplay among the three components results in the effective teaching/learning of mathematics (Ball & Bass, 2000).

2.9.1 Knowledge of content and students (KCS)

2.9.1.1 *Knowledge of students as learners of mathematics*

Hill et al. (2008, p. 375) define KCS as “content knowledge intertwined with knowledge of how students think about, know, or learn this particular content”. They also point out that KCS is widely believed to be an important component of teacher knowledge. The knowledge of students as learners is recognised as being one of the domains that effective mathematics teachers draw upon to plan and implement instruction. Within that domain, knowledge of students’ thinking is credited with significantly influencing instructional practices, and with improving students’ learning (Jenkins, 2010; Truran, 2001). The literature on the assessment of what currently constitutes PCK points to an increasing role for teacher knowledge of students with particular attention being paid to “knowing students’ thinking” (Jenkins, 2010, p. 142).

The dynamics of the interface between the teacher’s knowledge of content and knowledge of the student is illustrated by means of the didactics triangle in Figure 2.7 below.

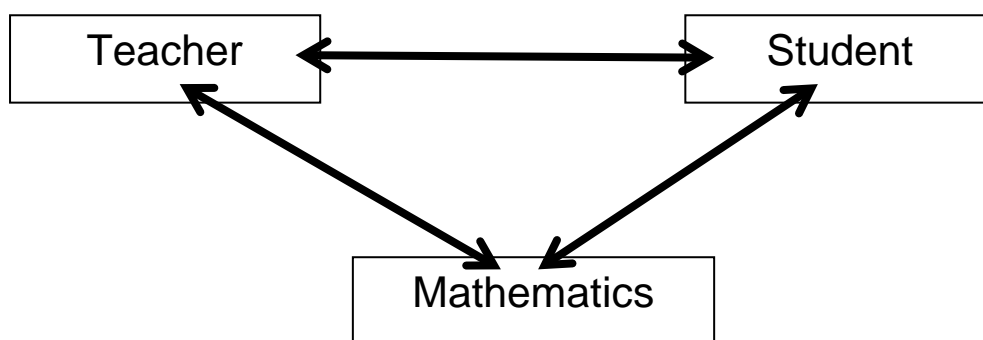


Figure 2.6: The didactic triangle (adapted from Steinbring, 2005, p. 314)

The teacher’s knowledge of content and teaching means that the currently desired mathematics teacher is one who is aware that, in the past, the functioning of the above didactic triangle was interpreted in a very schematic way. The teaching and learning of mathematics was described in mechanistic terms by means of, for example, the so-called ‘sender-receiver model’ (Steinbring, 2005, p. 314). As a result, the teacher was merely conveying mathematical knowledge to the learner. In contrast, the present-day teacher of mathematics, who is abreast with the dynamics of the didactic triangle, knows that this approach to teaching mathematics (which was based on behaviourist theories) has been strongly criticised in terms of mathematics education. Instead, it has been replaced by other ideas about the relationship of mathematics, learners and the teacher. The modern-day teacher knows that constructivism questions the assumption that mathematical knowledge can merely be ‘handed over’ from one person to another. This is the case because each

person must independently construct mathematical knowledge, and their own interpretations of this knowledge (Steinbring, 2005).

2.9.1.2 Learners' mathematical thinking

Learners' mathematical thinking involves how the learner makes sense of mathematics by means of the strategies that the learner applies in problem situations; the mathematical representations that the learner creates; the arguments that the learners makes; and the conceptual understandings that the learner demonstrates (Empson & Jacobs, 2008).

All the above is observable in terms of the learner's creations, actions and comments, by means of focusing on what the learner knows, and on what the learner can do, and by means of noticing what is useful and productive. This is also dependent on the teachers' ability to listen, to observe, to prompt, and to make sense of the learner's actions and comments. The teacher's knowledge of how the learner thinks provides the teacher with a framework that informs and guides the teacher's attempt to understand, and to explain, how the learner is making sense of mathematics (Jenkins, 2010; Southwell, 2002).

The mathematics teacher's knowledge of how a learner thinks and reasons about mathematics is a key component of PCK (Karp, 2011). The knowledge of content and learners

...informs instructional practices and guides instructional decision-making by providing an important lens through which to view and interpret how students respond to lesson activities generally and to assigned mathematics tasks in particular. Accordingly, it drives instructional modifications and interventions that are responsive to students' needs and result in improved student performance relative to quality mathematics standards (Jenkins, 2010, p. 149).

Adequate knowledge of the learners, coupled with the knowledge of the learners, enables the teacher to plan and modify their teaching to meet the learner according to their current levels of knowledge, in order to improve their performance, so as to meet the curriculum demands. The teacher is, then, able to decide at what level to pitch their instruction by using their knowledge of the content, and of the learners. The teacher also needs to know their content, and, of even more importance, is their need to know how particular mathematical content is best taught. Such knowledge of content and knowledge is discussed in the following section.

2.9.2 Knowledge of content and teaching (KCT)

The other component of PCK, which is called KCT, refers to the knowledge of mathematics, and to the knowledge of how to teach the mathematics. The knowledge of teaching consists of knowing how to prepare for instruction, and of mastering the different modes of delivering instruction. Such knowledge also includes the knowledge of how mathematical topics are connected (Brent & Simmt, 2006; Shuhua et al., 2004).

As has already been pointed out, there is, currently, an increased emphasis on developing teachers' capabilities to deliver high-quality learner results, and, thus, consideration is constantly given to the issue of what builds effective instruction in the classroom. Reynolds and Muijs (1999) provide valuable insights into the features of classrooms in which mathematics is taught successfully. Their study identified the following as being important attributes of effective teaching:

- a high level of opportunity to learn;
- an academic orientation emanating from the teacher;
- effective classroom management;
- a high proportion of whole-class teaching;
- heavily interactive teaching;
- the rehearsal of existing knowledge and skills; and
- the use of a variety of activities on a set topic (Reynolds & Muijs, 1999).

The classroom practices that were associated with low learner achievement included the placing of too much emphasis on repetitive work, too much individualisation, and too little fluency in calculations.

According to Reynold and Muijs' study, in addition to having the required content knowledge, the teacher needs to understand the process of teaching and learning. This is a lifelong process that involves the part played by the teacher's own school days, their relationship with significant other teachers, and their reflections on their own teaching experiences, and on staff development (Carroll, 2005).

Knowledge of content and teaching means that the current mathematics teacher is aware that, in the past, the functioning of the didactic triangle (see Figure 2.6) was interpreted in a very schematic way. The teaching and learning of mathematics was described in mechanistic terms by means of, for example, the so-called 'sender-receiver model' (Steinbring, 2005, p. 314). As a result, the teacher was merely handing over mathematical

knowledge to the learner. In contrast, the present-day teacher of mathematics, who is abreast with the dynamics of the didactic triangle, knows that taking such an approach to the teaching of mathematics, which was based on behaviourist theories, has been strongly criticised in mathematics education. Instead, it has been replaced by other ideas about the relationship of mathematics, learners and the teacher. The modern-day teacher knows that constructivism questions the assumption that mathematical knowledge can merely be handed over from one person to another. This is the case because each person must independently construct their mathematical knowledge, and their own interpretations of the knowledge (Steinbring, 2005).

2.9.3 Instruction delivery

The knowledge of content and teaching means that the teacher is in control of the effectiveness with which a lesson is delivered (Koning, Blomeke, Paine, Schmidt & Hsieh, 2011). They note four elements that add up to effective instruction delivery, namely:

- 1) **Quality of instruction:** This refers to the activities of teaching that make sense to the learners (such as presenting information in an organised way, noting transitions to new topics).
- 2) **Appropriateness of the level of instruction:** This refers to the teachers adapting their mode of instruction to the learners' diverse needs. Adaptability refers to the level of instruction (i.e. a lesson should neither be too difficult, nor too easy, for the learners), or to the different methods of within-class ability grouping.
- 3) **Incentives:** The learners should be sufficiently motivated to want to pay attention, to study, and to perform the tasks assigned. For a teacher, this means relating topics to the learners' experiences.
- 4) **Time:** This refers to the quantitative aspect of instruction and learning (e.g. strategies of classroom management enabling learners to spend a large amount of time on tasks).

The above-mentioned four elements are linked to one another, with teaching being effective only if all of them are applied. The development of a successful mathematics teacher is grounded in these principles that guide the current mathematics education reform (Koning et al., 2011; Prediger, 2010).

2.10 KNOWLEDGE OF THE CURRICULUM

Ball et al. (2008) define knowledge of the curriculum as being knowledge of the full range of programmes that have been designed for the teaching of particular subjects and topics at a given level, as well as of the various materials that are available in relation to these programmes. An instance of such knowledge would be knowing the teaching and learning materials that are available for teaching and learning the multiplication of two numbers. Shuhua et al. (2004), in their definition of knowledge of the curriculum, include selecting and using suitable curriculum materials, as well as fully understanding the goals and the key ideas of textbooks and the curricula. Textbooks “are an intricate part of what is involved in doing school mathematics: they provide frameworks for what is taught, how it might be taught, and the sequence for how it could be taught” (Crespo & Nicol, 2006, p. 331). The use of innovative mathematics curriculum materials has drawn considerable attention in recent years (Choppin, 2011), mainly with respect to the curriculum programmes that have been designed to support the instructional recommendations that have been made in terms of the educational reform programmes. Davis (2009) points out that the teachers’ successful use of curriculum materials largely depends on the aspects that are covered in the following subsections.

2.10.1 The teacher’s beliefs about the role of curriculum materials

A teacher who believes that learners learn from being told and shown how and what to do, and that it is the teacher’s responsibility to show and demonstrate what to do, is likely to find it difficult to use learner-centred materials, including textbooks. Such a teacher will most probably find such materials frustrating to use, as they are unlikely to provide the teacher with guidance on what to do with the learners, and on what steps to follow.

2.10.2 The teacher’s strategies and practices: the use of curriculum materials

When curricula reform takes place, a wide variety of materials and textbooks are usually on offer to meet the needs of a range of potential purchasers or users. Such resources are closely aligned with the goals of the curricula reform. Often, two practices pertain to the use of such materials and textbooks (Choppin, 2011). On the one hand, there are teachers who will take the time to go through the wide range available, and to select the materials that they find appealing and useful. Choppin (2011) notes that this kind of teacher is likely to adopt a more trusting and adherent view, using the tasks and recommendations in the curriculum materials to more comprehensively guide their instructional practices. On the other hand, some teachers will select materials containing elements with which they are familiar (Remillard, 1999). Such teachers will use the curriculum materials as a source of tasks,

without altering their teaching and learning practices, thus weakening the materials' ability to support the curriculum reform-related practices (Choppin, 2011; Remillard, 1999).

Professional development during curricula reform is a good strategy to employ towards prompting and encouraging the teacher to use the curriculum reform-based materials or textbooks. In South Africa, various mathematics textbooks and curricular materials for the Foundation Phase were developed during the introduction of OBE. The textbooks and curricular materials were intended to be aligned with the new curriculum. On the introduction of the New Curriculum Statement curricula, the textbooks differed from others that were available at the time, in that the former had different tables and graphs, new content, real-world contexts, and mathematical investigations, as well as encouraging increased learner involvement. The curricular materials and textbooks included activities appearing in the learner's textbook with suggestions about how to teach the activities, and about how to cope with the areas where learners might struggle to learn the content.

The teacher's interaction with the above-mentioned materials begins at the preparatory stage. How and which materials the teacher selects is influenced by the teacher's beliefs, content knowledge and knowledge of the curriculum (Davis, 2009; Nicol & Crespo, 2006). The rationale and meaningfulness for the teacher of a syllabus, or of a curriculum, the materials, media or textbooks (i.e. knowledge of the curriculum) also affects what is learned and taught (Hameyer, 2007).

2.10.3 The reason for the concern about knowledge of the curriculum

In the current study, the researcher concerned himself with the teacher's knowledge of the curriculum for the following reasons:

- **Contribution towards learners' achievement**

Curriculum materials have been known to contribute towards learners' achievement. Findings from studies by Yeping and Fuson (2011) suggest that curriculum materials are a key contributing factor to learners' achievement. However, learners' achievement cannot be explained solely by the differences in curriculum materials.

- **Instructional functions of curriculum materials**

The instructional functions of curriculum materials are one aspect that has received research attention recently. Yeping and Fuson (2011) have shown the importance of instructional features that are embedded in curriculum materials. Choppin (2011) also points out that, in studies of teachers' uses of an elementary standards-based curriculum programme, those

teachers who extensively adopted tasks were more likely to learn from the use of the programme....,

....with respect to expanding their instructional repertoires, generating insights into student thinking, and constructing the teacher's role in orchestrating student learning, results that indicate that teacher learning is associated with the extent to which teachers draw from the curriculum materials (Yelping & Fuson, 2011, p.175).

In a study examining how a reform-oriented textbook fosters the changes required by these reforms, and how it could contribute to such learning, Remillard (1999) found that teachers engaged with curriculum materials and textbooks in three different ways: exploring the content in preparation for teaching; examining ideas underlying student confusion; and engaging in mathematical thought interchange with students. These ways of engaging were found to lead to teachers learning about mathematics.

Grossman and Thompson (2004) expressed an interest in knowing how the materials helped the new teachers learn about teaching. Gamoran and Corey (2009) investigated patterns in teachers' use of reform-based elementary mathematics curriculum materials. Both studies found out that the curriculum materials that teachers encountered did, indeed, powerfully shape their ideas about teaching, as well as their ideas about classroom practice.

2.11 THE RELATIONSHIP BETWEEN MKT AND EFFECTIVE TEACHING

A study by Hill et al. (2005), on the effects of teachers' MKT on learner achievement, showed that it positively predicted learner gains in mathematics achievement during the first and third grades. Their surprise on observing this effect with first graders came from their conceptualisation of MKT as only showing its effects in grades involving more complex content. However, their study showed that MKT plays a role even in the teaching of very elementary mathematics content.

2.11.1 The role that teachers' MKT plays in their teaching

Studies by Hill et al. (2008) on the role that teachers' mathematical knowledge plays in their teaching of subject matter show the substantial link between MKT and 'high mathematical quality instruction'. Teachers with high MKT were noted as providing better instruction to their learners than did teachers with lower MKT, as the former were able to:

- avoid mathematical errors and missteps;
- deploy their mathematical knowledge to support more rigorous explanations and reasoning, and to better analyse and make use of learner mathematical ideas than would otherwise have been possible;
- create rich mathematical environments for their learners;

- be critical of their mandated curriculum, and to like to invest considerable time in identifying and synthesising activities from supplemental resource material;
- provide high-quality mathematical lessons;
- provide high-skill responses to learners; and
- choose examples wisely to ensure equitable opportunities for learning.

The above-mentioned elements were much more variable among lower knowledge teachers. Occasionally, a teacher with low-level MKT would exhibit some of the characteristics, but they were not constantly displayed across lessons observed.

There is growing recognition of the fact that mathematical knowledge alone does not guarantee improved teaching, and attempts are being made to define the various forms of knowledge that are required for teaching (Tirosh, 1999). As Ball, Thames and Phelps (2008) investigated MKT, they also began to notice its different domains (see Figure 2.5 above).

According to Ball, Lubienski and Mewborn (2001), the argument for the shift in focus from only mathematical knowledge to other types of knowledge is that, although these important lines of work describe what the teacher knows, they do not address the mathematical knowledge that teachers 'call up' as they teach mathematics. Hence, researchers have begun to undertake the difficult challenge of articulating the mathematical knowledge that a teacher must draw upon when teaching for understanding (Cavey et al., 2006). This is the knowledge that is related to mathematics that teachers call upon as they make decisions about the mathematical learning of the learners.

The research focus in the twentieth century has moved to examining the mathematical knowledge upon which the teachers draw to facilitate the learners' development of mathematical understanding. This research, in particular, considered the practice concerned, and explored the possibility of identifying those mathematical understandings that a teacher might call upon as they use different teaching strategies, and as they facilitate discussions about problem-solving, as well as when they highlight different mathematical ideas to support their learners' development of problem-solving skills.

The discussion, thus far, has acknowledged that teachers need to have a profound understanding of fundamental mathematics (as summed up in Figure 2.5). However, the literature has also shown that profound content knowledge alone is not sufficient for the effective teaching of mathematics. An effective teacher must also possess a deep and broad knowledge of the teaching process, the learners and the curriculum. Armed with this

knowledge, teachers are able to connect their knowledge of content to their knowledge of the learners, the curriculum, and the type of teaching that is required in a supportive network, so as ultimately to address the goal of enhancing learners' learning (Shuhua et al., 2004). As shown in Figure 2.6, this network of knowledge is influenced by the teachers' beliefs. Shuhua et al. (2004), and Fennema and Franke (1992), also point out the importance and the impact of teachers' beliefs on their knowledge. Different educational belief systems produce different attributes of PCK. The next part of the discussion focuses on this important aspect, namely teachers' beliefs

2.12 THE IMPACT OF TEACHERS' BELIEFS ON THEIR PRACTICE

The factors that influence teachers' practice are complex and numerous. In working towards the improvement of the teaching of mathematics, one has to consider teaching from a number of interconnected angles. One factor that has recently emerged as a central feature in understanding teacher practice is that of teacher beliefs. Beliefs are "mental constructions based on evaluation and judgment that are used to interpret experiences and guide behaviour" (Pedersen & Liu, 2003, p. 74). Cross (2009, p. 325) defines beliefs as "embodied conscious and unconscious ideas and views about oneself, the world, and one's position in it, developed through membership in various social groups: these ideas are considered by the individual to be true". Mathematics beliefs are personal judgements that are made about mathematics, formulated from experiences in mathematics, including beliefs about the nature of mathematics, learning mathematics, and teaching mathematics (Phillip, 2007; Raymond, 1997). The implication of this is that each individual teacher holds a range of beliefs that influences their perceptions of the teaching and learning of mathematics.

A careful consideration of the above definitions shows that beliefs are considered to be very influential in determining how individuals frame problems and structure tasks. In this regard, it is thought that how a teacher conceptualises mathematics has a direct impact on their teaching, and so, if there is to be any change in their instructional practices, their beliefs must first be addressed (Cross, 2009; Da Ponte & Chapman, 2006). This is no easy task, as beliefs develop over years of schooling and experiences, and as these beliefs seldom change dramatically without significant intervention (Liljedahl, Rolka & Rosken, 2007). The extent that new ideas are integrated into a teacher's knowledge and pedagogy largely depends on their prior beliefs (Bowyer & Meaney, 2007). Thus, because the opportunity for changing their beliefs is essential for teachers' development, it is important to understand not only what teachers believe, but also how their beliefs are structured and held.

Teachers' mathematics beliefs are classified into three groups: beliefs about the nature of mathematics; beliefs about mathematics teaching; and beliefs about student learning (Cross,

2009; Grootenboer, 2008; Harwood Wi, Hansen & Lotter, 2006) The beliefs reflect how teachers conceptualise their roles in the classroom, their choice of classroom activities, and the instructional strategies that they use. Beliefs are considered central to the way in which teachers conceptualise and actualise their role in the mathematics classroom, and, therefore, they are central to any efforts that are made towards improving their students' learning (Cross, 2009; Fosnot & Dolk, 2005).

The research evidence indicates that “teachers’ beliefs colour and influence their teaching practices, how they believe content should be taught, and how they think students learn” (Harwood, Hansen & Lotter, 2006, p. 69; Philipp, 2007, p. 261). The trainees in teacher education programmes learn how to develop both their subject matter knowledge, and their PCK. It has been observed, however, that, on completion of the programmes, many of the new teachers “revert to teaching methods that are more reflective of their own experiences as students than their experiences as prospective teachers” (Liljedahl et al., 2007, p. 319). The reason for this, according to Liljedahl et al. (2007), is that beliefs about mathematics, and its teaching and learning, influence the formation of attitudes, which, in turn, influence the teacher’s classroom practices. Cooney, Shealy and Arnold (1998, p. 306) suggest that these “teachers’ beliefs about mathematics and how to teach mathematics are influenced in significant ways by their experiences with mathematics and schooling long before they enter the formal world of mathematics education”.

Classroom practices must reflect reform recommendations for there to be improvement in mathematics achievement in South Africa. Such improvement requires a change in the instructional practices of many mathematics teachers: a change that can only be actualised if a better understanding is reached of not only the types of beliefs that the teachers have, but also how the beliefs are related to one another in practice.

Raymond’s (1997, p. 551) model of the relationships between mathematics beliefs and teaching practices suggests that mathematical beliefs stem from prior school experience, including experience as a mathematics learner, as well as the influence of prior teachers, and of teacher preparation programmes, in addition to prior teaching episodes (see Figure 2.7 below).

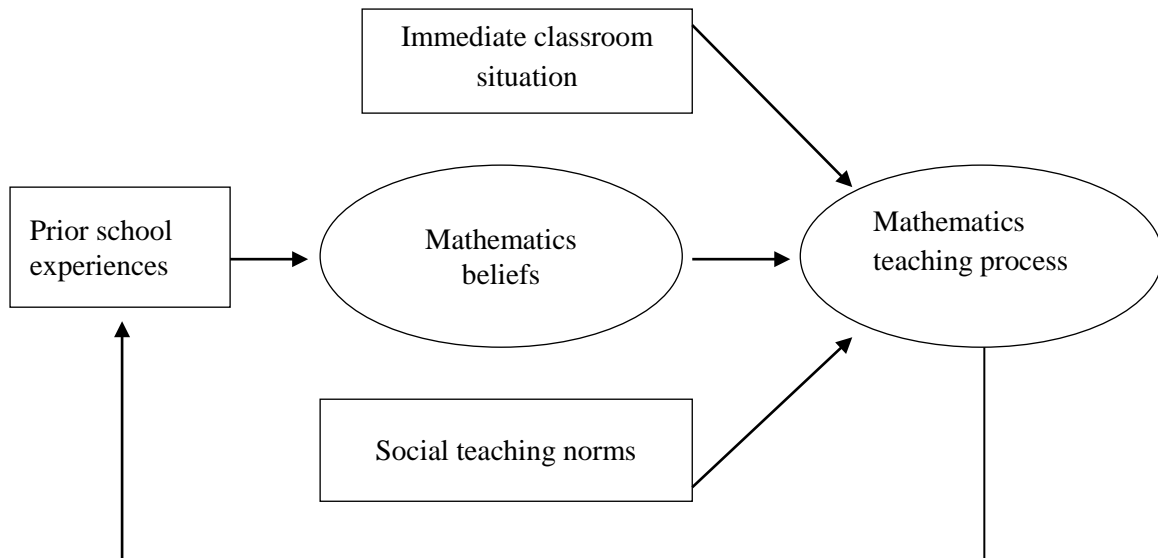


Figure 2.7: A model of the relationship between mathematics beliefs and teaching practices (adopted from Raymond, 1997, p. 551)

The model in Figure 2.7 above shows a direct relationship between mathematics beliefs and mathematics teaching practice. However, such beliefs and practices are not wholly consistent. The model suggests that other mediating factors are involved. Social teaching norms and the immediate classroom situation can affect the relationship between the beliefs and the practice of the novice school teacher, who is particularly vulnerable to outside influence (Raymond, 1997, p. 574).

2.12.1 Teachers' beliefs about the learning of mathematics

There are basically two opposing beliefs about mathematics learning. On the one hand is the belief that mathematics is learned by transmitting knowledge to learners. Teachers with this traditional view believe, among other things, that learners passively receive knowledge from the teacher. On the other hand, opposing this view is the belief that learners are active participants who construct their own knowledge.

Traditional teacher's beliefs about learning mathematics	Non-traditional teacher's beliefs about learning mathematics
The learner... <ul style="list-style-type: none"> passively receives knowledge from the teacher; learns mathematics by working individually; engages in repeated practice in order to master skills; primarily engages in practice for the mastery of skills; is primarily passive, raising questions on occasion; and works individually, except for homework. 	The learner... <ul style="list-style-type: none"> learns mathematics through problem-solving activities; learns mathematics without textbook or paper-and-pencil activities; learns mathematics through cooperative group interactions; is an active mathematics learner; and can learn mathematics in their own way.
The learning process... <ul style="list-style-type: none"> depends solely on the teacher; is based on the memorisation and mastery of algorithms providing primary evidence of learning; occurs primarily through textbooks and worksheets; consists of only one way in which to learn mathematics; and is more the responsibility of the teacher than of the learners. 	The learning process... <ul style="list-style-type: none"> occurs primarily through problem-solving tasks; involves learning mathematics from working with one another; is one in which the learners are, by and large, responsible for their own learning; Is one in which the learners are active, rather than passive, receipts; and Is one in which learning is evidenced more through the ability to explain one's understanding than it is through the expert memorisation and performance of algorithms.

Table 2.3: Teachers' beliefs about the learning of mathematics (adapted from Raymond, 1997, p. 559)

2.12.2 Teachers' beliefs about the teaching of mathematics

Beliefs about mathematics teaching can be generally characterised in terms of the teacher's view of their role in the teaching of the subject, and of the learner's role in learning it (Roesken, Pepin & Toerner, 2011, p. 453). On the one hand, there are teachers who hold the belief that the teacher is the provider of knowledge. On the other hand, there are teachers who hold the belief that the teacher is the facilitator of learning. Table 2.4 provides details of each view.

Traditional teacher's belief about the teaching of mathematics	Non-traditional teacher's beliefs about the teaching of mathematics
The teacher's role is to ... <ul style="list-style-type: none"> lecture and dispense mathematical knowledge; and assign individual set work. The teacher...	The role of the teacher is to ... <ul style="list-style-type: none"> primarily engage learners in the undertaking of problem-solving tasks; primarily present an environment in which learners are active, although they may occasionally play a more passive role; primarily evaluate learners using means

<ul style="list-style-type: none"> ▪ seeks the 'right answers', and is not concerned with explanations; ▪ approaches mathematical topics individually, one day at a time; ▪ emphasises the mastery and memorisation of skills and facts; ▪ instructs solely from the textbook; ▪ assesses learners solely through standard quizzes and examinations; ▪ primarily values right answers over process; ▪ emphasises memorisation over understanding; ▪ primarily (but not exclusively) teaches from the textbook; ▪ includes a limited number of opportunities for problem-solving; and ▪ plans and implements lessons explicitly, without deviation. Lesson activities follow the same pattern daily. 	<p>beyond the standard examinations;</p> <ul style="list-style-type: none"> ▪ encourage mostly learner-directed discourse; ▪ select tasks based on learners' interests and experiences; ▪ select tasks that stimulate learners to make connections; ▪ select tasks that promote communication about mathematics; ▪ create an environment that reflects respect for the learner's ideas, and structures the amount of time that is necessary to grapple with ideas and problems; ▪ pose questions that engage and challenge the learners' thinking; ▪ have the learners clarify and justify their ideas both orally and in writing; ▪ have the learners work cooperatively, encouraging communication among them; and ▪ observe and listen to learners when assessing learning.
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Table 2.4: Teachers' beliefs about the teaching of mathematics (adapted from Raymond, 1997, p. 560)

Confrey (1990, p. 107), who refers to the traditional approach to teaching mathematics as “the direct transmission approach”, suggests that three key assumptions about mathematics instruction underlie direct instruction, namely:

- Relatively short products are expected from the learners, rather than process-oriented answers to questions. Homework and tests are accepted as providing assessment of the success of instruction.
- Teachers, for the most part, simply execute their plans and routines, checking frequently to see whether their learners' responses are within desirable bounds.
- The responsibility for determining whether adequate levels of understanding have been reached lies primarily with the teacher.

Traditional mathematics instruction, focusing almost exclusively on the completion of instructional tasks aimed at achieving correct answers through a reliance on the memorisation of facts, rules, formulas, definitions, the use of algorithms, and the

reproduction of familiar material is an approach that has been repeatedly criticised in the United States (Hennings, McKeny, Foley & Balong, 2012). Beswick (2005, p. 40) categorises teachers' beliefs into the categories that are shown in Table 2.5 below.

Beliefs about the nature of mathematics (Ernest, 1989)	Beliefs about mathematics teaching (Van Zoest et al., 1994)	Beliefs about mathematics learning (Ernest, 1989)
Instrumentalist	Content focused, with an emphasis on performance	Skill mastery and the passive reception of knowledge
Platonist	Content focused, with an emphasis on understanding	Active construction of understanding
Problem-solving	Learner-focused	Autonomous exploration of own interests

Table 2.5: Categories of teacher beliefs (Beswick, 2005, p. 40)

The constructivist perspective, especially in mathematics education, fundamentally questions the direct transmission approach's assumption that "mathematical knowledge could be handed over from one person to another, because each person must construct mathematical knowledge and his or her own interpretations of this knowledge independently" (Steinbring & Scherer, 2006, p. 159). When Grootenboer (2008) attempted to facilitate belief change among prospective teachers, the teachers' responses seemed to fall into three categories: non-engagement; the building up of a new set of beliefs; and the reform of existing beliefs.

A substantial amount of evidence reveals "that teachers hold beliefs about learners that lead to differential expectations and treatment based on race/ethnicity social class, and gender differences" (Pohan & Aguilar, 2001, p. 160). It follows, therefore, that, if teachers are to better serve the needs and interests of all learners, but particularly those learners who are not performing well, then the low expectations, the negative stereotypes, the biases/prejudices, and the cultural misconceptions that are held by teachers must be identified, challenged, and reconstructed.

Li (2005) and Wilkins and Ma (2003) point out that the teacher's understanding of the teaching and learning process includes the teacher's beliefs regarding cognition, affect, and behavioural process. Such beliefs concern the following:

- purpose: what the teacher thinks learners gain from learning;
- process: what the teacher thinks it means to learn mathematics;
- personal regard: whether or not, and why, the teacher regards learning of mathematics as important; and
- social perception: what the teacher thinks of those who learn well, versus those who do not.

These beliefs underlie the teacher's approach to the teaching and learning of mathematics. Hence, teachers who believe that knowledge is reciprocal and shared by learners are likely to engage learners in cooperative group interaction activities (Love & Kruger, 2005).

2.12.3 Key issues about teachers' beliefs, and about the teaching and learning of mathematics

From the literature reviewed on teachers' beliefs, the following are considered to be key issues regarding teachers' beliefs, and their teaching and learning, both in general, and in relation to mathematics in particular:

- Teachers' beliefs, attitudes and priorities are linked closely to their classroom behaviour and practices (Rimm-Kaufman & Sawyer, 2004).
- Teachers' beliefs guide their thinking, meaning-making, decision-making, and behaviour in the classroom (Isikoglu, 2008).
- Teachers' beliefs play a filtering role in relation to new information. They play a moderating role regarding knowledge about the teaching and learning of mathematics (Grootenboer, 2008).
- Teachers' deeply held, traditional beliefs about the nature of mathematics have the potential to perpetuate mathematics teaching that is more traditional, even when teachers hold non-traditional beliefs about mathematics pedagogy (Raymond, 1997).
- Elementary school teachers often enter the teaching profession with non-traditional beliefs about how they should teach, but, when they are faced with the constraints of actual classroom teaching, they tend to implement more traditional classroom practice (Raymond, 1997).
- Teachers' beliefs tend to be well-established and resistant to change, with them having been found to influence their mathematical pedagogy (Grootenboer, 2008).

Cummings (2008) calls for beliefs to become an important focus of educational investigation, as they hold the key to understanding daily teacher decision-making. Hence, this study sought to identify centrally held beliefs of Foundation Phase teachers, whether or not the teachers' beliefs supported the establishment of classroom environments that were consistent with the principles of constructivism and with the development of learners' problem-solving skills.

In conclusion, one notes that "no one disputes the centrality of teachers' knowledge of the subject matter" (Zazkis & Zazkis, 2011, p. 250). However, one observes that for teaching

purposes, the knowledge of teaching mathematics is a “complex combination of pedagogy, psychology, didactics, research, constructs and theories, and curriculum” Zazkis and Zazkis (2011, p. 248). Such knowledge is influenced by the teacher’s beliefs about mathematics itself. All these aspects contribute to creating an effective teacher of mathematics. The interest in teachers’ beliefs in this research was generated by the supposition that what teachers do in their classrooms is, ultimately, a product of their beliefs (Beswick, 2007).

2.13 SUMMARY OF THE CHAPTER

In this chapter, an attempt was made to describe the concept of *effective teaching of mathematics*. In Section 2.2, an explanation was provided of what effective teaching is. The effective teaching of mathematics was aligned with the shifting away from the traditional emphasis on learning rules so as to be able to manipulate symbols, to practices in which learners are actively engaged with the mathematics itself. In Section 2.3, the discussion of two forms of understanding played a critical role in explaining and justifying the need to ground the teaching and learning of mathematics in constructivism. The detailed discussion of constructivism, which is the most current psychological approach to learning, in Section 2.6 was critical, because it is the theory of learning and development that is the basis of the current reform movement in teaching and learning.

In Section 2.7, MKT was discussed, focusing on its role, and on its importance in regard to effective teaching of mathematics, and, hence, the need for the development of the teacher’s MKT. This discussion was extended by exploring beliefs about what mathematics teaching and learning is. The central focus of the next chapter is problem-solving, with the focus being on teaching *via* problem-solving or PBL, because PBL orients learners toward meaning making over fact collecting.

CHAPTER 3: PROBLEM-SOLVING

“Problem-solving is important as a way of doing, learning and teaching mathematics. However, whether or how such ways of viewing problem solving get implemented in the classroom will depend on the teacher.”

(Chapman, 2008, p. 8)

3.1 INTRODUCTION

The term problem-solving “has become a slogan encompassing different views of what education is, of what schooling is, of what mathematics is and of why we should teach mathematics in general and problem-solving in particular” (Schoenfeld, 1992, p. 352). The review of related literature shows the term ‘problem-solving’ to have evolved from the traditional meaning of working on a mathematical task, to the current meaning, according to which problem-solving is viewed as a vehicle of learning (Schoenfeld, 1992). This chapter summarises the available range of literature on problem-solving, narrowing it down to the role that is played by problem-solving in mathematics teaching and learning today.

3.2 PROBLEM-SOLVING

3.2.1 The nature of problem-solving

Problem-solving is identified as one of the basic life functions of the natural intelligence of the brain (Polya, 1973; Zhong, Wang & Chiew, 2010). The decisions that individuals make in everyday life are related to certain problems that require solving, no matter how minor or critical they are. Within the mathematical context, problem-solving refers to “the process wherein students encounter a problem or a question for which they have no immediately apparent resolution, nor an algorithm that they can directly apply to get an answer” (Tripathi, 2006, p. 168). Problem-solving is a particularly intricate and interesting concept in mathematics education. It is not only seen as an important strand of mathematical skill, but it is also seen as a productive way of developing other mathematical competencies (Lester & Kehle, 2003; Pui Yee, 2008; Ryve, 2007; Tripathi, 2006). Problem-solving is generally considered to be the “most important cognitive activity in an everyday and professional context” (Jonassen, 2000, p. 63), and it is, thus, regarded as critical in the teaching and learning of mathematics (Gaigher, Rogan & Brown, 2006; Robabeh, Hassan & Farzad, 2012; Sepeng, 2011; Sepeng & Webb, 2012). The following sections of this chapter contain further discussion of mathematical problem-solving.

3.2.2 The history of problem-solving in the teaching and learning of mathematics

Problem-solving has been the focus of mathematics education reform for more than 20 years. Rickard (2005, p. 1) and Schoenfeld (1992, p. 334) describe the events (which are summarised below) that have brought problem-solving to its current status in the mathematics curriculum.

- The publication of *How to Solve It*, by George Polya in 1945, presented the author's well-known heuristic – understand the problem; devise a plan; carry out the plan; look back – as a coherent framework for problem-solving. He argues that problem-solving should be a legitimate topic in the teaching and learning of school mathematics.
- The production of The National Council of Teachers of Mathematics (NCTM) 1980 Yearbook, *Problem Solving in School Mathematics*, resulted in problem-solving being seen as the pillar of reform in mathematics education. The NCTM stressed that problem-solving ought to be at the centre of mathematics in schools in the United States.
- The production and tremendous impact of the NCTM Standards series, comprised of Curriculum and Evaluation Standards for School Mathematics in 1989, the Professional Standards for Teaching Mathematics in 1991, the Assessment Standards for School Mathematics in 1995, and the Principles and Standards for School Mathematics in 2000, resulted in problem-solving being declared a crucial element of change in the teaching and learning of mathematics. The NCTM, in 2000, observed that problem-solving was an essential element at the core of investigation and application, so that it ought to be intertwined all through mathematics teaching and learning, so as to provide a platform for the learning and application of mathematics.

Problem-solving has, therefore, traditionally been a part of the mathematics curriculum, having had a place in the mathematics classroom, although it was initially used as a starting point for obtaining a single correct answer, usually by following a single correct procedure. In the early 1980s, it was seen as important because it was viewed as the single vehicle for achieving the following three values of mathematics at school level (Taplin, 2011):

- 1) **Functional** (*the usefulness of problem-solving*): Mathematics is an essential discipline, because of the functional or practical role that it plays in relation to the individual and society. Teaching mathematics through problem-solving created

and simulated genuine real-life contexts that justified the mathematics involved, as opposed to treating it as an end in itself. Problem-solving became the emphasis of the teaching and learning of mathematics, because it incorporated skills and abilities that were an important part of everyday life. Thus, problem-solving contributed to the practical use of mathematics, by helping people to develop the facility to be adaptable when, for instance, technology breaks down. Problem-solving was advocated as a means of developing mathematical thinking as a tool for daily living. Problem-solving ability was said to be at the heart of mathematics, because it was the means by which mathematics could be applied to a variety of unfamiliar situations (Taplin, 2011).

- 2) **Logical** (*the analytic nature of problem-solving*): In addition to being a vehicle for the teaching and reinforcing of mathematical knowledge, and a means of helping to meet everyday challenges, problem-solving is also a skill that can enhance logical reasoning. Logical reasoning is a valuable skill in itself, as a way of thinking, rather than just as the means to an end, namely finding the correct answer. Hence, problem-solving was seen as an important means of developing the logical thinking aspect of mathematics that contributes to the development of intelligence (Polya, 1973, p. 1).
- 3) **Aesthetic** (*the appealing nature of problem-solving*): An additional reason for deeming the adoption of a problem-solving approach valuable is its aesthetic form. Problem-solving allows the learner to experience a range of emotions that are related to the various stages in the solution process. Mathematicians who successfully solve problems say that the experience of having done so contributes to an appreciation for the power and beauty of mathematics (Taplin, 2011).

It has been only fairly recently that problem-solving has come to be viewed as an important *medium* for the teaching and learning of mathematics (Stanic & Kilpatrick, 1989, p. 15).

3.3 APPROACHES TO PROBLEM-SOLVING

Schroeder and Lester (1989) distinguished three approaches to problem solving, namely teaching *about* problem solving; teaching *for* problem solving and teaching *via* or *through* problem solving. These approaches which are illustrated in figure 3.1 will now be discussed, with reference to their impact on learners' understanding.

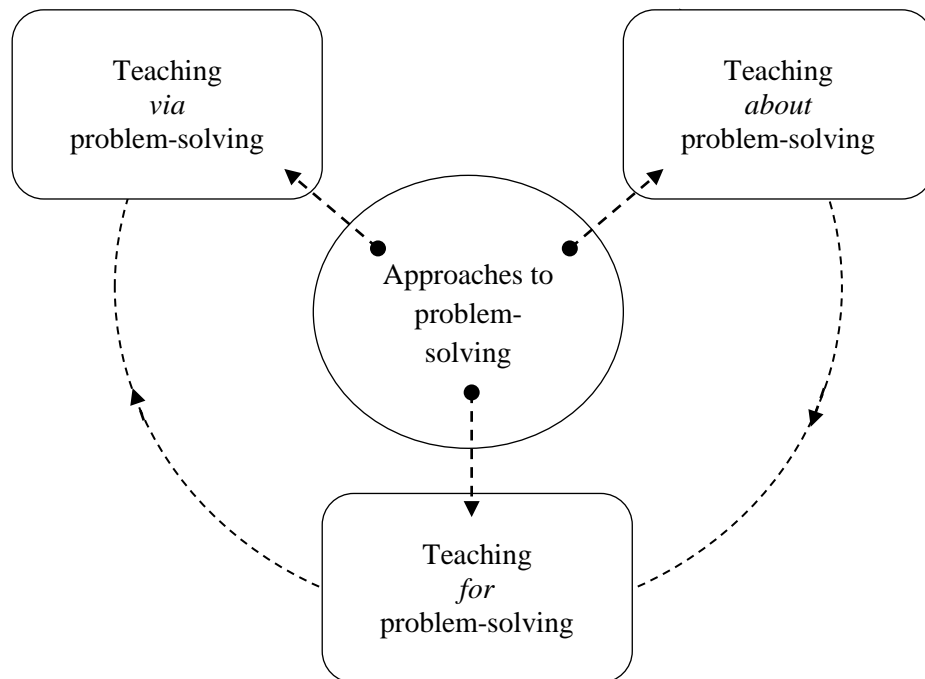


Figure 3.1: Approaches to problem-solving in the teaching process

Teaching about problem solving

In teaching *about* problem solving Polya's four-step model is the starting point. The four phases of this model are: understanding the problem; making a plan; carrying out the plan and reflecting on the results (see 3.4.4 for detailed discussion of Polya's model). The focus in teaching about problem solving is to directly teach these four problem solving steps together with a number of strategies from which learners can choose to solve the problem. This approach however reduces problem solving to yet another topic in the curriculum that may be taught in isolation. Teaching about problem solving does not foster learners' original thinking; because learners have to choose between a variety of solutions and problem solving just becomes an exercise of choosing one of the supplied strategies to use.

Teaching for problem solving

In teaching *for* problem solving, or to put it in another way “an *ends* approach” (Lester, 2013), the focus is on the application of the acquired mathematical knowledge to solve routine and non-routine problems. The teachers prepare learners to transfer the acquired mathematical knowledge to other contexts by exposing them to many instances of the mathematical concept and structures under study. In this approach learners get involved in problem solving only after they have learnt a new concept or algorithm. Teaching for problem solving limits the learners’ thinking, as they have to use the learnt algorithm and not their own thought out solutions.

Teaching via or through problem solving

In teaching *via* problem solving learning takes place during the process of attempting to solve problems in which relevant mathematics concepts and skills are embedded (Lester & Charles, 2003; Schoen & Charles, 2003) The approach of teaching via problem solving uses problems both as purpose for learning mathematics and as primary means of learning mathematics in other words it is “a *means* approach” (Lester, 2013). The problem-based tasks or activities are the vehicle by which mathematical concepts and understanding is developed.

The distinction in the three approaches is visible in its impact on the learner. Teaching about problem solving does not foster original thinking, because learners are given a variety of solutions to choose from and problem solving becomes an exercise of choosing one of the supplied solutions. Teaching for problem solving limits the learners’ thinking, as they have to recall and use the learnt algorithm rather than think out their own solutions. On the other hand the teaching via problem solving approach requires an inquiry-orientated classroom atmosphere where learners are encouraged to think out and reflect on their own solution and that of others. Such problem solving experiences foster higher order thinking processes.

While each of the approaches will be discussed in detail in the following sections, there is little doubt, as it shall be established in this chapter, that the teaching and learning of mathematics can be enhanced by the creation of teaching and learning environments in which learners are exposed to teaching *via* problem-solving, as opposed to the more conventional approach of teaching *about* problem-solving. The challenge for teachers at all levels is to create such an environment to enable the process of mathematical thinking to take place, and to search for opportunities to present mathematics in problem-solving settings.

Nowadays, the emphasis has shifted from teaching *for* or *about* problem-solving to teaching *via* problem-solving (Schroeder & Lester, 1989). The focus is on teaching mathematical topics through problem-solving contexts and enquiry-oriented environments that are characterised by the teacher “helping students construct a deep understanding of mathematical ideas and processes by engaging them in doing mathematics: creating, conjecturing, exploring, testing, and verifying” (Taplin, 2011).

As mentioned above, the approach to problem-solving has evolved over the years. Schroeder and Lester (1989, p. 33) point out the following approaches to problem-solving:

- teaching *about* problem-solving;
- teaching *for* problem-solving; and
- teaching *via*, or through, problem-solving.

The following section provides a brief descriptive analysis of each of these approaches to problem-solving.

3.3.1 Teaching *about* problem-solving

Teaching about problem-solving involves teaching the learners how to solve problems (Tripathi, 2009). The focus here is on the problem-solving process, or on the adoption of strategies for problem-solving. The emphasis is on the learners’ ability to understand the problem, to design a problem-solving strategy, to implement the strategy, and to look back on, or to check on, the correctness of their solution. Problem-solving in this approach is an exercise in selecting a solution from the supplied solutions. The weakness of the approach lies in it not promoting original thinking in the learners.

In teaching about problem-solving, the lesson also focuses on developing learners’ meta-cognitive behaviour. According to Ye, Doyle, Dias, Czarnocha & Baker (2011), a lesson about problem-solving consists of the following four phases:

- **Orientation phase:** In this phase, the teacher's principal objective is to help the learners understand the various features of the problem, including the significant information, the given circumstance(s), and the question.
- **Organisation phase:** By this phase, the assumption is that the majority of the learners have understood the problem, and, therefore, in this phase the teacher tries to assist the learners in coming up with strategies that would be likely to lead the learners to the likely solution. As an example, the teacher might have the learners discuss different plans, and past problem-solving experiences. The objective of such activities in the orientation and organisation phases is to prepare the learners for solving the problem.

- **Execution phase:** The learners are encouraged to work in small groups or individually to implement the plan that was agreed upon in the organisation phase in regard to solving the problem. The teacher's task is to observe what learners are doing, to provide them with clues through questioning, and to assist any groups that might have reached a stalemate. The teacher also prompts the learners to check the correctness of their calculations, and the reasonableness of their proposed solution.
- **Verification phase:** This phase involves learners personally scrutinising their solution, sharing their solution with other group members, or with the whole class, and providing their solution with a fitting name. Thoughtful debate about the accuracy of the strategies used, and the answers given, is held.

When a group cannot devise a solution to the problem, then a whole-class discussion of the possible solution to the problem is held. The group's errors, or misunderstandings of the problem, are discussed. During the class discussions and sharing of the different solutions, the teacher seeks to have the different groups reflect on their experience, and to pay special attention to the words or phrases in the problem that were helpful or confusing. The class also recalls, and reflects on, similar problems that have been solved in the past, or that have used similar strategies.

Each lesson focuses on the development of positive attitudes and beliefs about problem-solving in the learners, as well as in themselves as problem-solvers. The teacher complements the learners' ability in problem-solving, and encourages them to try different strategies to solve a problem. Group work is used to support learners, and to help them experience the fun and enjoyment of success in problem-solving tasks. One concludes that, in *teaching about problem-solving*, precise procedures or algorithms are being practised in the mathematics classroom, and that the emphasis is on teaching the procedures recommended in the syllabus, and on aiming them at solving certain nonroutine problems.

3.3.2 Teaching *for* problem-solving

The focus in teaching for problem-solving is on 'applying acquired mathematical knowledge', or on 'mathematics being taught' in the solution of 'routine or non-routine problems' (Salmon & Grace, 1984; Schroeder & Lester, 1989). The view is that problem-solving is the cornerstone of mathematics. Hence, without the ability to solve problems, the usefulness and power of mathematical ideas, knowledge and skills is limited. Therefore, unless the learner can solve problems, the mathematical facts, the concepts and the procedures they know are of little use. Problem-solving is the instructional goal in teaching for problem-solving (Wilson, Fernandez & Hadaway, 2011). Anderson (2009) points out that, in teaching

for problem-solving, the learner is required to have the following attributes (as are illustrated in Figure 3.2 below):

- deep mathematical knowledge;
- reasoning ability;
- heuristic strategies;
- good communication skills; and
- the ability to work in a group.

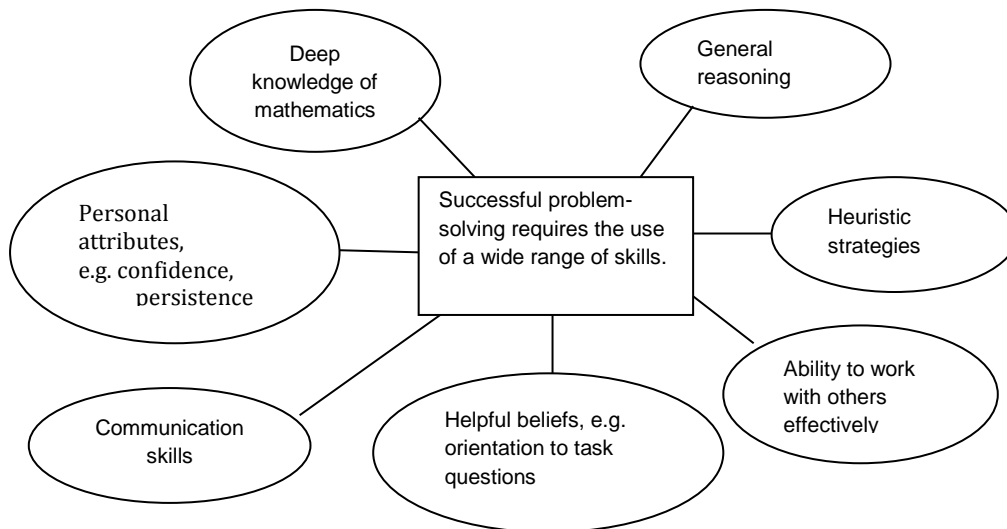


Figure 3.2: Factors contributing to successful problem-solving (Stacey, 2005, p. 342)

While early work in problem-solving focused mainly on describing the problem-solving process, more recent investigations have focused on identifying the attributes of the problem-solver that contribute to success in problem-solving. Voskoglou (2008, p. 13) discusses the attributes as follows:

- **Resources:** The conceptual understandings, knowledge, facts, and procedures used during problem-solving fall under this attribute.
- **Control:** This includes the selection and implementation of resources and strategies, as well as of behaviours that determine the efficiency with which facts, techniques and strategies are exploited, including planning, monitoring, decision-making, and conscious meta-cognitive acts, among others.
- **Methods:** The general strategies that are used when working out a problem, like constructing new statements and ideas, carrying out computations, and accessing resources all form part of the methods.
- **Heuristics:** More specific procedures and approaches that are used when working out a problem, like observing symmetries, using a graph or table, looking for counter

examples, and altering the given problem so that it is easier all form part of heuristics.

- **Affect:** Attitudes (including enjoyment, motivation and interest), beliefs (including self-confidence, pride and persistence, among others), emotions (including joy, frustration and impatience, among others) and values/ethics (including mathematical intimacy and integrity) all fall under affect.

For learners to be good problem-solvers, they would need to have the above attributes. Therefore when teaching for problem-solving, the teacher designs their teaching towards the development of such qualities.

3.3.2.1 Problem-solving skills

Salmon and Grace (1984, p. 21) identify the following as the skills that require introducing and development in learners during teaching for problem-solving:

- the identifying and use of the required operation;
- the guessing and checking of answers;
- the drawing and sketching of diagrams;
- the creating and writing out of word problems;
- the searching for relevant information;
- the determining of the reasonableness of the results; and
- the writing and solving of number sentences.

3.3.2.2 Problem-solving strategies

Problem-solving strategies are “tactics, action plans, general moves, methods of attack or executive schema for solving problems” (Salmon & Grace, 1984, p. 33). The literature shows that, when discussing strategies for problem-solving, reference is made to Polya’s (1973) four-stage model, which entails: (a) understanding the problem; (b) devising a plan; (c) carrying out the plan; and (c) reflecting on how the solution was reached. Salmon and Grace (1984) simplify the description of these stages to: (a) seeing; (b) planning; (c) doing; and (d) checking, while Ye et al. (2011, p. 32) refer to the four phases as: (a) orientation; (b) organisation; (c) execution; and (d) verification.

The importance or value of Polya’s (1973) four-phase model and of Ye et al.’s (2011) four phases in the teaching for problem-solving is that they facilitate an understanding of the process of problem-solving as consisting of many interrelated actions and decisions (Salmon & Grace, 1984, p. 34). In principle, the Salmon and Grace (1984) and Ye et al. (2011, p. 32)

strategies for problem-solving stated above are basically the same as Polya's (1973) strategies, apart from being expressed differently.

In teaching for problem-solving, one observes that teaching for problem-solving first starts with the teaching of the mathematics, and then progresses to problem-solving as a way of applying the mathematics learnt. For example, learners learn about how to calculate the fraction of a number, such as $\frac{1}{2}$ of 24. Once they have mastered the procedure for working out the fraction of a number, they are given word problems to solve that involve the application of this procedure, such as is shown in Figure 3.3 below.

Work Card 2	
Problem-solving	
1. Tom has 16 marbles. He gave $\frac{1}{4}$ to his brother Jake. How many marbles did Jack get?	2. There are 36 children in Grade 3B, of whom $\frac{1}{2}$ are boys. How many boys are in Grade 3B?

Figure 3.3: Problem-solving work card

The disadvantage of the above approach is that it assumes that the learners understand the mathematics being learnt, that the teacher only shows one way to solve the problem (giving the impression that there is only one way of solving a particular problem), and that the learners become accustomed to being shown how to solve a problem. In the process, the learners are not deepening their understanding in learning the mathematics; hence, they often tend to forget what they have learnt (Roedige & Karpicke, 2006). What they need is a more effective approach to the learning of mathematics.

3.3.3 Teaching *via* problem-solving.

The approach that is now popularly known as PBL uses problems for the purpose of learning mathematics. Schroeder and Lister (1989, p. 33) make a case for *teaching via problem-solving* by pointing out that:

In teaching via problem-solving, problems are valued not only for the purpose of learning mathematics but also as a primary means of doing so. The teaching of a mathematical topic begins with a problem situation that embodies key aspects of the topic, and mathematical techniques are developed as reasonable responses to reasonable problems.

Teaching mathematics via problem-solving is fairly a new concept, particularly at the Foundation Phase level. In contrast to teaching for problem-solving, in which the focus is on developing the learners' problem-solving skills and strategies, in the teaching via problem-

solving approach, problem-solving is viewed as a very powerful tool for helping the learners understand mathematical concepts, processes and techniques (Nunokawa, 2005; O'Connell, 2007; Schroeder & Lester, 1989). The teacher, therefore, uses problem-solving to develop mathematical concepts, skills and procedures. Mathematics is taught through problem-solving, and the problem-based tasks or activities are the vehicle by which the desired curriculum is developed, with learning being an outcome of the problem-solving process (Boaler, 1993). In teaching via problem-solving, the teacher focuses on external monitoring, the facilitation of problem-solving, and the modelling of problem-solving behaviour (Salmon & Grace, 1984, p. 11). A cooperative class environment that encourages active learning, either in terms of individual, small group, or whole-class activities is created.

A similar, but different, approach that is based on problem-solving is problem-centred learning (PCL). Comparing PBL and PCL is a very difficult task, especially when one compares what competencies each aims to develop in learners (Potvin, Riopel, Masson & Fournier, 2010, p. 2).

Both PBL and PCL are learner-centred pedagogies that are centred on constructivism (Roth, 1993, p. 113). The teacher takes the role of a facilitator in both approaches (Barrett, 2005; Hodgkin & Knox, 1975, p. 2).

From the above comparison, one concludes that there is a very thin line between PBL and PCL, with the latter appearing to be embodied in the former. The researcher illustrates this by means of Figure 3.4 below.

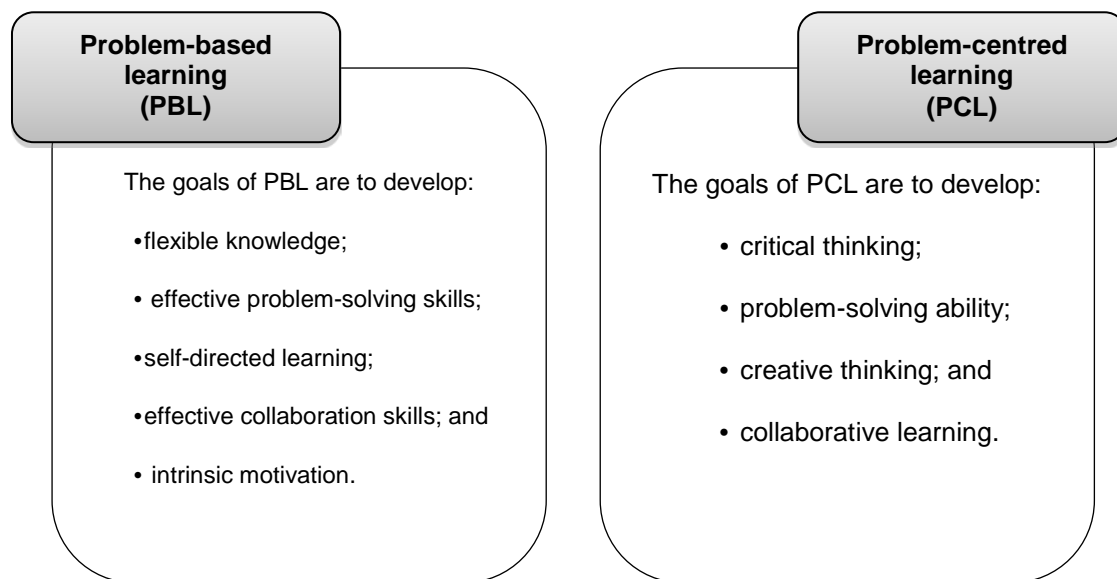


Figure 3.4: Comparison of the goals of PBL and PCL (Barrett, 2005)

Though worded differently, the goals of PBL and PCL are basically the same, and the theoretical basis for both is constructivism, as has been discussed above. Hence, in essence, PBL and PCL are different terms that are used in different disciplines, or in specific places in the world, to denote the same approach. However, the focus of the current study is on PBL, since its goals are broader than, and encompassing of, those of PCL.

3.4 MODELS OF PROBLEM-SOLVING

Real-life problems generally differ, as do their solutions. Sometimes a problem and its solution are clear, but, at other times, one may find it hard to state what the nature of the problem is, or how to solve it. Notwithstanding the above, one can use a problem-solving model to solve it. Below is an overview of various problem-solving models. The models concerned are highly flexible, and they can be altered to suit the different types of problems. A flexible set of tools is given to use at each step of the problem-solving. The models are designed to be followed one step at a time, although some steps might not need as much attention as others.

3.4.1 The FOCUS Model is a simple quality improvement tool that is commonly used in the health care sector. The tool can be used to improve any process, but it is particularly useful for processes that span different departments.

The five steps in FOCUS are as follows:

- 1) **F**ind the problem.
- 2) **O**rganise a team.
- 3) **C**larify the problem.
- 4) **U**nderstand the problem.
- 5) **S**elect a solution.

People often use the Focus Model in conjunction with the Plan–Do–Check–Act cycle, which allows teams to implement their solution in a controlled way.

3.4.2 The Productive Thinking Model provides a structured approach for solving problems creatively. It can be used individually, and in a group.

The six steps in the model are as follows:

- 1) Ask "What is going on?"
- 2) Ask "What is success?"
- 3) Ask "What is the question?"

- 4) Generate answers.
- 5) Forge the solution.
- 6) Align resources.

The advantage of the model is that it encourages one to use creative and critical thinking skills at each step of the problem-solving process. This means that one can take a well-rounded look at a problem, and that one can come up with better solutions.

3.4.3 The Six Steps of Problem-Solving Model is highly flexible, and it can be adapted to suit different types of problems. The steps in this problem-solving model are as follows:

- 1) Define the problem.
- 2) Analyse the problem.
- 3) Identify as many potential solutions as you can.
- 4) Choose the best solution.
- 5) Plan the action.
- 6) Implement the solution.

Although the model is designed to be followed one step at a time, some steps do not require as much attention as do others.

3.4.4 Polya's (1973) four-step model defines problem-solving as searching for an appropriate course of action to achieve an aim that is immediately attainable. The steps are as follows:

- 1) Understand the problem (Orientation).
- 2) Make a plan (organisation) – Make a general plan, and select relevant methods or appropriate heuristics for solving the problem. The learner relates their understanding of the problem in step 1 to their previous knowledge.
- 3) Carry out the plan – The learner in this step performs the computations that are required to implement the plan, and to obtain the solution to the problem.
- 4) Looking back – The learner reviews what they have done, and checks on the correctness of their solution.

The 4 models briefly discussed above have much in common, although they vary in the number of steps taken, as well as the wording used. The approaches are basically one and the same, in that they consist of a step-by-step approach to problem-solving. Although the number of the steps changes in each model, but the point of origin of these models, in all cases, was the study of Polya (1973). Polya, who states the problem-solving steps in his

famous book titled *How to Solve It*, was an authority on mathematical discovery, the understanding of learning, and the teaching of problem-solving.

In this study, Polya's (1973) model is the preferred model, as it was specifically developed for problem-solving in a mathematical context. The model also contains the 'looking back' step, which is used as a checking step, in which step the learners check their results arithmetically and logically. In addition, the meta-analysis of 487 learners in relation to problem-solving indicated that the learners who were trained to use such a heuristic model as Polya's model showed the largest gains in terms of problem-solving performance (Reys et al., 2001).

3.5 INTERNATIONAL APPROACHES TO PROBLEM-SOLVING

Approaches to problem-solving differ from country to country. Anderson (2009, p. 2) summarised approaches to problem-solving in terms of the mathematics curriculum, and in terms of the support provided for teachers in Singapore, Hong Kong, England and the Netherlands. The issues covered that were involved in the implementation of problem-solving are highlighted in the following country-by-country summary.

3.5.1 Singapore

Singapore's results to emerge from an early TIMSS study led to problem-solving becoming the primary goal of the learning and teaching of mathematics. The Singaporean mathematics curriculum, which was designed to be centred around problem-solving, is dependent on five interrelated components: skills; concepts; processes; attitudes; and metacognition (see Figure 3.5 below).

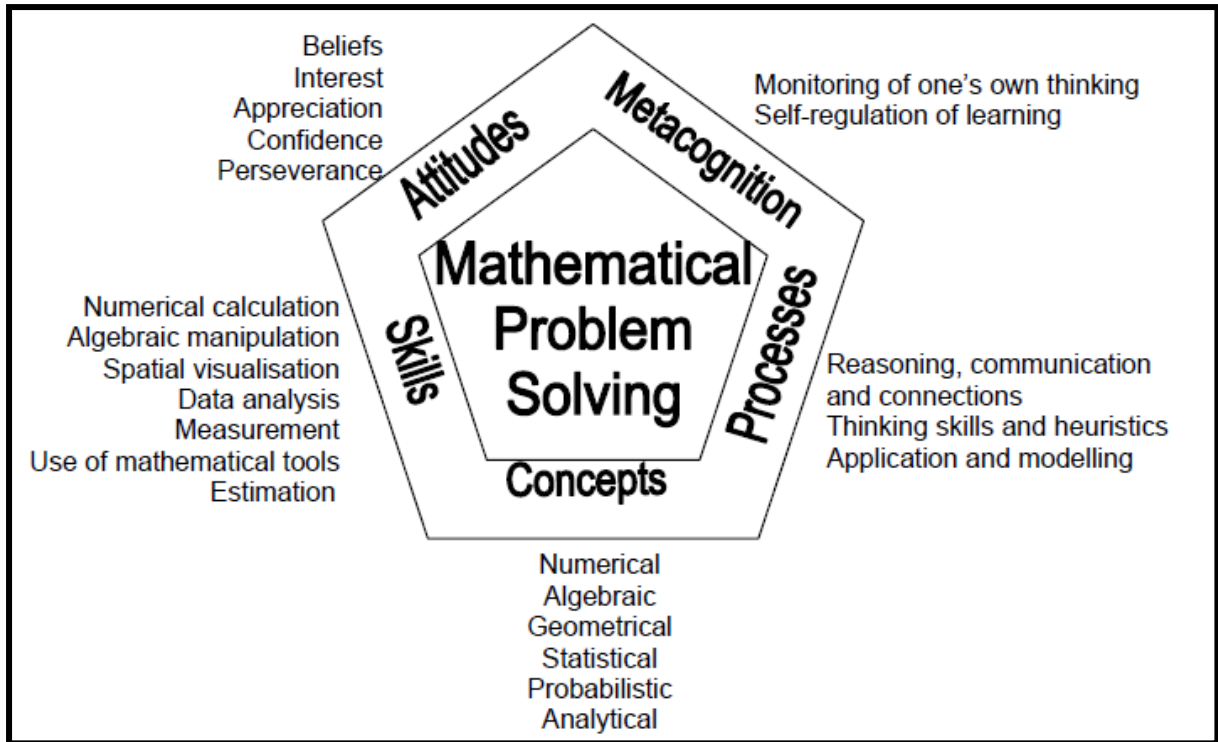


Figure 3.5: Framework of the Singapore mathematics curriculum (Fai, 2006, p. 4)

The mathematics content is presented as skills and processes, whereas the attitudes represent the affective extent of learning. Metacognition highlights the importance of self-regulation. The learning processes include the acquisition and application of mathematical knowledge. In addition to this, two new initiatives, called Thinking School, Learning Nation (TSLN) and Teach Less, Learn More (TLLM), were introduced with the objectives of reducing the curriculum content, and of creating sufficient time to engage learners in additional thinking and problem-solving tasks (Anderson, 2009).

3.5.2 Hong Kong

Although educators in Hong Kong tend to be more mindful of problem-solving approaches to the teaching and learning of mathematics than are the educators of some other nations, there is little evidence that shows the use of such approaches. Those educators who attempt to use these approaches continue leading or directing the learners towards preset solutions, as opposed to letting the learners explore and analyse the mathematical ideas themselves. It is only recently that the use of open-ended questions or groups has been observed in some of the mathematics lessons in certain secondary schools in Hong Kong (Anderson, 2009).

3.5.3 England

In England, problem-solving, which is portrayed as being at the core of mathematics, is described as being a process that includes representing, analysing, interpreting, and evaluating, as well as communicating and reflecting. To assist teachers, an extensive variety of helpful teacher/learner materials has been produced for district and school-based teacher professional development. Such materials contain examples of problems and rich activities that are designed for each content area. The teachers are generally encouraged to study the materials, and to see how they can be used for representing, analysing, interpretation and reflection processes. With the use of such materials, it is essential that the teachers concerned are supported if they are to include the processes in their mathematics lessons, and to provide meaningful problem-solving experiences for their learners. Notwithstanding, it is too early to say to what extent this change has had an effect. In future, assessment instruments will have to incorporate more open-end items so as to ensure the assessment of problem-solving (Anderson, 2009; Burkhardt & Bell, 2007).

3.5.4 The Netherlands

The Netherlands has developed a mathematics curriculum and a pedagogical approach that is known as Realistic Mathematics Education (RME) (Doorman, Drijvers, Dekker, Van den Heuvel-Panhuizen, De Lange & Wijers, 2007). The present-day RME is mostly determined by Freudenthal's (1977) view about mathematics. According to him, mathematics must be linked to reality, staying near to the learners, and being applicable to the rest of society, in order to be of human value. Instead of seeing mathematics as subject matter that has to be transmitted, the characteristics of RME include the following:

- well-researched activities that encourage learners to move from informal to formal representations;
- the use of realistic situations to develop mathematics;
- less emphasis on algorithms, and more on making sense;

- the use of 'guided reinvention'; and
- progress towards formal ideas, seen as a long-term process.

Treffeers (1987) describes the following five characteristics of RME:

- the use of contexts;
- the use of models;
- the use of students' own productions and constructions;
- the interactive character of the teaching process; and
- the intertwinement of various learning strands.

Instead of using the traditional pedagogy of demonstrating the formal mathematical skills and procedures, and then practising and applying the mathematical skills and procedures to the solving of problems, RME utilises real-life problems to develop the learners' mathematical skills and processes. Real-life problems are seen as the starting point for the application of the new mathematics.

The use of context problems is very significant in RME. This is in contrast with the traditional, mechanistic approach to mathematics education, which contains mostly bare, 'with no closes' problems. If context problems are used in the mechanistic approach, they are mostly used to conclude the learning process. The context problems function only as a field of application. By means of solving context problems, the students can apply what was learned earlier, in the bare situation.

In RME, the approach is different. The context problems function also as a source for the learning process. In other words, in RME, contextualised problems and real-life situations are used both to constitute and to apply mathematical concepts. While working on context problems, the students can develop their mathematical tools and understanding (Barnes, 2005).

Another notable difference between RME and the traditional approach to mathematics education is the rejection of the mechanistic, procedure-focused way of teaching, in which the learning content is split up into meaningless small parts, and in which the students are offered fixed solving procedures to be used in training exercises, which are often performed individually (Treffeers, 1991).

Although RME is expected to aid in the implementation of a problem-based curriculum, there is little proof of nonroutine problem-solving in Dutch classrooms. The absence of textbooks and examination items that focus on real-life problems has been proposed as the main limitation of the implementation of the RME curriculum. The above shows that there are

similarities and differences in the teaching and learning of mathematics and problem-solving initiatives in Singapore, Hong Kong, England, and The Netherlands. Singapore has made substantial changes by reducing the mathematics content of its curriculum, so as to create more time for problem-solving. The Dutch implementation of RME was designed for the teaching and learning of mathematics to be centred on real-life problems. England's latest curriculum affords increased flexibility and rich problem-solving teaching and learning materials, whereas Hong Kong is currently focusing on the development of the same levels of support for teachers as there are in England, Singapore and the Netherlands. A common feature in all these countries is the recognition that, for teachers to be able to incorporate more problem-solving experiences in their mathematics lessons, the teaching and learning materials used, such as textbooks, will have to include more examples of problems, and the examinations will need to test the problem-solving abilities of the learners involved.

Mullis et al. (2000) in relation to TIMSS, observed that Japanese learners were more successful than were US and Canadian learners. The report in question states that the factor behind that difference is that, whereas 49% of the teachers in Japan emphasise reasoning and problem-solving, the rate of such an emphasis among US teachers was 18%, with it being 13% for Canadian teachers. Therefore, a correlation between problem-solving and learner achievement in mathematics can clearly be seen (Mullis et al., 2000).

3.5.5 The case of Japan, Finland and Singapore

Often, in discussion of poor learner performance, the focus is on teaching. However, for a class to be successful, the focus has to be on learning, rather than on teaching. There is a need, therefore, to change the outlook involved in the modern classroom. The lack of focus on learning could be the biggest reason why most attempts that are made to improve learner performance fail. Whereas a number of factors are holding back mathematics teaching in South Africa, in spite of all the interventions that have taken place, much can also be learnt from the evidence that is provided by three high-performing education systems, namely those of Finland, Japan and Singapore.

- 1) Finland.** The country has managed to attract huge numbers of highly qualified young people to teaching. The national curriculum allows the freedom for teachers to use their own preferred methods, with beneficial results. In summarising the Finnish experiences of problem-solving in mathematics education, Pehkonen (2008) states:

...teachers in Finland are changing in the direction of a more favorable attitude to problem solving. But its use in teaching demands much from the teacher, and, therefore, they find excuses why not to use a problem-solving approach. (p. 4)

If we use the language introduced by Schroeder & Lester (1989), we might say that only few teachers are teaching via problem solving, while most of them teach something about problem solving. (p. 3)

Other studies of Finnish education had shown that Finnish teachers are rather traditional and pedagogically conservative (Savola, 2010). In addition, a British team that visited 50 primary schools did not see much evidence of either student-centred, or independent, learning (Norris et al., 1996).

In spite of all the evidence of Finland learners performing well in such international tests as PISA and TIMSS, as a result of the use of traditional methods, the following has been discovered:

- a) A new solution for teaching problem-solving within the curriculum is being considered. According to Pehkonen (2008, p.1), “[s]uch a reform is based on the use of problem solving as a teaching method that often is manifested by the use of open problems”.
- b) After noting the research evidence showing that traditional instruction prevailed in Finnish schools, Savola (2010, p. 33) asks, “so should Iceland and other countries aspiring to do better in international assessments start, or return to, teaching like the Finnish mathematics teachers?” The answer provided is:

While some of the classroom practices from the Finnish schools may prove effective elsewhere, we must keep in mind that there is no “one-size-fits-all” educational system; what works in Finland will not necessarily work anywhere else. Cultures—educational and other—are “situated contextual organisms” (Goldman, 2007, p. 33) that have the ability to adapt and morph only within certain limits. Finland is, after all, a bit different from its Nordic neighbors and other nations. It is a border country that has gone through three bloody wars in the last one hundred years. It fought against Russia, whose influences still permeate the Finnish culture at all levels. Perhaps it is because of this unique socio-historical background that the Finns are rather obedient and allow a sense of authoritarianism, all the while maintaining a democratic, Western society (Simola, 2005). This bodes well for the Finnish schools where traditional instructional methods are still prevalent.

- 2) **Japan** has similar characteristics. The Japanese primary school teachers have considerably higher levels of mathematics ability than do the primary school teachers of other nations. As in the case of Finland, the official Japanese course

of study is commendably brief, with teachers again having the freedom to innovate.

- 3) In **Singapore**, another strikingly common factor is the strong continuum between primary and secondary school mathematics. In this country, the foundations of mathematics teaching are introduced relatively early.

Burghes (2012), after observing mathematics education in Japan, Singapore and Japan, notes:

The crucial point is that all three countries have implemented national curricula in which knowledge of the fundamentals is stressed, but in which problem-solving also plays a central role. This provides motivation for teachers and ensures that pupils enjoy the subject and make progress.

The result is that students develop inquiring minds and a thirst for knowledge: they take responsibility for learning and how to learn. This contrasts strongly with the UK model, where teachers transfer knowledge to students through examples, and pupils spend much of the lesson in practice mode. Singapore, Finland and Japan still allow plenty of practice, but crucially teachers have the freedom to pose problems, and leave the responsibility to students to solve them.

From the above literature review of mathematics education in Finland, Japan and Singapore, one can conclude that, although no education system from other countries can be transported as is into another context, there is much to learn from the experiences of those who have addressed problems that South Africans are also currently meeting. As observed by Burges (2012) above, although the three countries concerned have implemented national curricula in which the knowledge of the fundamentals is stressed, and traditional instruction is prevalent, problem-solving also plays a central role in their educational systems.

Savola (2010, p. 33), who is quoted above, observes that the success of traditional instruction in Finland is perhaps due to the Finns' unique sociohistorical background that has resulted in the Finns being rather obedient as a race, and allowing a sense of authoritarianism to prevail, while, all the time, maintaining a democratic form of Western society. In the light of the above observation, other countries that aspire to improve their performance in international assessments might be encouraged to reverse their progress, and to return to teaching in the traditional methods that are used by the Finnish mathematics teachers. A similar situation exists as far as those observing the Japanese example goes.

3.5.6 The new South African approach

The current South African mathematics CAPS was finalised for implementation in schools in 2012. The Department of Education's Foundations for Learning campaign, which is aimed at creating a national focus on improving the numeracy abilities of children in South Africa, provides the following guidelines with regard to problem-solving:

Interactive group or paired work should follow where learners engage with problem solving or challenging investigation where they have to apply what they've learned in the earlier part of the lesson. Opportunities for learners to try out different ways to solve problems should be encouraged, e.g. rounding off or adding on to subtract as two possible strategies for adding 3- and 4-digit numbers. The teacher should once again leave time for whole class or group review where different learners share and explain their thinking, methods and answers. Sufficient attention shall be given to questions requiring higher order thinking and the solving of word problems in particular.

(Republic of South Africa. Department of Education, 2008, p. 19)

The new South African curriculum recognises the importance of problem-solving as a skill to be taught to learners. However, from the above extract, one concludes that teaching *for* problem-solving is the suggested approach, in terms of the new South African curriculum. The excerpt "...with *problem solving or challenging investigations where they have to apply what they've learned in the earlier part of the lesson*" implies that the learners first learn some mathematics, which they then apply in problem-solving. This implication is further illustrated by the guidelines that are given in the relevant section of the CAPS.



CONTENT AREA	CONTENT	CLARIFICATION / NOTES/ACTIVITIES/ EXAMPLES
	Problem solving involving subtraction	<ul style="list-style-type: none"> Solve word problems that involve subtraction and up to 10. Semi concrete level <p>Example: There are 4 birds sitting on a chimney. Three birds fly away. How many birds are on the chimney now?</p> <p>a) Teacher says: Tell me what the story is about?</p> <p>b) Draw a picture to show what the story is about?</p>   <p>Learners draw pictures or symbols to explain how they found their answer. Learners write the number sentence e.g. $4 - 3 = 1$</p>

Figure 3.6: Example of approach to problem-solving (Republic of South Africa. Department of Education, 2011, p. 45)

The major goal of the lesson that is indicated in Figure 3.6 above is that learners should solve the problems concerned using the subtraction skills and concept previously learnt. The above extract shows the adoption of a teaching **for** problem-solving approach, in that problem-solving is being used as a vehicle for practising the required application. The focus is on applying acquired mathematical knowledge to solve routine, or nonroutine, problems. The requirement that the learners have to read the problem carefully and to analyse it suggests that the focus is on acquiring mathematical skills and concepts, after which they are exposed to routine problems in relation to which they are required to apply the mathematical skills and concepts concerned.

The extract also shows the teacher's role as being that of helping the learners find the solution to the problem by providing an efficient strategy, namely to understand the problem, and to work out a plan, which they then have to execute. Anderson (2009, p. 2), in a summary of international trends in mathematics curriculum development, notes "an increased focus on problem solving and mathematical modelling in countries from the West as well as the East". Such nations have come to understand that problem-solving is key if learners are to have the capacity to utilise and apply mathematical knowledge meaningfully. The nations involved, which have also concluded that it is through problem-solving that learners can deepen their understanding of mathematics, are all the more captivated and intrigued by mathematics, appreciate its importance and usefulness all the more than they might otherwise have done.

Anderson (2009) also observes that numerous nations (including South Africa) have included problem-solving as an integral component of their mathematics curriculum. Its implementation in classrooms will, however, take more than mere 'speechifying' about problem-solving in the mathematics curriculum. While the provision of schools with valuable teaching and learning materials, and with sufficient time to do problem-solving, are important steps to take, problem-solving will only become valued in the teaching and learning of mathematics when it is also included in the relevant examinations.

In teaching for problem-solving, therefore, the emphasis is on learning the mathematics of a particular topic, so that it can be applied to solve problems. Very little scope for creativity exists if learners are just learning to use taught techniques.

There is compelling evidence in the literature reviewed above, namely that of Anderson (2009), Mullis et al. (2000), Nunokawa (2005), O'Connell (2007), Pundak and Rozener (2008), Schoenfeld (1992), and Schroeder and Lester (1989), that the traditional approach to the teaching of mathematics is not successful in helping learners to gain a conceptual understanding of the most basic mathematical concepts. PBL promises a variety of educational outcomes, including, but not limited to:

- skills in group work and information seeking;
- self-directed learning (SDL);
- communication skills; and
- the development of learners' knowledge base and reasoning skills (Sahin, 2010, p. 267).

3.6 PROBLEM-BASED LEARNING (PBL)

PBL is a learner-centred teaching method, in which learners collaboratively solve problems and reflect on their experiences. Kyeong Ha (2003, p. 1) points out that PBL "describes a learning environment where problems drive the learning". PBL as the constructivist answer to traditional learning theories, is based on three main preconditions for a successful and comprehensive learning process (Maurer & Neuhold, 2012):

- It is learner-centred.
- It follows an active process of knowledge construction.
- It is collaborative.

With PBL, learning is driven by challenging, open-ended problems. The learners work in small collaborative groups to identify what they need to learn in order to solve a problem, after which they engage in SDL, apply their new knowledge to the problem, and reflect on what they have learned. Teachers take on the role of 'facilitators' of learning. Thus, PBL is

- **Fact identification** – The learners formulate and analyse the problem by identifying the relevant facts in the scenario. This step helps the learners to represent the problem.
- **Hypothesis** generation – As the learners improve their understanding of the problem, they generate hypotheses about possible solutions.
- **Identification of knowledge deficiencies** – An important part of this cycle is the identifying of knowledge deficiencies that are related to the problem. Such deficiencies become what are known as the learning issues that learners research during their SDL.
- **Apply their new knowledge** – The learners apply their new knowledge, and evaluate their hypotheses in the light of what they have learned.
- **Abstraction** – On the completion of each problem, the learners reflect on the abstract knowledge that they have gained thereby.

The teacher assists the learners with learning the cognitive skills that are required for problem-solving and collaboration. As the learners are self-directed in taking care of their learning objectives and strategies to solve PBL's ill-structured problems (meaning those without a single correct solution), they also gain the skills that are required for their own lifelong education. The method of PBL, which was initially developed in medical schools, has since been used in a variety of settings, ranging from middle school to professional education.

3.6.1 Origins of problem-based learning

PBL was first used as a pedagogical approach in the 1960s at McMaster University Medical School, Ontario, Canada (Gijbels, Dochy, Van den Bossche & Segers, 2005, p. 28; Loyens, Magda & Rikers, 2008, p. 415; Rhem, 1998, p. 2). PBL is now used widely in elementary, secondary and tertiary education institutions worldwide, and it has also been adopted in various fields of professional training, such as nursing, engineering and education, among many others (Dolmans, De Grave, Wolfhagen & Van der Vleuten, 2005, p. 733). In education, PBL has gained prominence in a wide variety of disciplines, including, but not limited to, mathematics (Erickson, 1999, p. 518).

Opinions vary on whether PBL should be implemented for the entire mathematics curriculum, or whether it should be used merely to teach certain parts of the mathematics curriculum. Whether PBL will prove to be a totally successful teaching and learning innovation that becomes part of the teaching philosophy, or whether, as is more likely, it will have a brief moment of success, followed by disappointment and, eventually, abandonment,

is highly debatable. Numerous methods, such as discovery learning and programmed instruction, have had short periods of fame, but have, since then, been abandoned.

3.6.2 How PBL works

Rem (1998, p. 1) observes that “there must be something compellingly effective about problem-based learning, given the level of interest in it all through higher education”. PBL has the potential to support effective learning, because it is based on four modern insights into learning, namely that learning should be self-directed, collaborative, contextual, and a constructive process (Dolmans et al., 2005, p. 732).

PBL orients learners toward meaning making over fact collecting (Rhem, 1998, p. 1). In terms of such an approach, the learners learn through contextualised problems and situations. The dynamics of group work and independent investigation result in the learners concerned achieving higher levels of comprehension, and in them developing more learning and knowledge-forming skills, as well as more social skills than they might otherwise have done (Anderson, 2005, p. 54; Rhem, 1998, p. 2). PBL also brings prior knowledge into play more rapidly than do many other approaches, and it ends up fostering learning that is capable of adapting to new situations and related domains.

3.6.3 The PBL process and conditions

3.6.3.1 Learning should be a constructive process

Dolmans et al. (2005, p. 732) point out that “the constructive learning principle emphasises that learning is an active process in which students actively construct or reconstruct their knowledge networks”. In PBL, learners are provided with an opportunity to construct their own knowledge, as they actively work out the solution to a problem. When learners develop methods for constructing their own procedures, they are integrating their conceptual knowledge with their procedural skills (Kyeong Ha, 2003; O'Connell, 2007).

The limitations of the conventional methods of teaching mathematics are linked to teacher-oriented teaching, and to the ‘ready-made’ mathematical knowledge that is offered to learners who are not yet receptive to the ideas. In these conditions, the learners are likely to reproduce the procedures without having a deep conceptual understanding of the problems involved (Austin, 2001; Kyeong Ha, 2003). When mathematical knowledge or procedural skills are taught before the learners have conceptualised their meaning, the learners’ creative thinking skills are likely to be stifled by the instruction concerned.

3.6.3.2 Learning should be a self-directed process

The concept of SDL was initially asserted by the American adult educator Knowles (1975), who gradually reconceptualised self-directedness as being a contextually determined mode of learning ranging on a continuum from dependence to independence (Brookfield, 2009). SDL implies that the learners play an active role in planning, monitoring and evaluating the learning process concerned (Dolmans et al., 2005). In PBL the learners plan ways in which to approach the problem at stake, and they choose which strategies to adopt to solve the problem. The learners are involved in planning as they anticipate what ought to be done next, and as they look backward and forward. On solving the problem, they evaluate the situation, as they reflect on it, and return to the problem to check whether their solution makes sense.

Although some aspects of SDL do not apply at the Foundation Phase level, namely that the learners are able to set own learning goals, and to decide what is worthwhile learning, the learners are, however, able at this level to decide on how to approach a problem. Being at the dependence end of the SDL dependence to independence continuum, learners in the Foundation Phase are likely to:

- develop into more effective learners;
- become curious and willing to try new ways of approaching problems that they encounter;
- develop sufficiently to be able to view problems as challenges, to desire change, and to enjoy learning; and
- become motivated and persistent, as well as self-confident and goal-oriented (Abdullah, 2001; Morrow, Sharkey & Firestone, 1993).

For Foundation Phase learners coming through Montessorri preschools, SDL is actually not be a new concept, as it is the main feature of the Montessorri programme (see Table 3.1 below).

Montessori

Main Features

- The theoretical basis is the philosophy and beliefs of Maria Montessori.
- The prepared environment supports, invites, and enables learning.
- The children educate themselves, with their learning being self-directed.
- There is a set of curriculum regarding what children should learn. Montessorians try to stay as close to Montessori's ideas as possible.
- The children are grouped into multiage environments.
- The children learn by manipulating materials, and by working with others.

Table 3.1: Main features of the Montessori early-childhood programme (Morrison, 2010)

In early childhood education environments, SDL comes in the form of self-directed play, in terms of which there is clear separation between self-directed play and the work of the teacher, in which children may join. In the self-directed play, the children are the 'masters' in directing their own play, whereas the teacher has the role of preparing the environment, and of safeguarding the space and time for play, but not of becoming involved (unless a situation requires intervention for the purpose of safeguarding play and protecting the children) (Stipek & Byler, 2004). Hence, during PBL in the Foundation Phase the teacher gives the learners the problem to solve, but does not become involved in the problem-solving process. Instead, the teacher only intervenes when the situation requires them doing so for the purpose of facilitating the solving of the problem.

3.6.3.3 Learning should be a collaborative process

According to Jarvela, Volet and Jarvenoja (2010) and Dolmans et al. (2005), collaboration is a social structure, in which two or more people interact with each other and, in some circumstances, some types of interactions occur that have a positive effect. Collaboration is not a matter of division of tasks among the learners, but it involves mutual interaction, and the shared understanding of a problem (Kinch, 1998). In PBL, learners have a common goal of finding a solution to the problem put before them. Having worked together through collaborative discussions, the learners have to reach agreement on their final solution. The learners learn through "elaborations, verbalisations, co-construction, mutual support and

criticism and tuning in cognitively and socially”, with their interactions having the potential to influence learning positively (Dolmans et al., 2005, p. 733).

3.6.3.4 Learning should be a contextual process

Learning always takes place in a context. PBL, therefore, creates contexts that allow for “deeper and richer understanding and better use of knowledge” (Dolmans et al., 2005, p. 733). The objective of engaging learners in problem-solving involves not just solving specific problems, but also encouraging the exteriorisation and reorganisation of the schemes involved, due to the activity (Taplin, 2011). In addition to developing the learners’ confidence in their own ability to think mathematically, PBL is a “vehicle for students to construct, evaluate and refine their own theories about mathematics and the theories of others” (Lester, Masingila, Mau, Lambdin, Dos Santon & Raymond, 1994, p. 154). Barrett (2005, p. 57) points out that “problem-based learning is problem-based *learning* not problem-based *teaching*. It fits into the learning paradigm not the teaching paradigm and is part of a set of student centred approaches”. This implies that the teacher using PBL does not focus on what and how they are teaching, but rather on the learning of the learners.

3.6.4 Roles and procedures in PBL

Usually a class is split up into groups of approximately five learners each (Rhem, 1998, p. 3). The groups’ membership generally remains the same throughout the term. The teacher needs to know how to work with groups (as well as how to educate groups how to work with one another). The teacher has to know how to guide, without seeming to be unnecessarily keeping the answer hidden. The teacher also has to pose authentic problems, meaning that the problems should, to a certain extent, be open-ended.

Lester et al. (1994, p. 154) also point out several characteristics of PBL that the research has classified into teacher characteristics and learner characteristics (see Table 3.2 below).

Teacher characteristics	Learner characteristics
The teachers provide just enough information to establish background/intent of the problem.	The learners clarify, interpret, and attempt to construct one, or more, solution processes.
The teachers accept right/wrong answers in a non-evaluative way.	Interactions exist between learners and learners, and between the teacher and the learners.
The teachers guide, coach, ask insightful questions, and share in the process of solving problems.	Mathematical dialogue and consensus exists between learners.
The teachers know when it is appropriate to intervene, and when to step back and let the learners make their own way.	The learners make generalisations about rules and concepts, which is a process that is central to mathematics.

Table 3.2: Characteristics of problem-based learning (Lester et al., 1994, p. 154)

The striking differences between a problem-based approach and the traditional approaches to teaching mathematics are illustrated by the above characteristics that mark the role of the teacher. Rather than being the provider of knowledge, the teacher facilitates the construction of knowledge by the learners through guiding, coaching, and asking insightful questions. The teacher plays a less active role in a PBL lesson than in a conventional lesson, with their role in the former being one of stepping back and letting the learners make their own way. In contrast, the learner is the more active participant in a PBL lesson, unlike in a lesson in which the traditional approach is taken, with the learner playing a passive role of listening and watching the teacher show and demonstrate. In relation to PBL, the above characteristics illustrate a great deal of learner activity, in the form of clarifying, interpreting, engaging in mathematical dialogue, generalising, and interacting with the other learners and the teacher.

3.6.5 Cognitive effects of PBL

The attainment and organising of knowledge in PBL is believed to work through certain cognitive effects: the initial examination of the problem and the stimulation of prior knowledge through small group discussion; the explanation of prior knowledge and the vigorous processing of new information; the rearrangement of knowledge, involving the building of a semantic network; social knowledge construction; learning in context; and the stimulation of curiosity related to the presentation of a relevant problem (Borokhovski & Bethel, 2010; De Jong, 2010; Schmidt, 1993).

As observed above, the traditional method of talk-and-chalk or show-and-tell, which is very common in teaching mathematics, is not successful in helping learners to gain conceptual understanding of the most basic mathematical concepts (Sahin, 2010, p. 276). The poor performance of many learners in mathematics has led many mathematics teachers to search for alternative ways of teaching and learning mathematics. In this regard, due to the success of PBL in medicine and engineering, particularly in regard to motivating learners, the method is a strong option (Raine & Collet, 2003; Sahin, 2009a, 2010; Sahin & Yorek, 2009).

The key component of PBL is posing a 'concrete problem' to learners to initiate the learning process (Barrows, 1998; Libeskind, 1977, p. 168; Sahin, 2010, p. 268). According to Walker and Leary (2009, p. 14), the components shown in Figure 3.8 below constitute the minimum standards of PBL.

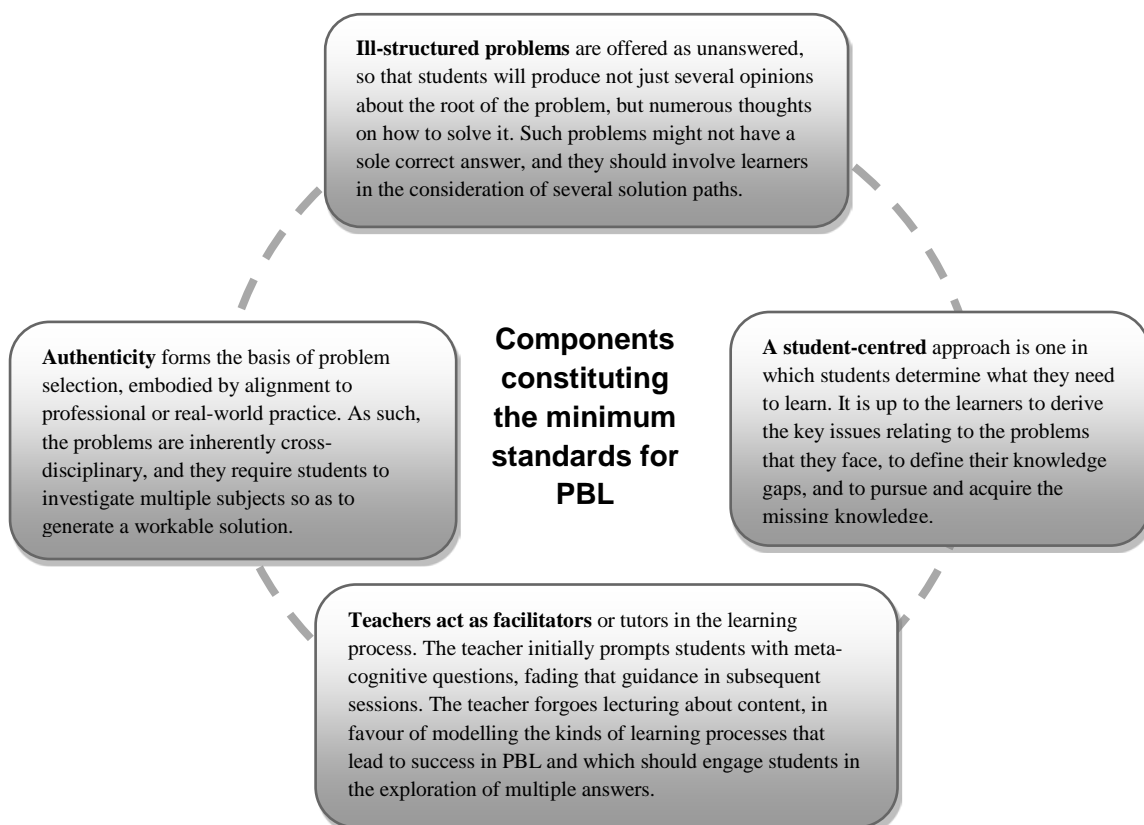


Figure 3.8: The components constituting standards for PBL (Walker & Leary, 2009, p. 14)

The researcher notes that, in terms of the 'student-centred' component, the learners are the ones who determine what they need to learn. It is up to them to derive the key issues relating to the problems that they face, to define their own knowledge gaps, and to pursue and acquire the missing knowledge. Within the Foundation Phase, such an approach would not be possible, meaning that the teacher would have to determine the knowledge gaps and

what the learners need to learn, as determined by the curriculum, or by CAPS, in the South African context. The teacher facilitates, through problem-solving, the pursuance and acquisition of the missing knowledge. The process of the pursuance and acquisition of the appropriate knowledge to fill the existing gap has to be child-centred. Rather than achieving this through the traditional teacher-centred approaches, the teacher does it through learner activity that involves clarifying, interpreting, having mathematical dialogue, generalising, and interacting with other learners and themselves, thereby making the learning process child-centred.

3.7 REASONS FOR THE PARADIGM SHIFT TOWARDS PBL

Why PBL has achieved its present success, and whether it will survive long enough to become 'the' way of teaching mathematics is debatable. The following subsections provide five reasons that are advanced as to why the use of PBL is growing so rapidly (Chapman, 2008; O'Connell, 2007).

3.7.1 Fostering the more positive attributes of learning in learners

In many ways, PBL was perceived to be the right response at a time when questions were being raised about problems with traditional methods of teaching mathematics. Those who want learners to learn to remember, to apply, and to continue to learn have often been disappointed by the results of the traditional methods, because, due to the adoption of such an approach, too many learners have come to memorise, to forget, and to fail to apply or integrate knowledge, ending up with a resistance to further learning. On the contrary, a PBL curriculum seems to foster the cultivation of the more positive attributes of learning in learners.

3.7.2 The success of PBL in the first few schools where it was attempted

PBL has been declared to be successful by both teachers and learners in the few schools in which it was first implemented. This professed success has been a factor in increasing the interest that has been expressed by those who have since heard about it. The successful implementation of PBL in the first few 'trial' schools led to PBL being adopted and implemented more widely in other schools.

3.7.3 The process of learning itself

Perhaps a more important reason is that which has to do with the process of learning itself. PBL, at least in its 'pure' implementation form, is in line with the learning theory that proposes that appropriate conditions for effective learning include the following:

- a learning atmosphere that is characterised by physical solace, common trust and appreciation, supportiveness, freedom of speech, and the tolerance of individual differences;
- learners identifying the learning objectives as their own objectives, and accepting shared accountability for planning and working to accomplish the relevant objectives, so that they become more committed to the learning process; and
- learners actively taking part in the learning process, and monitoring their own progress toward the achievement of the learning objectives.

The above conditions are all pertinent to the PBL group experiences, and to the whole atmosphere encompassing a PBL curriculum. Hence, PBL is a good match with conditions that are believed to facilitate learning, at least for the learners.

3.7.4 Consistency with current philosophical views

Constructivism assumes that

...knowledge is not an absolute, but is constructed by the learner based on previous knowledge and overall views of the world. Thus, the opportunity to find knowledge for oneself, contrast one's understanding of that knowledge with others' understanding, and refine or restructure knowledge as more relevant experience is gained, (all of which are done by learner in PBL curricula), seems to harness the reality of learning. The constructivist view of learning enables the adoption of PBL from pre-school to post-graduate training.

(Camp, 2010, p. 2).

Learning through problem-solving is consistent with current philosophical views of learning, particularly constructivism (Murray, Olivier & Human, 1998, p. 170).

3.7.5 Seizing the opportunity

Last, but not least, PBL is catching on as the approach that is desired by teachers. Therefore, schools that have not yet implemented PBL are enticed to take up PBL to identify with the other schools that already have, so as to avoid being seen as old-fashioned. Regrettably, however, such a rationale can lead to noncommitment to the whole-hearted implementation of PBL.

The above-mentioned reasons, which are alluded to by Camp (2010) and many others (Anderson, 2009; Artigue & Houdement, 2007; Clarke, Goos & Morony, 2007; D'Ambrosio, 2007; Doorman et al., 2007; Noddings, 1990; O'Connell, 2007) serve as a compelling case

for a paradigm shift to be made from the traditional knowledge transmission approaches of teaching *about* and/or *for* problem-solving to PBL, which involves teaching *via* problem-solving.

3.8 WHEN PBL IS NOT PBL

PBL, which was developed at McMaster University in Hamilton, Ontario, Canada, has spread to institutions beyond its original site. Adaptations have been made to account for differences between institutions, and for differences between the medical field (where it originated) and other fields, such as education (where it has become the substitute for the traditional approaches involved). Due consideration must be given to at what point the disparities become so drastic that the assertion can no longer be made that the PBL innovation still exists. The features that distinguish PBL from other approaches need to be determined.

The characteristics of 'pure' PBL (Camp, 2010, p. 3) are that it is:

- active;
- problem-centred;
- learner-centred;
- collaborative;
- integrated, and interdisciplinary; and
- applied in small groups.

When one considers the above characteristics, one can conclude that any teaching and learning is not pure PBL if it does not place learners in tutorial groups, and if it is teacher-centred, rather than learner-centred.

Often, teachers are hesitant to surrender their control of the learning process, resulting in PBL being implemented in a manner that keeps the teacher in control of what is learned, although the learning content is presented in packages, and the PBL is centred around small group discussions. A mathematics lesson that is developed for learners, and which consists of problem-stimulated learning, should not be regarded as PBL if it is not learner-centred (Brandt, Lunt & Rimmasch, 2012, p. 354; Camp, 1996, p. 3). It follows, therefore, that a mathematics lesson is not PBL when the following applies:

- It is problem-based, but not learner-centred.
- Much of the teaching is still in traditional formats, such as talk-and-chalk.
- The assessment of learner performance rests solely, or primarily, on content acquisition (Camp, 2010, p. 3).

PBL is an innovation that is definitely attracting much attention, for the reasons already discussed. It is becoming an alternative strategy for those who seek to improve learner understanding of mathematics, and to improve the teaching and learning of mathematics. In so doing, some teachers are likely to make increased use of the pure model while others will make all kinds of modifications which hybridise with traditional methods.

PBL will, without a doubt, change in its application from its original formation, because of both the new needs and the developments that are taking place in schools, and as a result of its poor, or lukewarm, implementation. Depending on whether the adaptations result in the continued improvement of the educational process for learners, or whether they are seen as 'we tried it, and it didn't work' attempts, will determine whether PBL is seen as a genuine paradigm shift, or as yet another one of those trends that comes and goes (Camp, 2010, p. 3).

3.9 THEORETICAL ADVANTAGES CLAIMED FOR PBL: AN ANALYSIS

One of the reported disadvantages of the PBL process is that it is a very different teaching process to the one that learners have already experienced. As a result, PBL can be stressful and disorienting (Mills, 2008). However, the following evidence that is behind some theoretical advantages that are claimed for PBL provide compelling reasons for adopting the method (Norman & Schmidt, 1992; Uden, 2006).

PBL facilitates the processing of new information: PBL leads to the activation of prior knowledge, which, in turn, facilitates the processing of new information, and elaboration, which enhances the use of knowledge. The activation of knowledge, and the subsequent facilitation of new knowledge, are closely linked to the key principles of constructive and collaborative learning (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

PBL stimulates the transfer of knowledge, as well as self-directed and lifelong learning: Learning in a PBL setting stimulates the transfer of knowledge. PBL also stimulates the other two advantages that are closely linked with the key philosophies of self-directed and contextual learning, which are self-directed and lifelong learning (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

Learners are better able to transfer concepts to new problems: A strong hypothetical premise exists in respect of the thought that PBL learners might be better able to transfer learnt concepts to new problems. There is some preliminary indication to this effect (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

PBL stimulates contextual learning: PBL invigorates contextual learning by increasing the transfer of concepts to new problems. This, however, does not intimate that the PBL curricula result in improved, generally content-free problem-solving skills, because the skills are content-specific, meaning that they are not independent of the acquisition of knowledge (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

PBL increases the retention of knowledge: There are indications that the group discussion in PBL results in the stimulation and explanation of prior knowledge, which facilitates the increased retention of knowledge (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006). Thus, PBL stimulates learners towards constructive and collaborative processes that influence learning positively

PBL enhances SDL skills: Such learning seems to improve SDL skills. PBL learners have been observed to make more frequent use of the library to access information, and to borrow more material from the library than do the learners who follow a traditional curriculum (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

PBL promotes the development of lifelong learning skills: PBL improves learners' lifelong learning skills. These incorporate communication and association abilities, research aptitudes, and, additionally, the capacity to handle issues, and to work in groups. The way that PBL challenges learners to learn through dynamic engagement with realistic problems makes learners remember what they have learnt for longer. The process of experiential discovery in which the learners take part also facilitates their reflection on their own thinking and learning processes. This greatly improves their understanding of a problem, since they are actively involved in the problem-solving process. All of the above advantages of PBL increase the growing enthusiasm of learners about PBL, and serve to raise their interest in the subject matter (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

PBL is an effective pedagogical approach: Considering all the above-mentioned advantages, the PBL procedure might be an extremely valuable pedagogical methodology, offering numerous useful impacts for the learners. As has been illustrated above, one of its greatest advantages is that it is an interdisciplinary strategy for learning. As a result, the departure from the more traditional system of learning, and from the traditional didactic mentalities, that PBL provides in all fields, enhances the professionalism of those involved (Barrett, 2005; Norman & Schmidt, 1992; Uden, 2006).

The literature review on effective teaching in Chapter 2 has shown that the use of a traditional old school model of learners passively learning facts and memorising them no longer suffices, for the simple reason that the approach fails to develop the full battery of

skills and abilities that are desired in a learner, and because the learner fails to retain much of what they have learned. In contrast, the literature just reviewed in Section 3.8 shows that PBL addresses the weaknesses of the traditional approach. All the above considered, the researcher was inclined to favour PBL, more so taking into account the comments that the teachers in the project schools he facilitated made in relation to their learners. The teachers complained that their learners did not remember what they had taught them, and that the latter did not show interest in their education.

3.10 TYPES OF KNOWLEDGE AND PROBLEMS IN PBL

The problems that the teacher presents to their learners in PBL should be based on the type of problem and knowledge that the former wants the latter to acquire (Rhem, 1998, p. 2). See Figure 3.9 below.

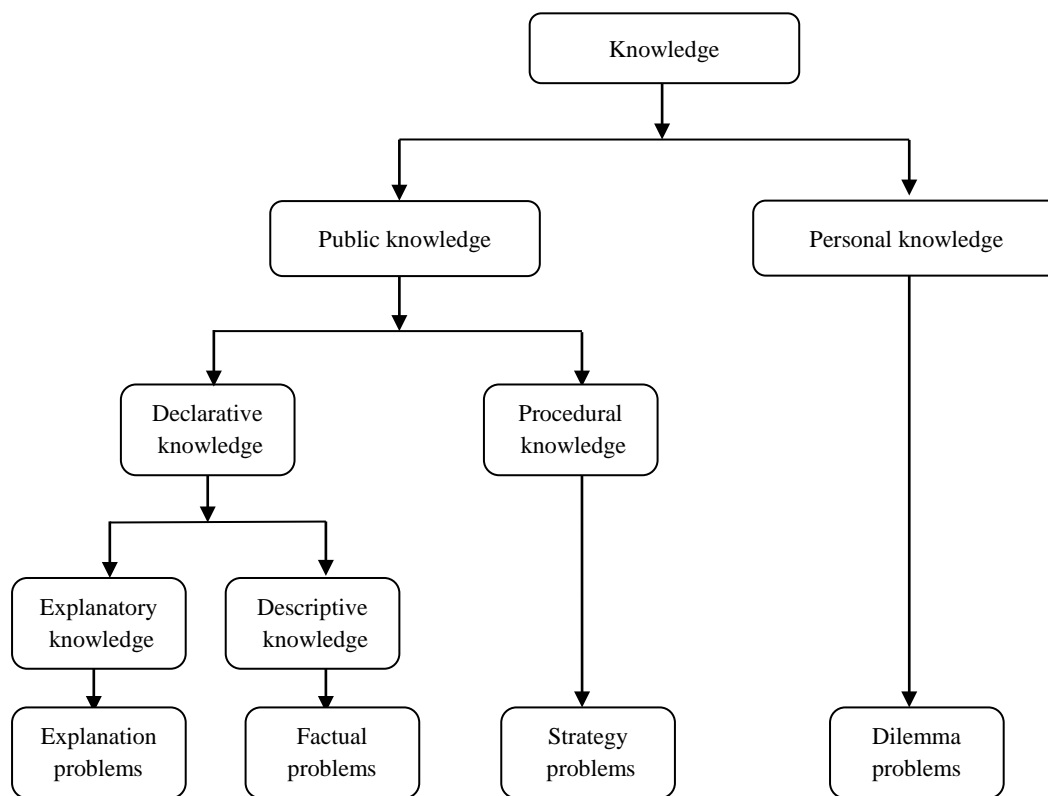


Figure 3.9: **The different kinds of knowledge and problems** (adapted from Rhem, 1998, p. 2)

3.10.1 Types of knowledge

Learners gain two categories of knowledge and four types of knowledge during the learning process. The two categories of knowledge are:

- **Personal knowledge** is made up of one's views, attitudes, opinions and values. The problems that are based on personal knowledge are those that focus on ethical issues.
- **Public knowledge** is knowledge that one can access from the different types of media, and which is accessible to all. Public knowledge can be divided into four groups, namely explanatory, descriptive, procedural and declarative.
 - **Explanatory knowledge** is the 'know why' type of knowledge.
 - **Descriptive knowledge** refers to knowledge of the facts.
 - **Procedural knowledge** refers to 'know how' knowledge.
 - **Declarative knowledge** refers to 'know that' knowledge.

The traditional approach to the teaching and learning of mathematics has, by its very nature, been heavily inclined towards procedural knowledge, in terms of which the teaching of mathematics, as has been discussed in Subsection 2.3.1, focuses on the transmission of procedural knowledge. In PBL, the provision of the correct type of problem provides an opportunity for learners to explore and construct all the forms of knowledge listed above. The next section, therefore, looks at how to construct problems for PBL. Such problems will ensure that the learners have an opportunity to construct all the forms of knowledge mentioned.

3.10.2 The construction of a problem

Problems for use in PBL are generally not found in traditional textbooks, hence, when adopting PBL for the primary school, designing effective problems is critical. The teacher should consider the following guidelines (Goodnough & Hung, 2008; Jonassen & Hung, 2008; Sahin, 2010; Savery, 2006; Sockalingam & Schmidt, 2011):

- 1) **Choosing the initial concept:** Select a central concept that is taught in a given course. Think of the end-of-chapter problem or assignment that is given to the learners to help them learn the concept. List the learning objectives that the learners should meet when they work through a problem.
- 2) **Real-world setting:** Think of a real-world setting for the concept that is under focus. Develop a narrative around the problem that will give a real-world feel during the development of the concept.
- 3) **Structure the problem:** As a problem is a basic part of PBL, it should be structured so that the learners can identify the learning issues concerned. The structure of a problem that is made up of a hook, a trigger, a scenario, and a problem brief is summed up in Figure 3.10 below.

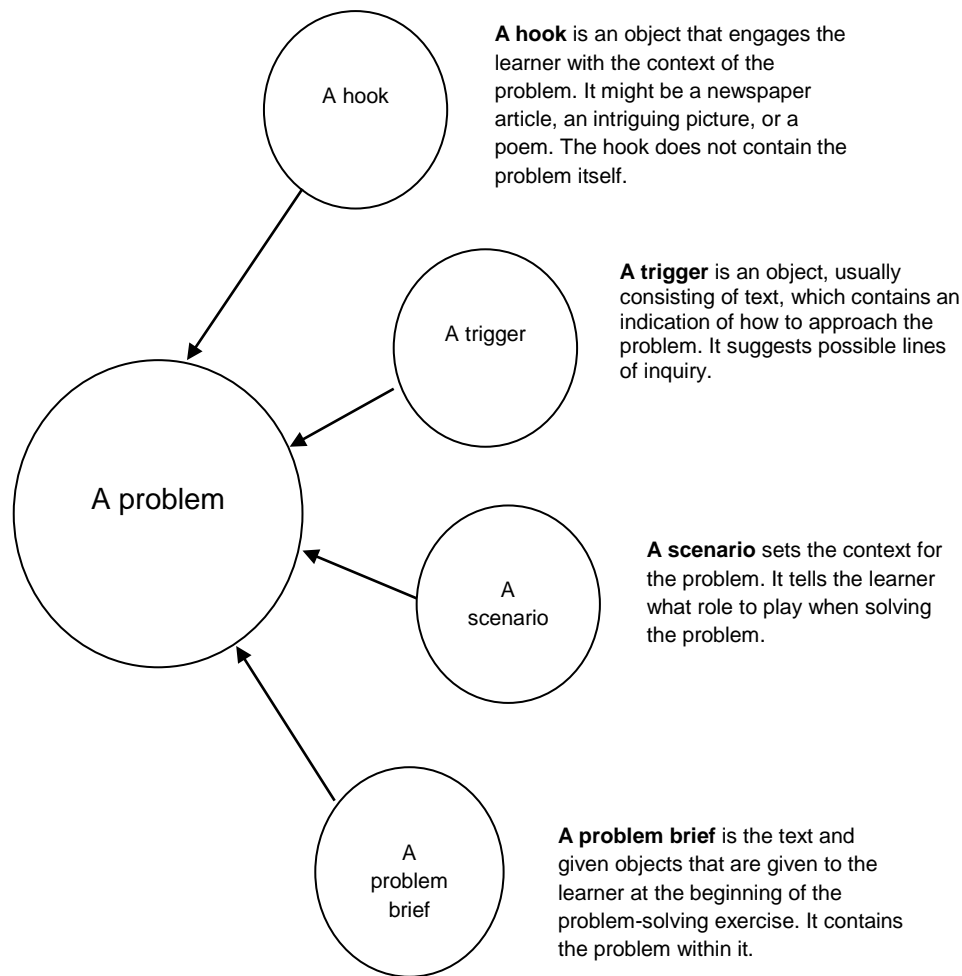


Figure 3.10: The structuring of a problem in PBL

A problem can be introduced in stages to the learners. Often problems in PBL are designed as multistage, and they may take the learners a week or more to complete.

- 1) **Write a teacher guide:** The teacher should compile a short guide detailing how they will use the problem, and how the given problem fits into the mathematics course structure.
- 2) **Identify resources for the learners:** While the learners are encouraged to identify the resources that they will need for solving the problem, it can also be helpful for the teacher to identify a few of the resources with which they can start problem-solving.

Jonassen and Hung (2008, p. 12) classify problems, and they show the level of structuredness of the various types of problems identified (see Figure 3.11 below).

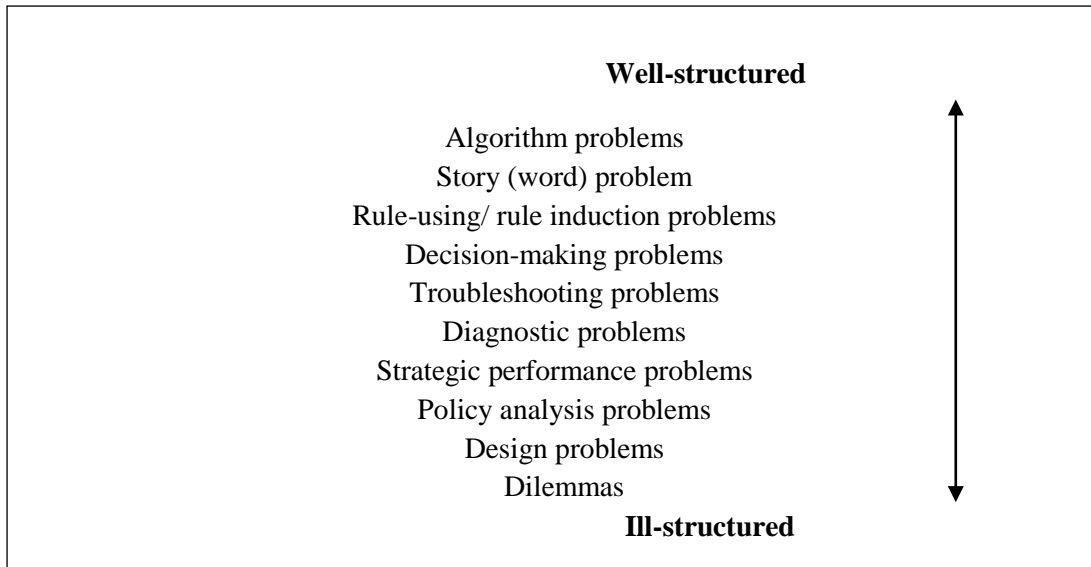


Figure 3.11: Typology of problem types (Jonassen & Hung, 2008, p. 12)

The problem type plays a critical role in the effectiveness of learning outcomes. The more ill-structured a problem is, the more likely it is to be effective in meeting the learning outcome. A good PBL problem inspires learners to solve it, and, through the process, they can construct their own understanding of it. The problem, thus, has to initiate the learners' learning process. The teacher, therefore, needs to construct or select a PBL problem that engages the learners, and that consequently achieves optimal learning. The following section looks at the characteristics of a good PBL problem.

A PBL problem is different from the traditional word problem, which is simply a translation of equations into mathematical sentences, and which requires the learners to ultimately focus on the mathematical operation (Chirico, 2009). Also, traditional word problems usually have only one right solution, and teachers usually teach only one approach to solving them.

3.10.3 Characteristics of a good PBL problem

The review of the literature (Duch, 1997; Goodnough & Hung, 2008; Portner, 2008; Sockalingam & Schmidt, 2011; White, 1997) regarding the characteristics of a good problem in PBL revealed several characteristics and their effects on the learner. The researcher has summarised such in Table 3.3 below.

<u>Nature of characteristic</u>		<u>Effects on the learner</u>
Realistic and resonates with the learners' experiences	➡	Supportive of intrinsic motivation.
Complex, ill-structured and open-ended	➡	The fostering of flexible thinking encourages the learners to develop higher level thinking and group collaboration skills than they otherwise might.
Affording of feedback	➡	Encouragement of the self-evaluation of the learners of the effectiveness of their knowledge, reasoning, and learning strategies.
Complex enough to require many interrelated pieces	➡	The learners' need to know and learn is motivated, and the targeting of desired learning outcomes is ensured.
Leads learners to generate hypotheses, and to defend them to others in their group.	➡	Facilitation of the public articulation of their current state of understanding, the enhancement of their knowledge construction, and the setting of the stage for future learning
Often requires multidisciplinary solutions.	➡	The use of knowledge and skills from several content areas is required to solve the problem.
Necessitates the gathering of knowledge from a wide range of sources.	➡	The learners come to see how knowledge is a useful tool for problem-solving.
Relates to the real-life world.	➡	The understanding of content is facilitated.

Table 3.3: Characteristics of a good problem

A study that examined learners' views of the characteristics of good problems used in PBL (based on their own experiences), divided the above characteristics into two categories – the feature characteristics and the function characteristics (Schmidt & Sockalingam, 2011). The following subsection presents and discusses the two categories.

3.10.4 Feature and function characteristics of problems

Feature characteristics refer to characteristics that are “the design elements of the problems...such as problem format, clarity, familiarity, difficulty, and relevance (application and use)” (Schmidt & Sockalingam, 2011, p. 22).

In contrast, function characteristics refer to the potential, or desired, outcomes resulting from working on the problems, for example the extent to which the problem stimulates critical reasoning. These functional characteristics are reflective of the five principles of constructivist learning and the objectives of PBL (Schmidt & Sockalingam, 2011, p. 22). Figure 3.12 below shows the classification of the feature and function characteristics.

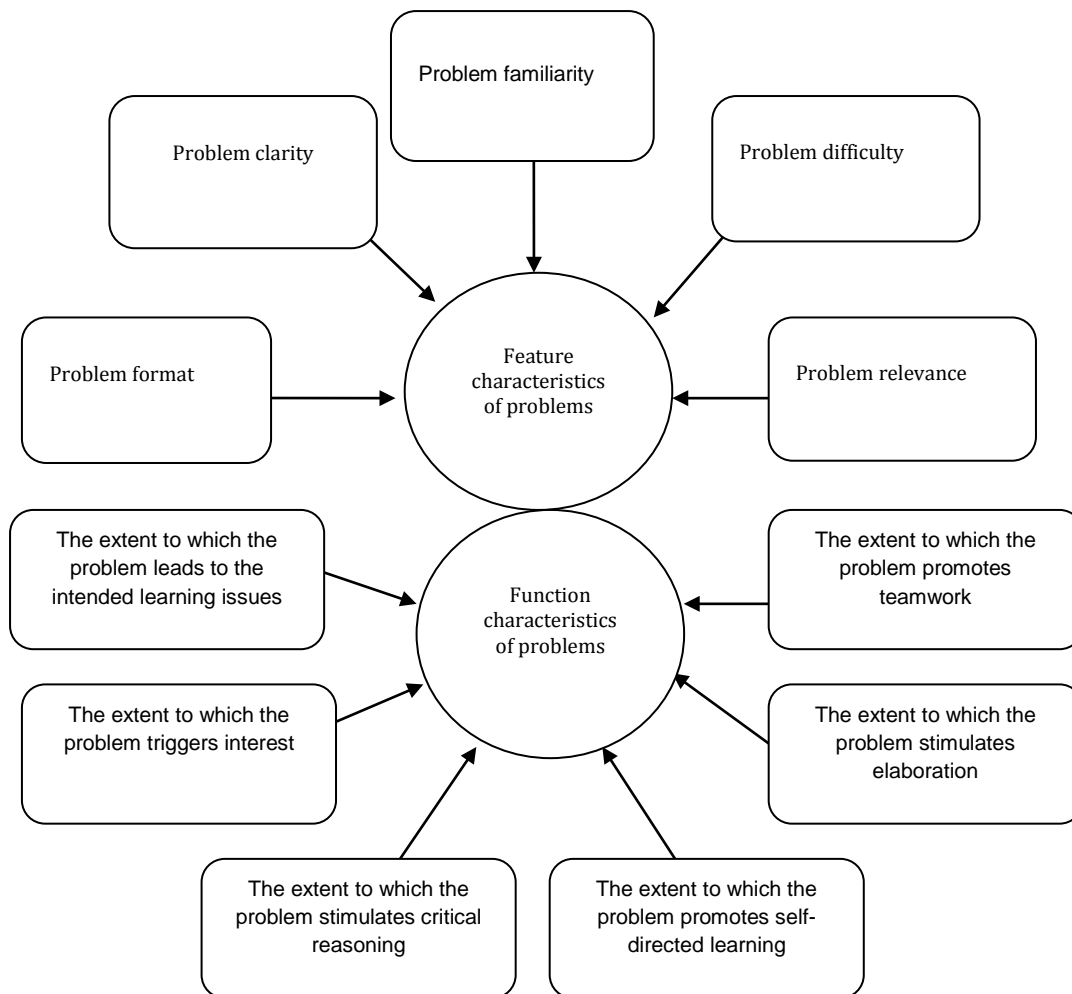


Figure 3.12: Functions and feature characteristics of problems (Sockalingam & Schmidt, 2011, p. 21)

The teacher needs to consider the above characteristics when assessing the effectiveness of PBL problems. The functional characteristics of a problem (as discussed above) are likely to be suitable pointers of its effectiveness, as they address the PBL goals concerned.

Problems are considered to be one of the three key components of PBL; the other components are the learners, and the teacher. The following subsection discusses the role of the teacher as the third key element of PBL.

3.10.5 The role of the teacher in PBL

Although good problems are vital, they are not the sole condition for viable PBL. The teacher's role is also essential in rendering PBL effective. In PBL, the teacher is a master learner, who is able to model good learning and thinking strategies, rather than a master of the content itself. In this way, the teacher's part is one of a facilitator of the learning process,

instead of that of a supplier of information. Hmelo-Silver (2004, p. 244) identifies five roles of a teacher in PBL that the researcher has summarised and illustrated in Figure 3.13 below.

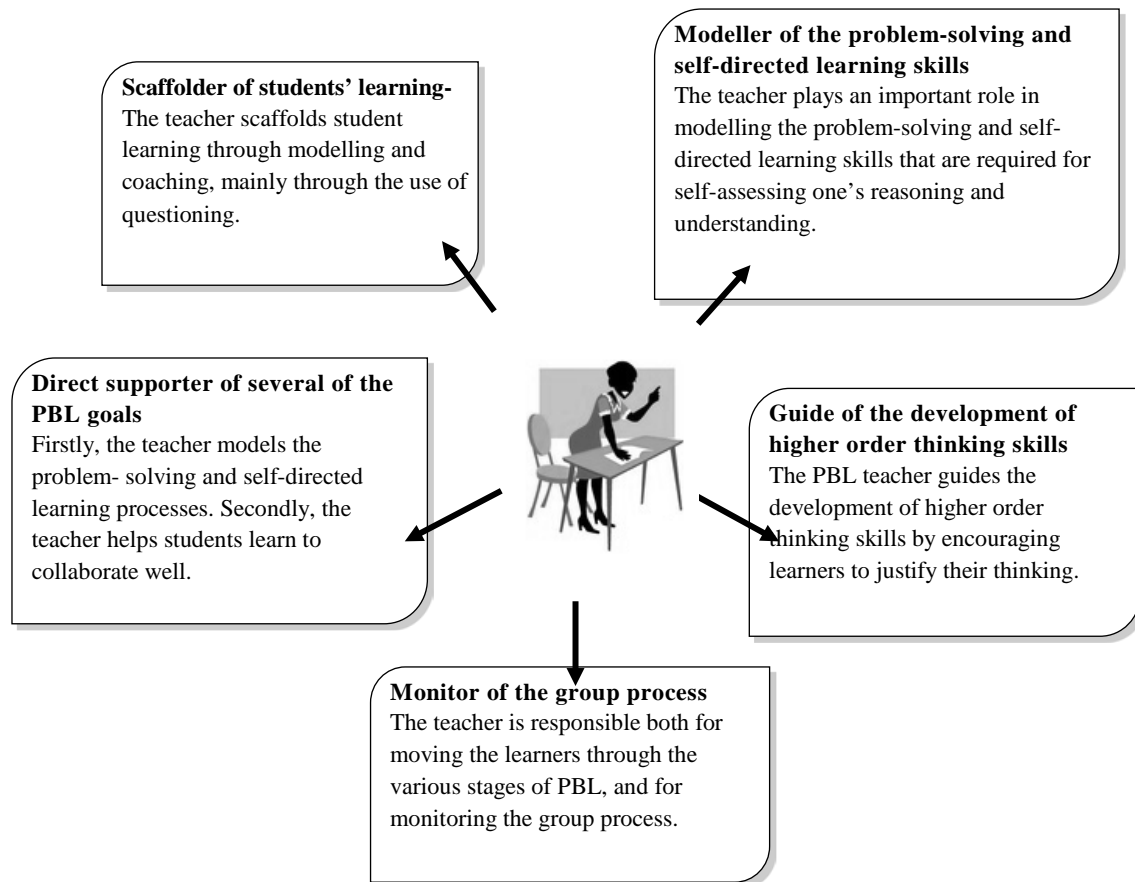


Figure 3.13: The five roles of the teacher in PBL

Hmelo-Silver (2004, p. 244) emphasises that the teacher needs to progressively fade their scaffolding role, as the learners become more experienced with PBL, until such time as the learners adopt many of the teachers' roles. The teacher rotates from group to group, adjusting the time spent with each of the groups in the classroom, according to the latter's needs. Facilitation, which is a subtle skill, involves knowing when an appropriate question is called for, when the learners are going off track, and when the PBL process is stalled (Hmelo-Silver, 2004).

In the above literature, one notes that the PBL teacher abandons direct instruction, and the learners assume greater responsibility for their own learning than they would otherwise do. The dilemma for the teacher is to provide affordances for constructive processing (Hmelo-Silver & Barrows, 2006). The teacher's role becomes one of a resource aide, and a group mentor. This organisation stimulates group information processing, rather than dispensing of information by the teacher. The teacher inspires and motivates the learners to be

participants in the new learning environment, backing the learners to become more self-directed in their learning. The teacher becomes a content and procedural asset, who enables group processes, and guides the learners to additional learning materials, as well as becoming a sounding board for the learners. The PBL teacher encourages learners' self-direction in the learning process, and raises questions that inspire the learners to be involved in discussion, instead of directing learners in how to solve problems. The teacher does not become much involved in the learners' discussions, but, rather, seeks to inspire the individual learner to participate in group work, desisting from either lecturing or directing the learners. Essentially, the PBL teacher provides the learners with learning guidance only when it is desirable to do so. Their guidance does not provide a solution to the problem on which the learners are working, but it involves giving additional clarifications to inspire learners to work towards possible solutions to the problem.

3.10.6 The three-phase PBL lesson format

Hartweg and Heisler (2007) elaborate on how they use the before, during and after threepart PBL lesson format in a year-long professional development project involving the integration of problem solving into the third-grade mathematics curriculum and implications for learning and teaching Foundation Phase mathematics. The PBL lesson format elaborated is different from a non PBL lesson format. The PBL lesson consists of three parts: before, during and after as shown in figure 3.14

BEFORE	DURING	AFTER
Getting ready <ul style="list-style-type: none"> • Activate prior knowledge. • Be sure problem is understood. • Establish clear expectations. 	Students work <ul style="list-style-type: none"> • Let go • Listen actively. • Provide appropriate hints • Provide worthwhile extensions 	Class Discussion <ul style="list-style-type: none"> • Promote maths community of learners. • Listen actively without evaluation • Summarise main ideas and identify future problems.

Figure 3.14: The three-phase structure for PBL lessons (Hartweg & Heisler, 2007)

The before phase refers to what the teacher does with the learners before they enter into the actual problem solving and not before as in lesson planning and preparation. At this stage 99 the teacher, through carefully chosen activities, activates specific prior knowledge that provides the foundational knowledge to start embarking on the problem. It is critical that the teacher ensures that the learners understand the problem during this phase before setting them to work on the problem.

A teacher with a deep traditional approach might find it difficult to let go in the 'during' phase. The teacher needs to listen actively to understand the learners' approach to solving the problem, and to provide hints where the learners are encountering difficulties, rather than to provide hints that serve as starters for them finding their own solutions. The value of the learners solving a problem in this manner is that it shifts the value system from the answers to the processes and thinking involved (Hartweg & Heisler, 2007).

The value of the 'after' phase lies in its creation of an opportunity for the learners to work as a community of learners, and for them to discuss, justify and challenge the various solutions to the problem that are presented. By so doing, the teacher creates yet another learning opportunity, as the learners reflect as a class, or as individuals, on the ideas that they have presented (Hartweg & Heisler, 2007).

3.11 MODELS OF PBL

Arguably, across the various PBL literatures (De Graaf & Kolmos, 2003; Duch, 2001; Irby, 1996; Newman, 2005; Tennessee Teaching and Learning Centre, 2010), five models of a PBL can be distinguished that may be utilised when opting to incorporate PBL into classroom teaching. Which model is utilised depends on the subject, on the difficulty of the problem to assess, on the size of the class, on the course level, and on the available resources.

3.11.1 Medical school model – In terms of this model, the learners are split up into groups of 8 to 10 with one faculty leader per group to assist in the discussion, as a group discussion is a primary school activity. This model is recommended most for upper level, seminar-type classes. The model is a good choice for highly motivated experienced learners.

3.11.2 Floating facilitator model – In this model, it is best to limit group sizes to four or five students. The instructor, in acting as a floating facilitator, moves from group to group to check for student understanding. The facilitator model, which is a generally structured format, has a greater degree of instructor input in the learning issues and resources than do the other models presented here. Other activities within this model are group reporting, whole-class discussion, and mini lectures. This model is a good choice for the less experienced learner, and for classes of all sizes.

3.11.3 Peer tutor model – This model, despite being similar to the medical school model, instead of a faculty or graduate student, uses an undergraduate peer as a tutor/leader to the group. The peer tutor helps in monitoring group progress and dynamics, as well as serving as a role model for novice learners. The model is a good choice for all class sizes.

3.11.4 Large class model – PBL can work in large classes, as long as they are instructor-centred. In this case, the instructor acts as a discussion leader, generating questions, and guiding students in ranking learning issues and reporting results, as well as in sharing resources.

3.11.5 Hybrid PBL – This model consists of the non-exclusive use of problem-driven learning in a class. The model may include lecture segments, or other active learning components. Floating or peer facilitating is common in such a model. This model is often used as entry point into PBL in course transformation processes.

In this study, the floating facilitator model was chosen as the model that was applicable to the Foundation Phase as it allows the teacher who, in this case, is the facilitator to move from group to group to check for learner understanding. This model is most appropriate for use in the Foundation Phase, as it allows for a relatively structured format, and for a greater degree of teacher input than usual in relation to the learning issues and resources that are required, considering the level of the learners involved. Other activities that are viable in terms of this model are group report out, whole class discussions and mini lectures. As this model is a good choice for use with less experienced learners, it is the most appropriate option for learners in the Foundation Phase.

3.12 ASSESSMENT IN PBL

Assessment is a valuable tool for making instructional decisions, and for enhancing student learning (Leatham, Lawrence & Mewborn, 2005). The assessment, which assumes an imperative part in PBL, could be a multifaceted action and a key element, influencing the manner in which learners learn and react to teaching. Teachers, peers, and self-assessment should all be appropriately involved in the final assessment.

Assessment, as a vital piece of the learning methodology, necessitates redirecting attention from teaching approaches that define learning in terms of separate morsels of information that are distributed and passively absorbed, to approaches that create environments for the learner to make meaning, and to understand the world around them (Van Glaserfeld, 1996).

Widely varied methods have been used to assess students' learning in PBL, from "traditional multiple-choice exams and essay exams to new assessment techniques such as case-based assessment, self- and peer assessment, performance-based assessment, and portfolio assessment" (Gijbels, Dochy, Van den Bossche & Segers, 2005, p. 32).

3.12.1 The reasons for learner assessment

The main fundamentals of assessment are, firstly, to support learning through engaging learners in learning activities and through providing feedback; secondly, to measure learning against the stated learning outcomes; and, finally, to assure the standard of awards (Macdonald, 2005).

Assessment that focuses on the learners' choices of correct or incorrect solutions is not suitable for use in PBL. Lafer and Tchudi (1996) claim that traditional assessment is a game that asks the learner to guess what the teacher wants, rather than a means of urging them to accomplish the best that they possibly can. It is problematic to use traditional tests in assessing such PBL learner outcomes as problem-solving skills, critical thinking, creativity, SDL, teamwork, and communication, as such tests are basically designed to enable knowledge assessment to take place.

A widely used approach to assessment in PBL is that of **authentic assessment**, which is generally categorised into the following types of assessment (Tai & Chan, 2007):

- **Performance assessments** test the learners' capability to apply acquired knowledge and skills in a diversity of realistic circumstances, and to work collaboratively to solve complex problems.
- **Portfolio assessment** includes creating a portfolio that archives learning over time. The portfolio can consist of diary entries, peer reviews, artwork, diagrams, charts, graphs, multimedia presentations, group reports, learner notes and outlines, rough drafts, and example of more polished writing.
- **Reflection and self-assessment** requires learners to reflect on, and to assess, their own contribution and learning progress, which are characteristics that form an essential part of SDL.
- **Peer assessment and self-assessment** require learners to assess one another (O'Grady & Choy, 2008), and to judge the nature of their own work. While there is some deliberation about whether peer assessment and self-assessment are feasible in PBL, it is generally acknowledged that helping learners to create an understanding and judgment of the nature of their own work and that of others is to be much desired (Woods, 1994).

An effective PBL assessment and evaluation programme can ensure that learners are gaining the maximum benefits that they can from engaging in PBL, and that the PBL process itself is being implemented effectively.

3.13 CRITICISMS AND MISCONCEPTIONS ABOUT PBL

As is the case with all learning theories, there are critiques of, and misconceptions relating to, the implementation of PBL in the primary school. Since this approach, which began in the arena of medical education, began to spread to other disciplines, strong opinions have been expressed, and questions have been raised, about the wisdom, effectiveness and educational efficiency of such learning. Regrettably, many of the criticisms have been based on emotion, tradition, and faculty perceptions and preferences, rather than upon a careful analysis of the benefits with which PBL can provide learners (Vernon & Blake, 1993). Some of the criticisms and misconceptions that have been raised against PBL are described in the following subsections.

3.13.1 The use of PBL results in low levels of academic achievement

Whereas few critics of PBL doubt the ability of learners who are schooled in the method to be able to exhibit strong reasoning powers, PBL learners have sometimes failed in terms of the recall of factual knowledge (Berkson, 1993). Concerns have also been raised over the breadth of content covered. Due to the focus of PBL on the solving of specific problems, academic achievement scores often favour the use of traditional teaching methods when standardised tests are used, but they favour neither method when nonstandardised forms of assessment are employed (Vernon & Blake, 1993). Such measures include the evaluation of problem-solving ability, interpersonal skills, peer relationships, the ability to reason, and self-motivated learning (Albanese & Mitchell, 1993; Vernon, 1995), as well as the assessment of learners' knowledge content. Although the use of PBL tends to reduce performance at the initial levels of gaining knowledge, it tends to improve long-term retention (Farnsworth, 1994). Such criticism will be strongest among the instrumentalists who are content-focused, with an emphasis on performance skill mastery, and on the passive reception of knowledge.

3.13.2 PBL is time-consuming

It is true that PBL takes time, but it is time well spent. A problem in PBL is not meant to 'cover' a long list of contents and skills, but it is meant, rather, to teach selected important content in greater depth than might otherwise be possible. The key is to design a problem well, so that it aligns with the curriculum objective, and to manage it well, so that the amount of time that is available is used efficiently. Not all problems need to take several lessons to

complete -- some can be solved in a single lesson. A teacher's approach also does not have to be continuously PBL – the solving of even one or two PBL problems in a week is better than none. Some teachers are concerned that the planning of PBL takes too much time. PBL requires significant advance preparation, but planning for PBL becomes easier the more that one applies it. One can also save planning time by means of collaborating with other teachers, in the sharing of PBL problems, in the adapting of PBL problems from other sources, and in using the same PBL problem with other classes at a later date.

3.13.3 Role of the learner

An unanticipated problem with the introduction of PBL is the traditional assumptions that are made by the learner about what teaching and learning mathematics entails. Most learners would have spent their previous years assuming that their teacher was the main disseminator of knowledge, or they might initially have come to school with that assumption, in the case of learners in the Foundation Phase. Because of this orientation towards the subject matter expertise of their teacher, and the memorisation of facts that has traditionally been required of learners, many learners appear to have lost the ability to "simply wonder about something" (Reithlingshoefer, 1992). Such an outlook is especially seen in Foundation Phase learners who have difficulty with SDL (Schmidt, Henny & De Vries, 1992).

3.13.4 Role of the teacher

The teachers who implement PBL need to change their traditional teaching methods of lecturing, holding discussions, and asking learners to memorise material for tests. In PBL, the teacher acts more as a facilitator than as a disseminator of information. As such, the teachers concerned tend to focus their attention on questioning learners' logic and beliefs, on providing hints to correct erroneous learner reasoning, on providing resources for learner research, and on keeping learners on task. Because this role is foreign to some teachers, they might find it difficult to break away from their past habits.

3.13.5 The creation of appropriate problems

Creating the best possible problems is the most problematic part of PBL. Without problems that have the feature and function characteristics, as discussed in 3.9.4 above, there is a good chance that important information will not be learned. In a study that correlated learner-directed study and curriculum objectives, it was found that learners did not stay on track, and many important objectives were omitted (Dolmans, Gijsselaers & Schmidt, 1992). If learners become distracted from the expected objectives during their problem-solving, they might completely miss out on the main content, if they are not redirected by their teacher (Mandin, Harasym & Watanabe, 1995). Problem formulation is a science and a skill that can either be

present, or not. This criticism is highly applicable in the Foundation Phase, in which the learners have yet to acquire SDL skills. At this level, the teacher needs to be an active facilitator in the consideration of ways in which to approach the problem at hand, and in the choosing of strategies to solve a problem.

3.13.6 PBL requires having access to resources

PBL is grounded in such resources as parents, teachers, libraries, friends, the internet, computers, and smart phones, among others. Although the implementation of true PBL requires the use of all these resources, schools should not hold back from implementing PBL, since, ideally, the aforementioned resources are required no matter which instructional approach is adopted. The literature review has shown that PBL can be a successful approach to the teaching and learning of mathematics with understanding, because it can be implemented in under-resourced schools, starting with the professional development of the teachers involved, and being strengthened by means of ongoing classroom support. Extra resources can enrich the process concerned, but their unavailability should not be an excuse for not implementing PBL.

3.13.7 PBL focuses on critical thinking and collaboration, at the expense of content knowledge and academic skills

In well-designed PBL, learners gain content knowledge and academic skills, as well as learning how to solve problems, how to work in groups, how to think creatively, and how to express their ideas. When constructing a PBL problem, the teachers should align the problem with the targeted curriculum objective, and they should use rigorous assessment practices to record the evidence of achievement. PBL links the development of critical thinking skills to certain content, because the learners involved need something to think about critically. Critical thinking can, therefore, not be developed independent of content.

3.13.8 PBL is only for older learners

Foundation Phase learners can benefit from engaging in authentic PBL problem-solving, just as much as high school learners can do. Although the teachers of Foundation Phase learners might have to manage a PBL problem differently with younger learners, PBL can be implemented successfully in the phase. Solving content-rich PBL problems helps to build background knowledge that influences the understanding of mathematics positively. PBL can increase the learner motivation to read, to write, and to learn mathematics, because the learners involved are engaged in solving problems, and so have an immediate, meaningful reason for learning and applying the skills. PBL is, therefore, effective even for young learners of mathematics, because the mathematics concerned is purposeful, as well as

being connected to personally meaningful experiences. Learners with learning difficulties also benefit from the peer interaction for which PBL allows. For students with disabilities, teachers can use the same support strategies during PBL as they would use in other situations, with such strategies including differentiation and modelling, as well as the providing of more time, and scaffolding. Since PBL involves working in small groups, it gives teachers more time and opportunities to meet individual learning needs than they might otherwise have.

3.13.9 PBL does not fit with the current CAPS curriculum and teaching style

Although some teachers might find the use of PBL 'disordered', as, with the use of such a method, they are not in total control of their learners every step of the way, they can still use PBL group management practices to make the group work involved productive in terms of the amount of time available. It is important to train the learners how to work well in groups, as well as how to manage their time and tasks, and to be investigative. For teachers who are only used to direct instruction, it might be challenging, at first, to have to manage the learners' group work, and to have to cope with the open-endedness of PBL. However, with more experience, doing so should become easier than it might be at the beginning. Besides which, teaching in a PBL environment does not mean giving up all traditional practices; there is still room for the use of teacher-directed lessons, mini lectures, textbooks, and even worksheets.

3.13.10 The culture of traditional direct teaching is still strong in countries doing well in the conducting of international tests

Despite the fact that traditional direct teaching has in research studies been shown to be a less successful approach in the teaching of mathematics for understanding (Mji & Makgato, 2006; Muijs & Reynolds, 2000; Nkhoma, 2002; Orton & Forbisher, 1996), this approach can however be fruitful if the focus is on understanding. Examples can be found in some of the countries that are doing well in international tests but have educational systems that are shaped by cultures of traditional direct teaching. In Finland for example, both teachers and students adopt a traditional position, but highly qualified teachers come through a research-based teacher education system where reflection on own teaching is emphasised (Ahtee, Lavonen & Pehkonin, 2008). Despite the traditional roles of teachers and students, application of knowledge and problem solving skills are characteristics of the Finnish system, featuring alternative teaching methods like "Models from everyday life, Activity tasks, Mathematical modelling, Learning games, Problem solving, and Project work." (Ahtee et al, 2008: 22). Savola (2010 p.11) points out that "perhaps the most important reason behind the Finnish success story is the respect that the teaching profession enjoys within the society.

These cultural characteristics—the innate obedience and the relatively high esteem of teachers—have helped propel Finnish schools to the top.”

Many of the countries where mathematics is taught in a more traditional way are however now taking steps to transform their teacher-centred educational approach into learner-centred learning in order to enhance learning motivation and learning outcomes (Huichun Li & Xiang-Yun Du, 2013). The establishment of a more learner-centred approach requires restructuring the traditional teacher-directed teacher-centred relationship. The teacher-student relationship is not formulated simply in an educational context but is heavily imbedded in a particular social and cultural context (Huichun Li & Xiang-Yun Du, 2013). Hence the teacher-learner relationship can be regarded from both pedagogical and social point of view. Whether the student-teacher relationship is equal and democratic largely depends on the dominant situation of the interpersonal relationship in the society rather than the educational context (Huichun Li & Xiang-Yun Du, 2013).

The inferences that one can make from the above is that traditional direct teaching is successful in countries where the teaching profession enjoys respect within the society, and where teaching for relational rather than instrumental understanding is an important goal in the teaching of mathematics.

The above average learners survive the traditional approach to the teaching of mathematics better than the rest as they are able to construct their own knowledge or understanding from the teacher's examples, explanations or direct teaching. The average and below average learner however struggles to construct own knowledge and understanding based in these circumstances.

3.14 SUMMARY OF THE CHAPTER

In this chapter, problem-solving was discussed. From Section 3.5 onwards, PBL was discussed in detail, as the current study deals with PBL as an approach that is grounded in constructivism as the best alternative to the traditional approach to teaching mathematics. The traditional approach has not been successful in helping learners gain conceptual understanding of the most basic mathematical concepts. Most important in this chapter is the discussion of how PBL conforms to constructive learning principles that emphasise that learning is an active process, in which learners construct, or reconstruct, their knowledge networks. The chapter has discussed how, in PBL, learners are provided with an opportunity to construct their own knowledge, as they actively work out the solution to a problem. PBL is regarded as the preferred approach to the teaching and learning of mathematics in this

research study, as it is in line with the most current psychological conceptualisation of learning.

This research was driven by the desire to improve the teaching and learning of mathematics in the Foundation Phase of five schools taking part in an intervention. The data gathered in the investigation were used as the basis for intensive staff development. The following chapter concerns the professional development of mathematics teachers.

CHAPTER 4: PROFESSIONAL DEVELOPMENT

“Professional development for teachers is now recognised as a vital component of policies to enhance the quality of teaching and learning in our schools... As investment increases, policy makers are increasingly asking for evidence about its effects not only on classroom practice, but also on student learning outcomes. They are also looking for research that can guide them in designing programs that are more likely to lead to significant and sustained improvement in students’ opportunities to learn.”

(Ingavarsen, Meiers & Beavis, 2005, p. 1)

4.1 INTRODUCTION

Professional development can be defined as the improvement in the quality of services that are provided by an individual professional (Kester, Dengerink, Korthagen & Lunenberg, 2008, p. 567). Kennedy (2005, p. 236) presents a framework in which the main characteristics of a range of models of professional development are identified and categorised. This chapter looks at some of the key issues of professional development, in relation to which guidelines are provided for the professional development phase that was incorporated in this study. It was important that the professional development input be guided by a literature review to ensure that it was an effective intervention.

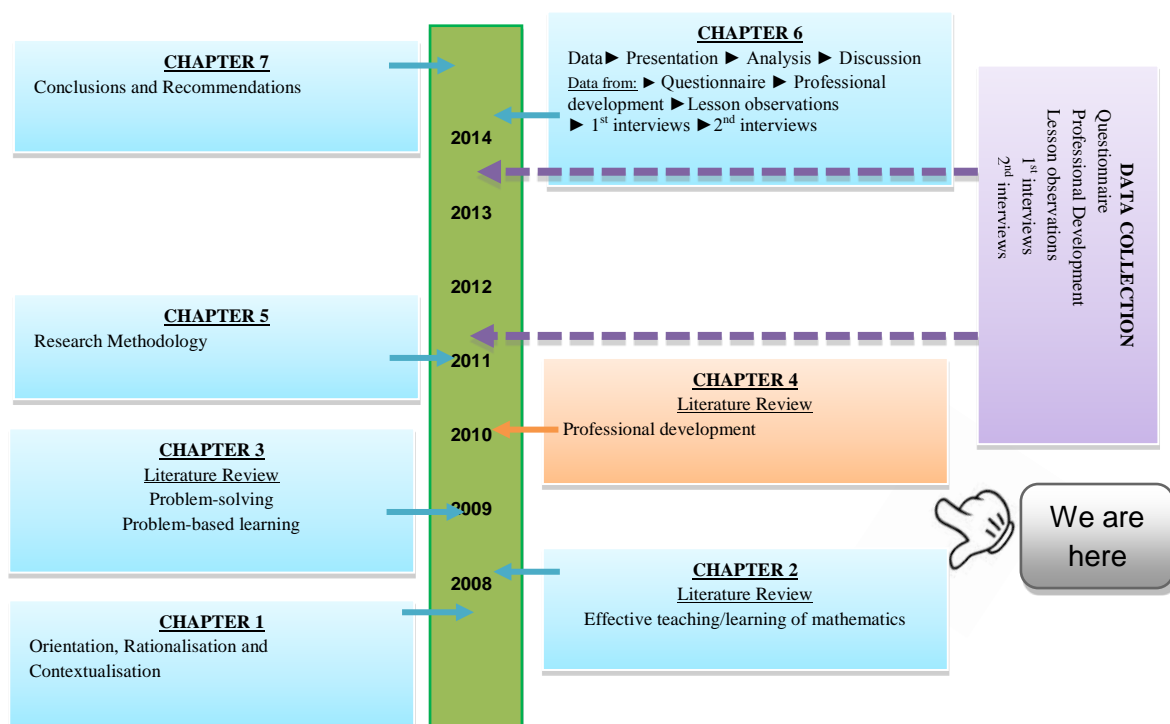


Figure 4.1: The research timeline

In the following section, the researcher considers the circumstances under which each particular model might be adopted, as well as exploring the form(s) of knowledge that can be developed through each particular model.

4.2 PROFESSIONAL DEVELOPMENT MODELS

The research took into account the nine key models of professional development (Hayes, 2000; Kennedy, A., 2005; Little, 1993; Thompson & Goe, 2009). Kennedy (2005, p. 236) discusses each of the models at length. Table 4.1 below summarises each of the models concerned.

MODELS OF PROFESSIONAL DEVELOPMENT	
The cascade model	The cascade model involves individual teachers attending training events, and then cascading, or disseminating, the information to colleagues. This model is normally used in circumstances where resources are inadequate.
The action research model	In the action research model, the participants themselves are involved as researchers, with a view to improving the quality of action within the research. The quality of action can be viewed as the participants' understanding of the circumstances, as well as the practice within the given circumstances.
The training model	This model has been the leading form of professional development for teachers. In addition to supporting a skills-based, technocratic view of teaching, it provides teachers with the opportunity to update their skills, so that they are able to demonstrate their competence. The training is generally delivered to the teacher by an expert, by means of a programme that is determined by the deliverer, and in terms of which the participants are placed in a passive role.
The community of practice model	The added value of learning in communities pertains to the viewing of the existence of individual knowledge, and to the combinations of several individuals' knowledge through practice as being a powerful site for the creation of new knowledge. A clear relationship exists between communities of practice and the mutually supportive and challenging form of the coaching/mentoring model.
The deficit model	In the deficit model, professional development can be designed specifically to address an alleged deficit in teacher performance. Such development may be set within the context of performance management as a means of raising standards, or as an element of intervention to achieve greater efficiency or effectiveness.
The standards-based model	The standards-based model represents a desire to create a system of teaching, as well as teacher education that can generate and empirically validate connections between teacher effectiveness and student learning. The model also relies greatly on a behaviourist perspective of learning.
The coaching/mentoring model	The defining characteristic of this model is the importance of the one-to-one relationship, which generally occurs between two teachers, and which is designed to support professional development. Both coaching and mentoring share this characteristic, although most attempts to distinguish between the two suggest that coaching is more skills-based than is mentoring, which involves an element of counselling and professional friendship.
The award-bearing model	This model depends on, or emphasises, the completion of award-bearing programmes of study that are usually, but not exclusively, validated by universities. This external validation can be viewed as a mark of quality assurance, but it can equally be viewed as the exercise of control by the validating organisations.
The transformative model	The transformative model of professional development involves the combination of a number of processes and aspects that are drawn from the other models outlined above. The central characteristic is the combination of practices and conditions that support a transformative agenda.

Table 4.1: Models of professional development for teachers (adapted from Kennedy, 2005, p. 236)

After carefully considering the above models of professional development, the researcher decided to adopt the transformative model by incorporating the following aspects from each of the models:

- 1) ***The training model:*** In terms of this model, the professional development that was provided to the teachers allowed them an opportunity to update their skills, in order that they might demonstrate their competence. Although the development was 'delivered' by the researcher, the participants were not placed in a passive role in relation to it.
- 2) ***The deficit model:*** The professional development was designed specifically to address a deficit in teacher performance. It was set within the context of improving the low mathematical performance of learners, as reflected by the ANAs and the Western Cape Education Department (WCED) systemic test results.
- 3) ***The cascade model:*** The individual teachers attending the professional development were encouraged to cascade, or to disseminate, the information to their colleagues.
- 4) ***The community of practice model:*** The researcher encouraged the development of communities of practice by the individuals who attended the professional development.

4.3 THE PROFESSIONAL DEVELOPMENT PHASE OF THE RESEARCH

During the project's professional development phase, the focus was on how the teachers could meaningfully implement PBL. The examples that were used during the professional development were related to the CAPS Foundation Phase, Mathematics grades R to 3, in order to make them relevant.

The continuation of support to teachers after professional development during the implementation stage has long been identified as a critical component of more effective professional development programmes (Corcoran, 1995; Elmore, 2002). The strongest criticism of many professional development programmes over the years has been the lack of built-in provision that has been provided for follow-up support for the teachers in their classrooms, as they apply new ideas and skills acquired during professional development (Ingvarson, Meiers & Beavis, 2003).

Thus, as part of the professional development follow-up support, the researcher visited the individual participants in their schools, where he interviewed them to determine their individual support needs in the following specific areas:

- in identifying problems that were suitable for use in PBL;
- in using problem-solving as a vehicle for learning; and
- in designing activities that provide learners with opportunities to solve problems, and to learn through problem-solving.

On-site follow-up support for the professional development participants was, thus, provided in their classrooms, as they applied the new ideas and skills in using PBL. The findings of the interviews, and the on-site 'elbow' support, are reported on in Chapter 6.

Active learning, in which the teachers have opportunities to become actively engaged in the meaningful analysis of teaching and learning, is one feature of successful professional development (Barko, 2004; Birman, Desimone, Porter & Garet, 2000; Garet, Porter, Desimone, Birman & Yoon, 2001). It is important that teachers are actively engaged in their own learning. Effective professional development programmes encourage teachers to analyse their current practice in relation to professional standards for good practice. In this research project, it was intended that the participants should be actively engaged in their own learning. This was made possible by means of the provision of activities that required them to reflect on and to analyse their current practices, in relation to the professional standards of good practices. Figure 4.2 below is an example of such an activity from the module that was used during the professional development.

First, let's do an exercise:

1. Individually, write down five words or short phrases that come to mind when you think of:

Student-centred learning
2. In pairs or small groups, select three 'most important' words/ short phrases.
3. Finally, report on just one.

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Figure 4.2: Extract from the professional development module

Feedback has long been recognised as being a vital requirement for professional development programmes that aim to help teachers develop new skills and integrate them into their practice. The effective integration of new skills requires programmes to have a clear theoretical foundation that is supported by research, modelled in real settings, and which allows opportunities to practise the new skills, and to receive feedback from a facilitator or colleagues. Feedback for participants as they practised the use of problem-solving as a vehicle for learning was provided as part of the professional development. The researcher observed the participants' lessons and provided feedback with regard to their choice of problems and their role as facilitators during the problem-solving process.

Ham and Davey (2010, p. 231) point out that today's professional development differs from traditional professional development in that "it is often charged with facilitating teacher (self)-inquiries into their own practices [rather] than...with 'teaching' new pedagogical techniques". While focusing on how problem-solving can be an effective vehicle for teaching, the researcher took into account the shift in professional development, factoring in the knowledge of the principles and practice of 'reflection' and 'critically reflective practice', and how these may empower teachers to improve their own practice (Ham & Davey, 2010, p. 233).

Bredeson (2000, p. 66) points out the indicators for assessing the quality of professional development activities and programmes, including professional development design, delivery, content, context, and outcomes. The indicators are helpful in assessing the connections between teachers' learning and understanding of content in their fields, and how they organise and help their learners deal with that content (Bredeson, 2000, p. 65).

Sheerer (2000, p. 31) suggests a model of professional development for in-service teachers and teacher educators, based on five guiding principles that are linked to particular theoretical positions espoused in the educational literature. These guiding principles and their theoretical underpinnings are presented below.

- Professional development models need to be designed by, and to impact on, both teacher educators and teachers, so as to ensure that the changes that are required in educational practice really do take place.
- Teachers should be viewed not only as implementers of reform initiatives, but also as investigators and problem generators.
- Models of professional development are needed that go beyond training, so as to allow teachers to act as well-informed critics of reforms.

- Professional development must be designed in ways that deepen the relevant discussion, as well as in ways that promote the building up of discourse communities, and that are supportive of innovations.

Taking the above matters into consideration, the researcher had pre-professional development discussions with the teachers during the planning of the professional development. The teachers were assisted with analysing the current learner performance by identifying areas of poor performance through the systematic analysis of both the ANAs and the WCED systemic test results. This assisted the teachers to become informed critics of learner performance, placing them in a position to be able, equally, to criticise the professional development in terms of how it met the need to address the issue of improved learner performance.

Figure 4.3 below shows the theory of change of Bruce and Ross (2007), which was developed in relation to a qualitative study of teachers experiencing professional development. At the centre of this theory is self-assessment, in which the teacher observes their effect on learner achievement, makes a judgment about how well they attain their instructional goals, and reflects on the level of satisfaction obtained thereby.

Teacher reflection is vital to teacher development. It is through reflection that the teachers generate knowledge that is grounded in practice, and that they become able to describe the extent of their improvement and personal effectiveness in facilitating the level of mathematics reform that is implemented in classrooms (Ricks, 2011, p. 251).

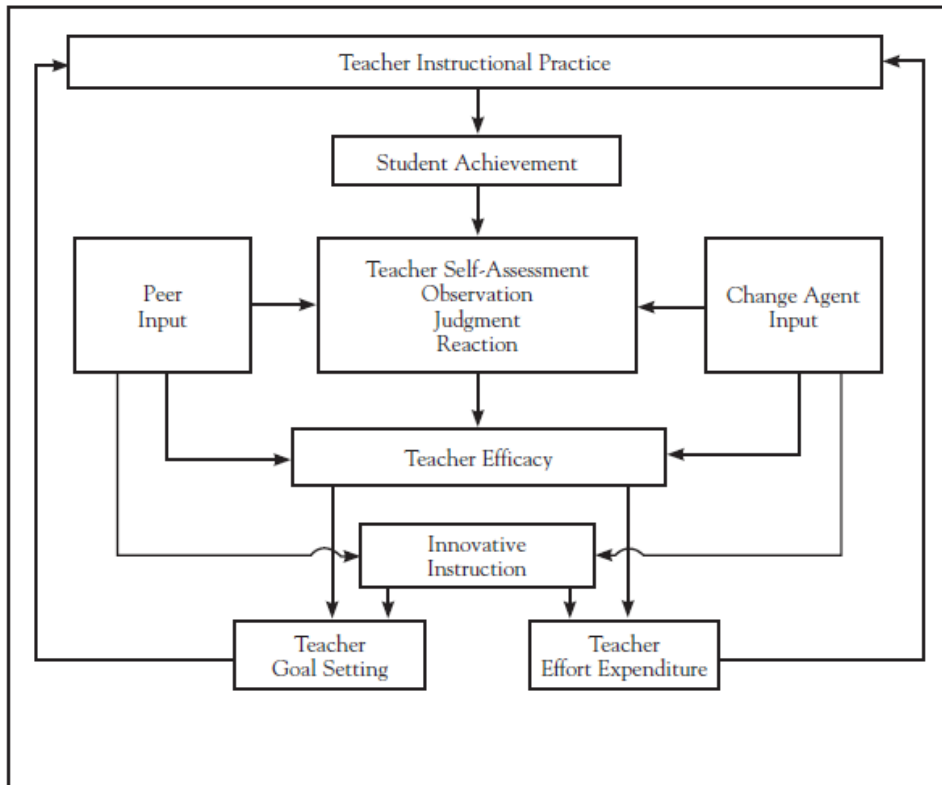


Figure 4.3: Model of teacher assessment as a mechanism for teacher change (Bruce & Ross, 2007, p. 51)

During lesson observation, the teachers were asked to conduct a brief critical reflection on their lessons. The reflections were analysed on the basis of the three points of self-assessment, using ATLAS.ti. The data from, and the observation of, these reflections are presented in Chapter 6.

Bruce and Ross (2007, p. 52) point out that individual processes can be influenced by other agents, particularly peers and change agents (such as the researcher as a professional development presenter, in the context of this research). The model suggests that peers and professional development presenters provide teacher efficacy information that influences the self-assessments that are made by the professional development participants. The teacher self-assessments, together with the information on innovative instruction, serve to increase teacher effectiveness, which influences teacher goal-setting and effort expenditure. In the model, changes in goals and effort contribute to improved instructional practice, which results in higher levels of learner achievement than might otherwise have been attained (Bruce & Ross, 2007).

According to Guskey (2002, p. 382), what teachers hope to gain through professional development are specific, concrete and practical ideas that directly relate to the day-to-day operation of their classrooms. It follows, therefore, that professional development programmes that fail to address such needs will probably not succeed. What attracts teachers to professional development is their belief that it will expand their knowledge and skills, contribute to their growth, and enhance their effectiveness in relation to their learners.

In consideration of the above, the teachers were asked to make a brief assessment of their professional development experience at the end of the professional development by briefly answering the following four questions:

- How has the workshop been helpful to you?
- Will you be able to apply PBL in your class?
- What further assistance will you require to be able to apply PBL?
- Do you wish to make any other comments?

The responses were analysed to assess the impact of the workshop with regard to the expansion of their knowledge and skills, to the contribution to the teachers' growth, and to the enhancement of the teachers' effectiveness in relation to their learners.

Guskey (2002, p. 382) points out that an important factor that many professional development programmes fail to consider is the process of teacher change. Professional development activities are frequently designed to initiate change in teachers' attitudes, beliefs and perceptions. Professional development leaders, for example, often attempt to change teachers' beliefs about certain aspects of teaching, or about the desirability of a particular curriculum, or instructional innovation. They presume that such changes in teachers' attitudes and beliefs will lead to specific changes in their classroom behaviours and practices, which, in turn, will result in improved learner learning.

The literature review (Section 2.12) showed that the research evidence indicates that teachers' beliefs tend to colour and influence their teaching practices, how they believe content should be taught, and how they think learners learn (Harwood et al., 2006, p. 69; Philipp, 2007, p. 261). Literature on teacher change, beliefs and practices shows that teacher change does not necessarily follow on professional development (Guskey, 2002; Harwell, 2003; Ingaverson et al., 2004; Tschannen-Moran & McMaster, 2009). The whole is an interconnected, nonlinear change sequence (Clarke & Hollingsworth, 2002). Significant change in teachers' attitudes and beliefs occurs primarily after they gain evidence of

improvements in learner learning. The improvements typically result from the changes that the teachers have made in their classroom practices (Guskey, 2002, p. 383).

Ernest (1989) argues that mathematics teachers' beliefs have a powerful impact on the practice of teaching. He illustrates the relationships between beliefs, and their impact on practice, in Figure 4.4 below. He gives the example that one teacher's view of the nature of mathematics, or personal philosophy of mathematics, might be that mathematics is a Platonist unified body of knowledge, in which case the teacher is the explainer, and learning is the receptor of knowledge. In contrast, another teacher can view mathematics as problem-solving, in terms of which the teacher is a facilitator, and learning is the active construction of understanding, and possibly even an autonomous form of problem posing and solving.

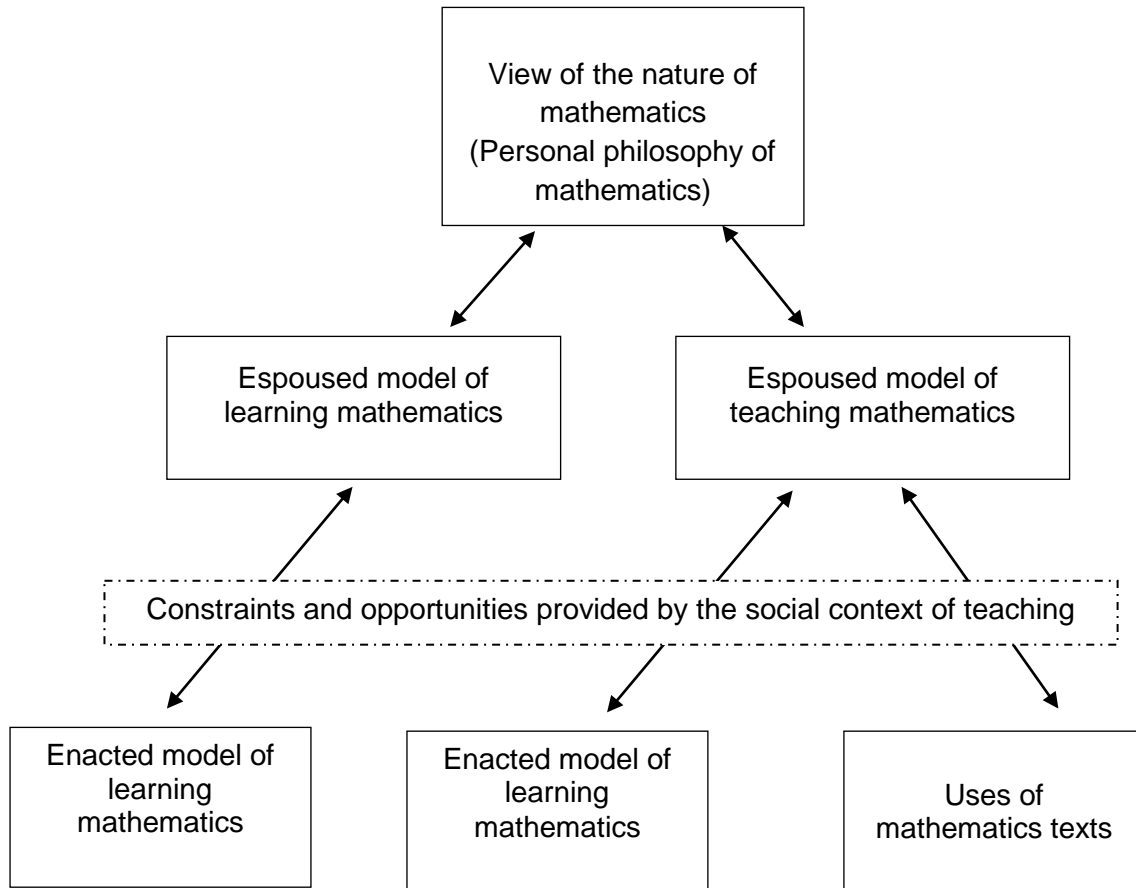


Figure 4.4: Relationship between beliefs and their impact on practice (Ernest, 1989)

The three major goals of professional development programmes are, therefore, achieving change in the classroom practices of teachers, in their attitudes and beliefs, and in the learning outcomes of learners. Of particular importance, however, is the sequence in which the outcomes most frequently occur. After professional development, the teacher changes their classroom practices by implementing the innovations that have been learned during the staff development. Only once the teacher has experienced change in the learner's learning outcomes will the teacher change their beliefs and attitudes towards the professional development innovations, thereby coming to change their daily practice. The point that the above model makes is that:

Evidence of improvement or positive change in the learning outcomes of students generally proceeds, and may be a pre-requisite to, significant change in the attitudes and beliefs of most teachers.

(Guskey, 2002, p. 384)

Guskey (2002, p. 383) also points out:

professional development programmes that are based on the assumption that change in attitudes and beliefs comes first are typically designed to gain acceptance, commitment, and enthusiasm from teachers and school administrators before the implementation of new practices or strategies.

The teachers who are involved in planning sessions need surveys to ensure that the new practices or strategies are well aligned with what they want. However, as important as the procedures are, they seldom transform attitudes, or prompt strong commitment from teachers (Guskey, 2002; Harwell, 2003; Ingavarsen et al., 2004; Tschannen-Moran & McMaster, 2009).

The new perspective on professional development (Villegas-Reimers, 2003, p. 13) has the following characteristics:

- **It is based on constructivism, rather than on a transmission-oriented model.** Hence the participants, meaning the teachers, are treated as active learners who are engaged in the concrete tasks of teaching and reflection.
- **It is perceived as a long-term process, as it acknowledges that teachers learn over time.** As a result, a series of related experiences, rather than a once-off presentation, is seen to be the most effective process, as it allows the teachers to relate their prior knowledge to their new experiences. In the current study, the series of related experiences was achieved through the provision of regular follow-up support that was provided to assist the change process. Hence, the teachers were visited in their schools after the professional development had taken place, so as to provide regular on-site support to help the process along.
- **It is a process that takes place within a particular context.** Hence, the professional development in this study related to actual classroom experiences, and to the daily activities of teachers and learners.

Supovitz and Turner (2000, p. 964) point out that high-quality professional development should include the following essential elements:

- immersion of the participants in inquiry, questioning, and experimentation, and, therefore, in model inquiry forms of teaching;
- engagement of the teachers in concrete teaching tasks that are based on teachers' experiences with learners;
- a focus on subject matter knowledge, and on the deepening of teachers' content skills;

- a grounding in a common set of professional development standards, and the showing of teachers how to connect their work to specific standards for learner performance; and
- connection to other aspects of school change.

Effective professional development should, thus, include the above elements, with the participants being questioned regularly throughout the professional development, as to how the issues under discussion are applicable to their own classroom situations. The tasks that are given during the staff development are to be concrete teaching tasks that are based on the teachers' experiences with learners. For example, in an exercise on the construction of problems for PBL, the teachers could be grouped according to the grades that they teach, and they should be asked to construct a problem for their specific grade, based on the CAPS curriculum content. The relevant problems could then be brought before the whole group. Each problem should then be interrogated in terms of whether the problems set satisfied the five function characteristics (as illustrated in Figure 3.21) which are: the triggering of learning issues; interest; SDL; the promoting of teamwork; and critical thinking. By so doing, the professional development would then be taking place within the CAPS context.

4.4 CONCLUSION

This chapter looked at the models of professional development, and it took a close look at the characteristics of high-quality professional development, and the theory of teacher change resulting from professional development. The theoretical background established in this chapter informed the planning and the nature of the professional development that was held to introduce the group of 15 teachers to the PBL methodology.

The following chapter focuses on the research methodology of this study. It combines the discussion of the research methods applied in the study with the philosophy underlying the methods. It also focuses on the theoretical research issues that form the basis of the study, critically examining the issues involved, as well as their validity, procedures and scope in terms of the study.

CHAPTER 5: RESEARCH DESIGN AND METHODOLOGY

5.1 INTRODUCTION

This chapter provides a theoretical description of the research, in terms of the research design, the population, the sample, the data generation, the analysis, and the interpretation.

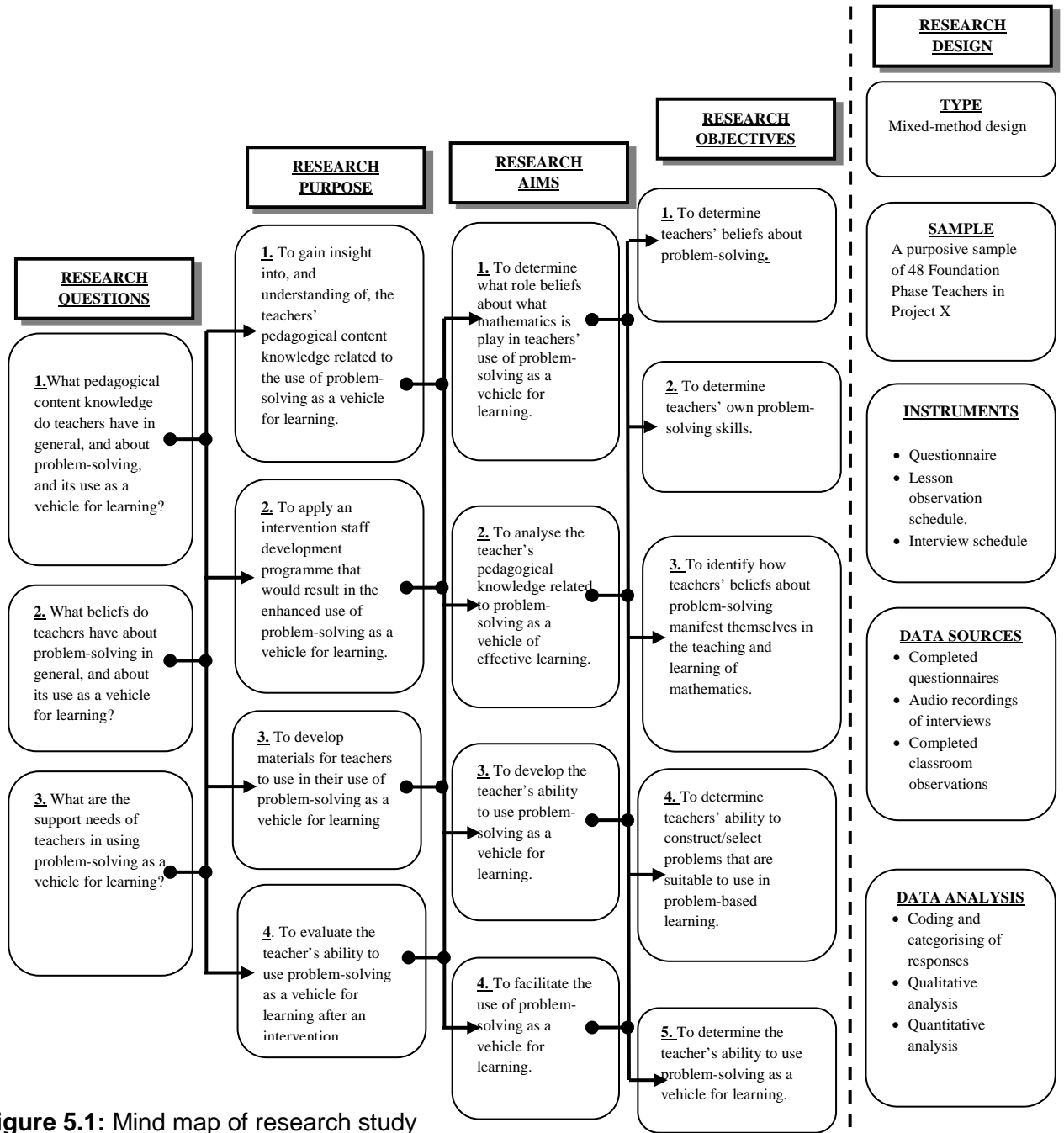


Figure 5.1: Mind map of research study

The mind map in Figure 5.1 above provides an outline of the critical components of the research, and the process of the empirical study. The research design components of this mind map, which are the focus of this chapter, are: the research questions; the purpose; the aims; the objectives; and the design.

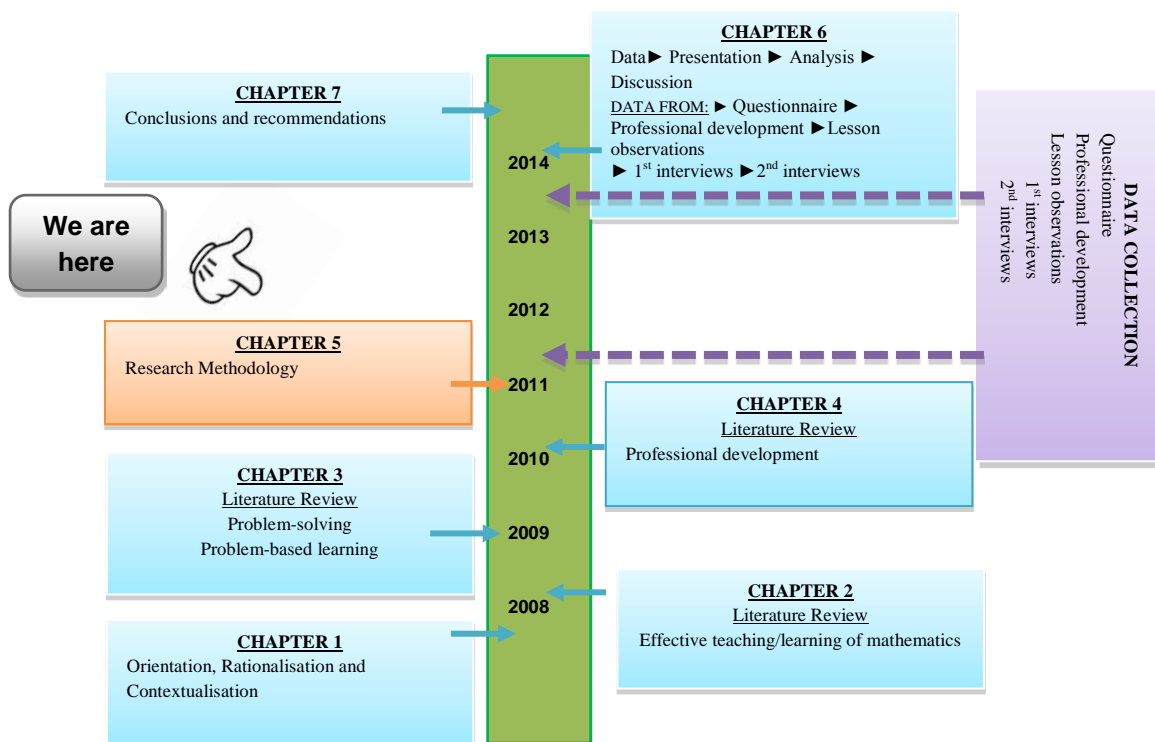


Figure 5.2: The research timeline

5.2 REVISITING THE RESEARCH QUESTIONS

The main research questions (which were earlier outlined in Chapter 1) focus on the Foundation Phase teachers' PCK of, and skills regarding, how to use problem-solving as a vehicle for learning. The researcher explored the use of problem-solving as a vehicle for learning in the project schools.

The research questions asked were:

- 1) What PCK do teachers have in general, and about problem-solving and its use as a vehicle for learning?
- 2) What beliefs do teachers have about problem-solving in general, and in relation to its use as a vehicle for learning?
- 3) What are the support needs of teachers in their use of problem-solving as a vehicle for learning?

The literature study on MKT, teacher beliefs about the teaching and learning of mathematics, problem-solving in mathematics, and PBL provided the theoretical basis for

addressing the research questions concerned. This was done in order for the researcher to understand the developments that have emerged in the areas involved. The understanding obtained in this way provided the researcher with a framework on which to base the construction of research instruments that were aimed at finding answers to the above questions. The theoretical understanding established in the literature study also provided the researcher with a framework that was the basis for the analysis of the data generated, as well as for the justification of any conclusions made in the empirical study in relation to the above questions. It also formed the foundation of any comparison of the results of the research with that of other researchers.

5.3 RESEARCH POPULATION AND SAMPLE

The research population was a group of Foundation Phase teachers participating in an intervention known as Project X. Due to time and resource constraints, studying every member of a population is generally prohibitive, so a sample is required from the population. In the case of this research, it was possible to investigate the entire project population with whom the researcher was involved. The population of the Project X schools were 48 Foundation Phase teachers, who were distributed across the phase's grades, as is shown in Table 5.1 below.

Schools	Grade 1	Grade 2	Grade 3	Total
A	2	2	2	6
B	4	4	4	12
C	3	3	3	9
D	4	4	4	12
E	3	3	3	9
Total	16	16	16	48

Table 5.1: Summary of the project population

The first research instrument, which was a questionnaire, was distributed to the whole project population, namely to the 48 Foundation Phase teachers in the five schools in which Project X took place. However, when it came to the professional development that was to introduce the project schools to PBL a sample of 15 teachers had to be selected, as the project lacked sufficient funding to provide staff development for the entire project population of 48 teachers.

5.3.1 The study sample

A sample is a subset of the population that is taken to be representative of the entire population (Babbie, Mouton, Voster & Prozesky, 2009, p. 164; McMillan & Schumacher, 2006, p. 119). An important word in this definition is *representative*. A sample that is not representative of the population, regardless of its size, is inadequate for testing purposes, as the results cannot, then, be generalised to the entire population (McMillan & Schumacher, 2006, p. 67). In this study, the major focus was on the group of 48 Foundation Phase teachers who were involved in the intervention known as Project X. The researcher, therefore, used a purposive sampling technique to select the study sample when it came to the in-service training of teachers regarding the introduction of PBL.

Teddie and Yu (2007, p. 80) define purposive sampling techniques as non-probability, purposeful, or qualitative sampling. They identify the following three broad categories of purposive sampling techniques:

- **Sampling to achieve representativeness, or comparability:** “[T]hese techniques are used when the researcher wants to (a) select a purposive sample that represents a broader group of cases as closely as possible or (b) set up comparisons among different types of cases” (Teddie & Yu, 2007, p. 80).
- **Sequential sampling:** Such sampling “uses the gradual selection principle of sampling when (a) the goal of the research project is the generation of theory (or broadly defined themes) or (b) the sample evolves of its own accord as data are being collected. Gradual selection may be defined as the sequential selection of units or cases based on their relevance to the research questions, not their representativeness” (Teddie & Yu, 2007, p. 80).
- **Sampling special, or unique, cases:** Such sampling is “employed when the individual case itself, or a specific group of cases, is a major focus of the investigation (rather than an issue)” (Teddie & Yu, 2007, p. 80).

The sampling of special, or unique, cases can, again, be subdivided into four types: revelatory case sampling; critical case sampling; the sampling of politically important cases; and the complete collection. The 48 Foundation Phase teachers with whom the researcher was working in the intervention project provide a ‘total environment’ for the study, thus the purposive sampling type used was that of complete selection. From a rational point of view, purposive sampling was best for the research concerned, as the main purpose of the research was to address a specific purpose (i.e. to determine professional development and other intervention support needs), as opposed to the rationale for probability sampling, which is representativeness. Whereas the results of this research might not be generalisable (thus

excluding the possibility of external validity), some form of generalisability might be possible (namely, transferability). Accordingly, a questionnaire to determine in-service professional development needs was administered to all of the 48 teachers present at the beginning. Later, a stratified random sample was selected in relation to professional development on PBL, and individual direct interviews and classroom observations were undertaken to determine other intervention and support needs for the teachers in the project.

The Investopedia Dictionary (2012) defines stratified random sampling as a method involving:

...the division of a population into smaller groups known as strata. In stratified random sampling, the strata are formed based on members' shared attributes or characteristics. A random sample from each stratum is taken in a number proportional to the stratum's size when compared to the population. These subsets of the strata are then pooled to form a random sample.

The researcher used stratified random sampling for the following reasons:

- It would have been both time-consuming and costly to staff develop the entire population in the project, and later to observe and interview the entire project population of 48 Foundation Phase teachers.
- Such sampling enabled the selection of a sample that would be collectively representative of the project population.
- The sample was representative enough for generalisability in relationship to the project population, as it constituted 27% of the project population.

5.3.2 The sampling process

The research used both purposive sampling and probability sampling. Figure 5.3 below summarises the sampling process used in the research.

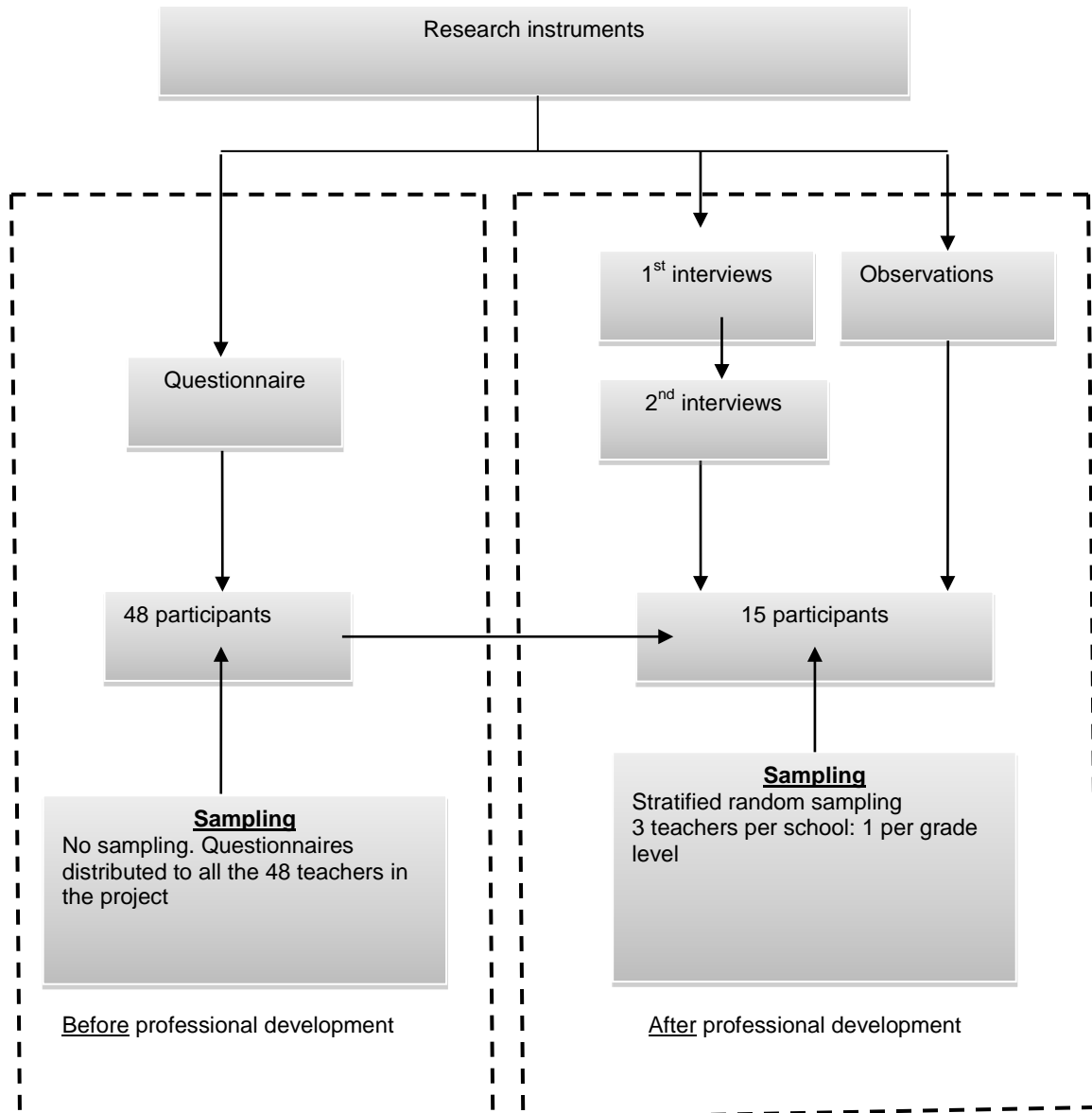


Figure 5.3: The sampling process

Purposive, or non-probability, sampling was used in the administration of the questionnaire, which was administered before the professional development. The researcher administered the questionnaire at this time because he was working with 48 teachers in Project X, and felt that, to address the specific purposes related to the research questions, he would learn the most by involving the entire project population. A stratified random sample of 15 teachers were interviewed and observed while using problem-solving as a vehicle for learning.

5.4 RESEARCH DESIGN

Johnson and Onwuegbuzie (2004, p. 17) define mixed methods research as "the class of research where the researcher mixes or combines quantitative and qualitative research techniques, methods, approaches, concepts or language into a single study". Tashakkori and

Creswell (2007, p. 4) also define a mixed methods research design as consisting of “research in which the investigator collects and analyses data, integrates the findings, and draws inferences using both qualitative and quantitative approaches or methods in a single study or a program of inquiry”. They point out that a mixed methods research design utilises qualitative or quantitative approaches where there are:

- two types of research questions (with qualitative and quantitative approaches);
- two different manners in which the research questions are developed (participatory vs. pre-planned);
- two types of sampling procedures (e.g. probability and purposive);
- two types of data collection procedures (e.g. focus groups and surveys);
- two types of data (e.g. numerical and textual);
- two types of data analysis (statistical and thematic); and/or
- two types of conclusions (emic and etic representations, ‘objective’ and ‘subjective’).

In this research the following of the above aspects of mixed methods research design were utilised:

- 1) The research questions utilised both qualitative and quantitative approaches.
- 2) The questionnaire was used for the collection of both qualitative and quantitative data. The qualitative data were used to arrive at a description of the teachers’ beliefs and practices, whereas the quantitative data helped in explaining the extent of such beliefs and practices. Hence, both textual and numerical data were obtained in this way.
- 3) The data analysis was both statistical and thematic. For example, question 2 provided statistical data on how often the teachers concerned asked the learners to do pen-and-paper calculations, and to practise; question 3 asked how often teachers demonstrated to the class a procedure on the chalkboard, which they then let their learners practise; and question 4 asked the teachers how often they taught the learners mathematics by means of focusing on rules and procedures. Statistical data from the 3 questions were analysed thematically into two themes, on the one hand those using the *direct transmission approach*, and, on the other hand, those using the *constructivist approach*. The two approaches were discussed in full in Chapter 2.
- 4) Two types of conclusion were drawn: objective and subjective. Objective conclusions were made based on, or supported by, the statistical data. The objective conclusions led the researcher to make subjective conclusions, which were not directly substantiated by means of the research data obtained.

Grafton, Lillis and Mahama (2011, p. 6) identify two key components to the mixed methods approach, being the integration of methods, and the fact that the research should concern a single study or programme of enquiry (as opposed to parallel studies or programmes). In arguing for mixed method design, Cresswell (2003, p. 5) writes:

...the use of quantitative and qualitative approaches in combination provides a better understanding of research problems than either approach alone.

In combining qualitative and quantitative approaches, the mixed methods research design utilises the strengths of both approaches. Castro, Kellison, Boyd and Kopak (2010, p. 343) observe the strength of each approach to be as described in the following paragraphs:

The strengths of the *quantitative* approach include:

- the accurate operationalisation and measurement of a specific construct;
- the capacity to conduct group comparisons;
- the capacity to examine the strength of association between variables of interest; and
- the capacity for model specification and the testing of research hypotheses.

By using the quantitative approach, this research was able to accurately operationalise and measure such constructs as the extent of the use of traditional methods in the teaching of mathematics, as well as the levels and use of the constructivist approach. The research was able to use the quantitative data to examine the association between the traditional beliefs about teaching mathematics, and their manifestation in the teaching of mathematics.

The strengths of the *qualitative* approach include the following:

- the capacity for generating richly detailed accounts of human experiences (emotions, beliefs, and behaviours);
- the ability to provide narrative accounts that are examined within the original context in which observations occur; and
- the possibility of supplying an in-depth analysis of complex human and family systems, as well as cultural experiences, in a manner that cannot be fully captured with the use of measurement scales and multivariate models.

Using the qualitative approach enabled the researcher to augment the quantitative findings with richly detailed accounts of the teacher's approach to the teaching and learning of mathematics, as well as their approach to problem-solving and the nature of problems they

use during their problem-solving lessons. The qualitative data collected provided narrative accounts that supported the teaching behaviours and practices that were observed during lesson observations. The qualitative approach allowed for an in-depth analysis of the teachers' views on what the teaching and learning of mathematics entails, and how their views impacted on their role in the classroom.

Bergman (2011, p. 274) recently made a similar affirmation when he pointed out that it is often argued that mixed method research design is better than mono-method research, because it presents a supplementary perspective, that is neither merely qualitative or quantitative in nature, but both. Sosulski and Lawrence (2008, p. 121), in their support for mixed methods research, argue:

the power of numbers and an aim of generalizing quantified outcomes balanced with the rich context of lived experiences captured in qualitative inquiry can yield results that are quite distinct from single method designs.

In discussion of the merits of, and the warrants for, considering mixed methods social inquiry as a distinctive methodology, Greene (2008, p. 20) points out:

A mixed methods approach to social inquiry distinctively offers deep and potentially inspirational and catalytic opportunities to meaningfully engage with the differences that matter in today's troubled world, seeking not so much convergence and consensus as opportunities for respectful listening and understanding.

Three types of mixed-method design exist: explanatory; exploratory; and triangulation (McMillan & Schumacher, 2006, p. 28). In this study, a *triangulation design* was used, because it allows for the simultaneous collection of both qualitative and quantitative data, and the strengths of each approach can be applied to provide not only a more complete result, but also one that is more valid than it might otherwise have been (Cresswell, 2003; McMillan & Schumacher, 2006).

5.4.1 The research instruments

The research instruments consisted of a questionnaire, a semi-structured interview schedule, and a classroom observation schedule. The instruments were used to generate data that provided:

- insight into the teachers' MKT related to problem-solving in general, and to its use as a vehicle for learning;
- insight into what beliefs the teachers had about problem-solving as a vehicle for learning, and how the beliefs manifested in the classroom; and

- clarification on the need for a staff development programme that would result in the introduction of the use of problem-solving as a vehicle for learning (PBL) in the classroom.

According to Winterstein and Kimberlin (2008, p. 2280), before constructing an instrument with which to elicit the required data, researchers are supposed to ask themselves the following questions:

- Do research instruments exist that can elicit the data sought?
- How well do the identified instruments match the construct that the researcher has conceptually defined for the study?
- Has the instrument been validated in a population similar to the one being studied?

In the current study, the researcher subsequently conducted a literature search for such instruments, with the following five sources providing the basis for the development of the questionnaire:

- Krebs (2005): *Analysing Student Work as a Professional Development Activity*
The source describes a professional development activity, in which the teachers analysed mathematical tasks, predicted students' achievement on tasks, evaluated students' written work, and assessed students' understanding.
- Watson (2001): *Profiling teachers' competence and confidence to teach particular mathematics topics: The case of chance and data*
The source presents an instrument that was designed as a profile of teacher achievement and teacher needs, with respect to the probability and statistics strands in the mathematics curriculum.
- Ball, Hill and Schilling (2008): *Unpacking pedagogical content knowledge: Conceptualizing and measuring teachers' topic-specific knowledge of students*
The source contains items that measure the teachers' combined knowledge of content and students' writing.
- Ball and Hill (2008): *Developing measures of teachers' mathematics knowledge for teaching*
The source contains survey items measuring MKT, focusing on elementary content knowledge, the elementary knowledge of students and content, and middle school content knowledge.
- Goldin (2004): *Problem Solving Heuristics, Affect, and Discrete Mathematics*
The source focuses on discrete mathematics opportunities for mathematical discovery, and on interesting nonroutine problem-solving. Question 19 of the questionnaire was taken from this source.

The research instruments used in these sources provided very useful guidelines, examples and some items that could be adapted as instruments for use in the research. In the process of constructing the research questionnaire, the items in Table 5.2 below were adapted from the indicated sources.

Questionnaire item	Source	Adaptations made
20	Goldin (2004, p. 56)	Added pictures of 2 buckets.
21	Ball & Hill (2008, p. 20), item 24	D adapted to South African Foundation Phase language description of a triangle

Table 5.2: Adaptation of other questionnaires

The rest of the questions were constructed as follows:

Question 1: This question was an open-ended mind map that was aimed at investigating the teachers' views on problem-solving.

Questions 2–4: These questions were set to investigate the teachers' approach to the teaching and learning of mathematics. The questions were constructed based on the characteristics of traditional teaching that are mentioned in Table 2.3.

Questions 5–8: The questions in this section sought to provide information about how teachers rate themselves. The questions were constructed basing on the knowledge that is required to teach mathematics effectively, based on the literature review contained in sections 2.7 to 2.11.

Questions 9–14: These questions were constructed on the basis of the literature reviewed in sections 2.2 to 2.5. Questions 15 to 17 was constructed on the basis of the traditional and non-traditional beliefs about, and the teaching and learning of, mathematics (see Table 2.4). The questions were designed to investigate the teachers' approach to problem-solving.

Questions 18–19: The questions in this section of the questionnaire sought to provide information about the teachers' problem-solving lessons. These two items involved the classroom problem-solving scenario that was constructed based on the literature review in the problem-solving sections 3.2 and 3.3.

Questions 23–30 were constructed to gather demographic data regarding the knowledge of the essential Foundation for Learning Campaign.

Questions 31–36 were constructed to gather biographical data on the participants.

5.4.2 The validity and reliability of the questionnaire

Golafshani (2003, p. 601) and Morse et al. (2002, p. 14) point out that validity and reliability are two factors with which any researcher should be concerned when designing a study, analysing results, and judging the quality of the study. In essence, validity entails the question, “does your measurement process, assessment, or project actually measure what you intend it to measure?” (Handley, 2002, p. 1). The related topic of reliability addresses whether repeated measurements, or assessments, provide a consistent result, given the same initial circumstances (Handley, 2002, p. 1).

5.4.2.1 Validity

Validity has three essential parts:

1) **Internal validity** encompasses whether the results of the study are legitimate because of the way in which the groups were selected, the data were recorded, or the analysis was done. According to Merriam (1995, p. 54) the following strategies are taken to strengthen internal validity.

- *Triangulation* involves the use of multiple investigators, multiple sources of data, or multiple methods for confirming the emerging findings. For example, if researchers hear of a phenomenon in the interviews that they undertake, and they see the same phenomenon taking place in the classrooms, as well as read about it in pertinent documents, they can be confident that the phenomenon reflects the reality of the situation.
- *Member checks* involve taking data from the study participants and from the tentative interpretation of the data, back to those from whom they were derived, whom they can ask if the interpretations are plausible, to see whether they ring true.
- *Peer/colleague examination* entails asking the peers or colleagues to examine the data available, and to comment on the plausibility of the emerging findings.
- The *stating of researcher's experiences, assumptions, and biases* is a means of presenting the orientation and biases, among other aspects, of the researcher's outlook at the outset of the study. This enables the reader to better understand how data might have been interpreted in the manner in which they were.
- *Submission/engagement in the research situation* relates to the collecting of data over a long period of time, so as to ensure an in-depth understanding of the phenomenon.

Using any of the strategies discussed above helps to ensure that the interpretation of the reality being presented is as true to the phenomenon as possible (Merriam, 1995, p. 55).

- **External validity** (which is often called generalisability) asks whether the results that are given by the study are transferable to other groups (i.e. populations) of interest.
- **Content validity** addresses how well the items developed provide an adequate and representative sample of items that measure the construct of interest. Winterstein and Kimberlin (2008, p. 2278) point out that, because there is no statistical instrument that determines whether a research instrument adequately covers the content, content validity usually depends on the judgement of experts in the field. Hence, once the researcher has completed the draft, experts in the field are consulted to determine whether the constructed research instrument adequately covered the area being investigated.

5.4.2.2 Reliability

Reliability is concerned with the extent to which one's findings would be the same if the study were to be replicated.

Thanasegaran (2003, p. 35) defines reliability as “the degree or extent to which measures are free from error and therefore yield consistent results (i.e. the consistency of a measurement procedure)”. If the results of a study can be “reproduced under a similar methodology, then the research instrument is considered to be reliable” (Golafshani, 2003, p. 598). The significant point that is raised in both definitions is that a reliable research instrument should produce the same results if the research is repeated at another location or time.

Merriam (1995, p. 55) points out that, while yielding consistent results is not problematic in the pure sciences, doing so in the social sciences is, as studying people is not the same as studying non-living substances. In other words, human behaviour is never stagnant. For example, classroom interaction is not the same day after day, and neither is a person's understanding of the world around them. Instead, reliability in social science research strives for *dependability* or *consistency* in the results obtained. The real question in social science research is not whether the results of one study are the same as the results of a second or third study, but whether the results of the study are consistent with the data collected (Merriam, 1995, p. 56). To achieve consistency, Merriam (1995) suggests that the following three techniques be applied:

- *Triangulation* uses more than one method for data collection. For example, one could use interviews, a questionnaire, or observations to collect data on how teachers use particular computer software in their teaching of mathematics.
- *Peer examination* can be used to achieve consistency in the interpretation of the data collected. In this instance, the researcher asks another person to verify whether the results are in line with the data collected.
- *An audit trial* operates in the same way that accounting auditors operate. However, in this instance the researcher must describe in detail their data collection procedures, so that a third party can trace the research data from the construction of the data collection instrument, through to the data analysis and interpretation.

Reliability is often at risk when assessments are taken over time, performed by different people, or are highly subjective (Healy & Perry, 2000, p. 23). In the current study, reliability was not at risk, as the research was conducted over a short period, and the data concerned were collected by the same person.

From the literature consulted so far, one observes that reliability in natural sciences research is concerned with obtaining the same results when the research is replicated. In contrast, reliability in social science research is concerned with the consistency and the dependability of the research process and findings, and how these factors optimally reflect the researcher's ability and the data collected. In summarising the issue of validity and reliability in relation to the present research, one can conclude that validity and reliability were ensured by means of triangulation.

5.4.3 Establishing the questionnaire's validity and reliability

Radhakrishna (2007, p. 1) points out that the reliability of a questionnaire can be established by means of using a pilot test to collect data from 20 to 30 subjects who are not included in the sample taken. The data collected from a pilot test can be analysed using SPSS. A reliability coefficient (alpha) of .70 or higher is considered to be an acceptable indication of reliability.

The validity of a questionnaire can be established using a panel of experts who address the following questions:

- Is the questionnaire valid? In other words, does the questionnaire measure what it is intended to measure?
- Does the questionnaire represent the content of the study?

- Is the questionnaire appropriate for the sample/population at hand?
- Is the questionnaire comprehensive enough to collect all the information that is required to address the purpose and goals of the study?
- Does the instrument look like a questionnaire?

Figure 5.4 below shows how the validity and reliability of the questionnaire that were used in the research project were established in line with the above literature review.

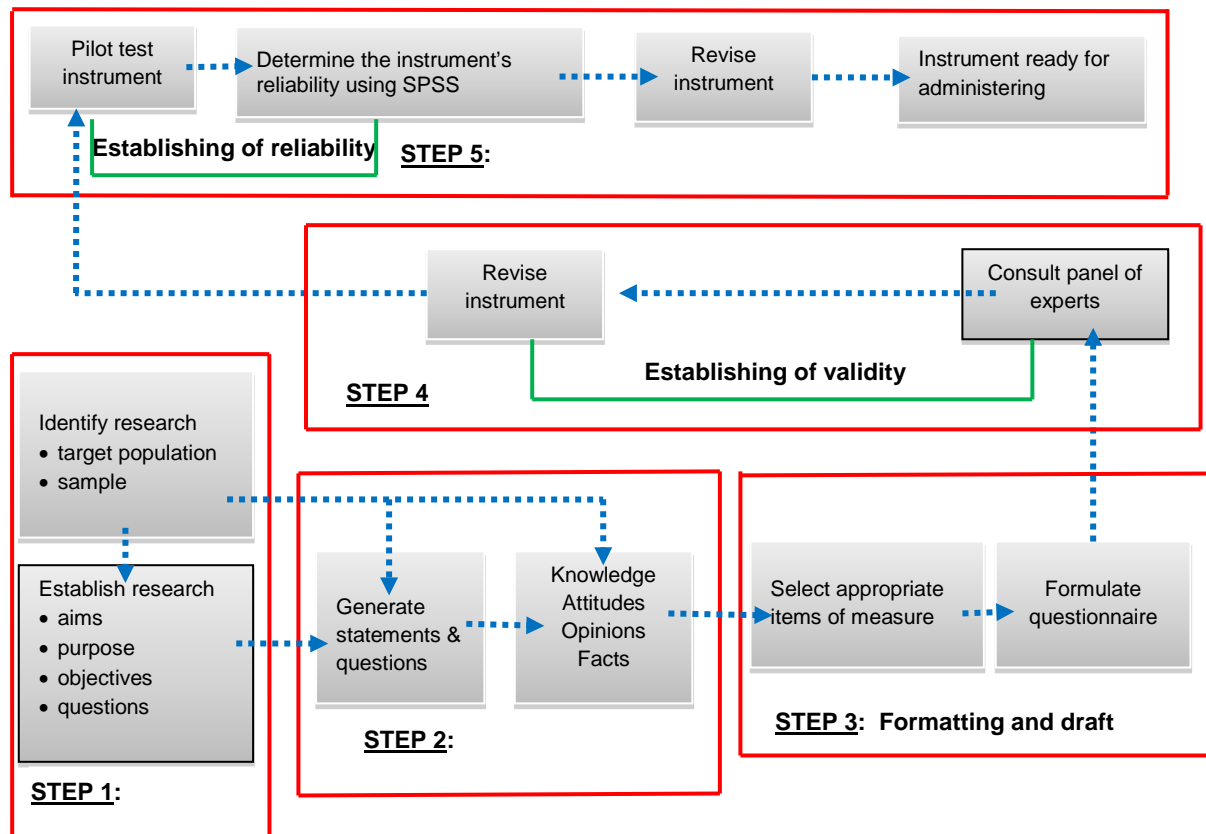


Figure 5.4: Establishing research instruments' validity and reliability (adapted from Radhakrishna, 2007)

5.4.3.1 Establishing questionnaire validity

In many cases, the evidence that is most critical to evaluating the usefulness and the appropriateness of a test for a specific purpose is based on content validity (Rico, Dios & Ruch, 2012). Content validity refers to the degree to which a test appropriately represents the content domain that it is intended to measure (Salkind, 2007). This concept of match is sometimes referred to as alignment, whereas the content or subject are referred to as the performance domain. Experts in the given performance domain generally judge the content

validity involved. The experts ensure that each test covers the content that matches all relevant subject matter. Both face validity and curriculum validity may be used to establish content validity.

Face validity is an estimate of the degree to which a measure clearly and unambiguously taps the construct that it purports to assess. Thus, face validity refers to the 'obviousness' of a test, being the degree to which the purpose of the test is apparent to those taking it (Bornstein, 2011). According to Mogari (2004, p. 1):

Face validity is commonly used in educational research, particularly by postgraduate students and other novice researchers, to validate research instruments...One identifies credible experts in the area being studied, who are then expected to studiously apply their knowledge and skills to scrutinise the instrument to determine the degree to which the scale measures what it is meant to measure.

In line with the content and face validity procedures described above, the research questionnaire, after being drafted (step 3), was passed on to two experts (step 4) in the field of mathematics education to determine the construct validity involved. This was done so as to ascertain their opinion as to whether the researcher had "addressed all relevant issues and formulated the questions in an understandable and unambiguous way" (Eiselen & Uys, 2005, p. 4), as well as to consider Radhakrishna's five questions referred to above. The questionnaire was also given to an expert in questionnaire design at the Centre for Statistical Consultation, Department of Statistics and Actuarial Sciences, Stellenbosch University for verification of the formulation of the questions, and the response format. The questionnaire was then revised to incorporate the comments and suggestions of the consulted experts.

5.4.3.2 Establishment of the reliability of the questionnaire

The undertaking of pilot testing (step 5) is imperative in any study, since it enables the researcher to identify, and to rectify problems, prior to the survey being conducted (Eiselen & Uys, 2005, p. 4). Secondly, it provides an indication of the response rate that can be expected. It is highly recommended that researchers pilot test their questionnaires on subjects with characteristics similar to those who will be used in the study (McMillan & Schumacher, 2006, p. 202). The sample size should be greater than 20. Accordingly, in the current study the researcher carried out a pilot study (step 5, see Figure 4.3) among a group of 25 respondents who matched the target population. The data collected from the pilot study were analysed using SPSS, and their reliability was calculated.

A description of the run and rerun process that was carried out to establish the reliability of the questionnaire's five subscales is given below.

SUBSCALE 1: Approach to the teaching and learning of mathematics**SPSS Pretest output 1**

Cronbach's alpha	Cronbach's alpha based on standardised Items	No. of items
.577	.630	4

Table 5.3: Reliability statistics for the *approach to the teaching and learning of mathematics*

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item – total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q2	23.83	14.917	.378	.366	.504
Q3	23.98	14.538	.336	.189	.523
Q4	23.50	16.513	.503	.380	.496
Q5	24.50	14.821	.184	.226	.618

Table 5.4: Item total statistics for the *approach to the teaching and learning of mathematics*

The above outputs show which items to remove in order to create the best items to measure the *approach to teaching and learning of mathematics*. In this case, the greatest increase in Cronbach's alpha was to come from deleting item Q5 from the instrument, which was, accordingly, done. The subscale *approach to the teaching and learning of mathematics* would have a Cronbach's alpha of 0.618.

Questionnaire	No. of items	Cronbach's alpha	No. of items deleted
Pilot test questionnaire	4	0.577	1
Final research questionnaire	3	0.618	-

Table 5.5: Summary of the run and rerun for the *approach to the teaching and learning of mathematics*

George and Mallery (2003, p. 231) provide the rules of thumb for the Cronbach's alpha that are contained in Table 5.6 below.

Cronbach's alpha	Interpretation
> 0.9	Excellent
> 0.8	Good
> 0.7	Acceptable
> 0.6	Questionable
> 0.5	Poor
< 0.5	Unacceptable

Table 5.6: The interpretation of Cronbach's alpha (George & Mallery, 2003, p. 231)

According to DeVellis (2003, p. 95), the range of Cronbach's alpha values generally are classified as minimally acceptable (0.60–0.64), acceptable (0.65–0.70), good (0.70–0.75), very good (0.75–0.80), and excellent (0.80 and above). Peter (1979, p. 15) points out:

[I]n early stages of research, modest reliability in the range of 0.5 to 0.6 will suffice. For basic research, it is argued that increasing reliability beyond 0.8 is unnecessary because at that level correlations are attenuated very little by measurement error.

In line with Table 5.6 above, Hanneman (2006, p. 3) points out that, although the most common 'rule of thumb' is that the alpha should exceed 0.8, "in practice, scales with lower reliabilities are often used (and productively so)". Field (2006, p. 1) points out that although a cut-off point of 0.7 is more suitable than is one of 0.8, "values below 0.7 can realistically be acceptable because of the diversity of the constructs being measured." Schmitt (1996, p. 352) also concurs with the use of a low alpha in pointing out that, due to "...other desirable properties, such as meaningful content coverage...low reliability may not be a major impediment to its use". Yu (2005) also allows for the use of a low alpha in pointing out:

[I]t is a common misconception that if the alpha is low, it must be a bad test. Actually your test may measure several attributes/dimensions rather than one and thus the Cronbach's Alpha is deflated.

Tan (2009, p. 102) gives the following interpretation for the reliability coefficient as an internal consistency index:

- Below 0.50, the reliability is low.
- Between 0.50 and 0.80, the reliability is moderate.
- Greater than 0.80, the reliability is high.

According to Tan (2009), reliability of 0.50 to 0.80 is considered to be moderate. Although the Cronbach's alpha of 0.6 that was obtained for the scale *approach to the teaching and learning of mathematics* is considered 'low', 'questionable', or 'moderate', according to the above rules of thumb, this scale was used in the current research due to the diversity of the construct being measured (Field, 2006), and because of its other desirable properties, such as its meaningful content coverage (Schmitt, 1996, p. 352). It is common, for these reasons, to have a lenient cut-off point of 0.60 (Maizura, Masilamani & Aris, 2009, p. 220) in social sciences research. Examples of such instances are as follows:

- Maizura, Masilamani and Aris (2009, p. 220) in their study with scales, decision latitude, an psychological job demands, obtained a low Cronbach's alpha of 0.64 on a psychological job demands scale.

- Mokkink, Knol and Uitdehaag (2011, p. 1502) obtained low Cronbach's alphas on the group factors mental of 0.56 and bulbar of 0.48.
- Gregorich, Helmreich and Wilhelm (1990, p. 682) had reliability scales ranging from 0.47 to 0.67.

SUBSCALE 2: Approach to problem-solving

SPSS Pretest output 2

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of items
.542	.567	7

Table 5.7: Reliability statistics for *approach to problem-solving*

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item – total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q19	13.16	8.251	.194	.224	.533
Q20	13.54	7.922	.302	.413	.492
Q21	12.92	7.854	.270	.178	.504
Q22	13.16	7.584	.171	.245	.563
Q23	13.57	7.252	.422	.445	.441
Q24	13.62	8.631	.319	.296	.500
Q25	13.59	7.692	.307	.290	.489

Table 5.8: Item total statistics for *approach to problem-solving*

The above SPSS outputs show which item to remove so as to create the best items to measure *approach to problem-solving*. In this case, as the greatest increase in Cronbach's alpha would have come from deleting item Q22, the item was removed from the instrument. The subscale *approach to problem-solving* would have a Cronbach's alpha of 0.563 (0.6 to the nearest one decimal point). Although the Cronbach's alpha of 0.6 is questionable, according to George and Mallery's (2003, p. 231) rule of thumb, this subscale was used because, as has already been observed in Schmitt (1996, p. 352), "when a measure has other desirable properties, such as meaningful content coverage of some domain...low reliability may not be a major impediment to its use". It also falls in the minimally acceptable range of 0.6 to 0.64 (DeVellis, 2003).

Questionnaire	No. of items	Cronbach's alpha	No. of items deleted
Pilot test questionnaire	7	0.5	1
Final research questionnaire	6	–	–

Table 5.9: Summary of run and rerun for *approach to problem-solving*

SUBSCALE 3: Self-rating

SPSS: Pretest output 3

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of Items
.801	.809	4

Table 5.10: Reliability statistics for *self-rating*

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item – total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q11	10.44	1.798	.396	.291	.863
Q12	10.13	1.527	.766	.785	.676
Q13	10.11	1.737	.561	.487	.775
Q14	10.18	1.513	.788	.752	.665

Table 5.11: Item total statistics for *self-rating*

The above SPSS outputs show which item to remove to create the best items to measure *approach to problem-solving*. In this case, all the items identified appeared to be worth retaining, as the alpha was 0.801, which is very good (DeVellis, 2003).

Questionnaire	No. of Items	Cronbach's alpha	No. of items deleted
Pilot test questionnaire	4	0.8	0
Final research questionnaire	4	0.8	0

Table 5.12: Summary of run and rerun for *self-rating*

SUBSCALE 4: The problem-solving lessons

SPSS: Pretest output 4

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of Items
.346	.377	3

Table 5.13: Reliability statistics for *the problem-solving lessons*

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item – total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q17	4.87	2.114	.212	.099	.257
Q19	4.87	1.636	.292	.118	.067
Q20	4.55	1.557	.124	.023	.469

Table 5.14: Item total statistics for *the problem-solving lessons*

The above SPSS outputs show which item to remove to create the best items to measure the teachers' *problem-solving lessons*. In this case, the greatest increase in Cronbach's alpha would come from deleting item Q20, hence the item was removed from the instrument. The subscale *problem-solving lessons* would have a Cronbach's alpha of 0.469 (0.5 to the nearest one decimal point). Though the Cronbach's alpha of 0.5 is questionable, according to George and Mallery's (2003, p. 231) rule of thumb, this subscale was maintained as it was in the instrument, because it had desirable properties of the meaningful content coverage of this domain. Hence, a low reliability alpha was not a major impediment to its use (Schmitt, 1996, p. 352).

Questionnaire	No. of items	Cronbach's alpha	No. of items deleted
Pilot test questionnaire	3	0.346	1
Final research questionnaire	2	0.469	0

Table 5.15: Summary of run and rerun for *the problem-solving lessons***SUBSCALE 5: Teacher beliefs about mathematics and its teaching****SPSS: Pretest output 5**

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of Items
.612	.615	3

Table 5.16: Reliability statistics for *teacher beliefs about mathematics and its teaching*

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item – total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q26	5.00	2.296	.499	.253	.406
Q27	5.32	2.004	.438	.218	.496
Q28	5.75	2.713	.342	.124	.616

Table 5.17: Item-total statistics for *teacher beliefs about mathematics and its teaching*

The above SPSS output shows which item to remove to create the best items to measure *teacher beliefs*. In this case, all the items appeared to be worthy retaining, as the alpha concerned was 0.612. Deleting item Q28 would have increased the alpha by only 0.004. Once again, although the Cronbach's alpha of 0.612 was, according to the rule of thumb, considered to be questionable (George & Mallery, 2003, p. 231), all the items of this subscale were retained, because "when a measure has other desirable properties, such as meaningful content coverage of some domain....low reliability may not be a major impediment to its use" (Schmitt, 1996, p. 352).

Questionnaire	No. of items	Cronbach's alpha	No. of items deleted
Pilot test questionnaire	3	0.612	0
Final research questionnaire	3	0.612	0

Table 5.18: Summary of run and rerun for *teacher beliefs about mathematics and its teaching*

After the SPSS run and rerun were summed up above, the questionnaire was revised in line with the observations resulting from the pilot testing and alpha run.

Babbie et al. (2009, p. 276) point out:

Although we should strive with everything in our power to do truly valid, reliable, and objective studies, the reality is that we are never able to attain this completely. Rather, it remains a goal, something thing to be striven towards, although never to be fully attained.

Conducting the alpha run of the results of the pilot study, and the pilot study itself, fell within the researcher's efforts to conduct a truly valid, reliable and objective study.

5.4.3.3 Missing values

Prior to conducting the data analysis, it is essential to address the issue of missing values. Such values are frequently the result of the reluctance of a respondent to answer particular items in the questionnaire administered to them.

The different methods that were considered for addressing missing values are discussed below:

- **List-wise deletion** is one of the most popular methods for dealing with missing values. Such deletion involves excluding all the cases that contain missing values from the analysis (Byrne, 2001). The final sample to be used in the analysis,

therefore, only comprise complete data records. One of the disadvantages of using the method is that it decreases the sample size.

- **Pair-wise deletion** refers to the deletion of only those cases of variables for which the values are missing. The case is, therefore, not deleted for the entire set of data analysis, but it is only deleted in the analysis involving variables for which there are missing values (Byrne, 2001).
- **Mean imputation** is a strategy in terms of which the arithmetic mean is substituted for a missing value. This method can be problematic, as it might reduce the variance of the variable, because the arithmetic mean represents the most likely score to be obtained (Byrne, 2001).
- **Regression-based imputation** is a strategy in terms of which every missing value is replaced by a predicted score, using multiple regressions based on the values of the other variables (Kline, 2011).
- **Imputation by matching procedure** is a method in terms of which the missing values are replaced by substitute values that are derived from other cases, with similar response patterns (Theron, Spangenberg & Henning, 2004). The PRELIS program can be used for this purpose (Jöreskog & Sörbom, 1996).

In this study, pairwise deletion was the preferred method for dealing with missing data.

5.5 DATA SOURCES AND DATA GENERATION

5.5.1 Data sources

The current study's data sources were completed questionnaires, transcribed interviews, and the notes that were taken during classroom observations (see Figure 5.5 below).

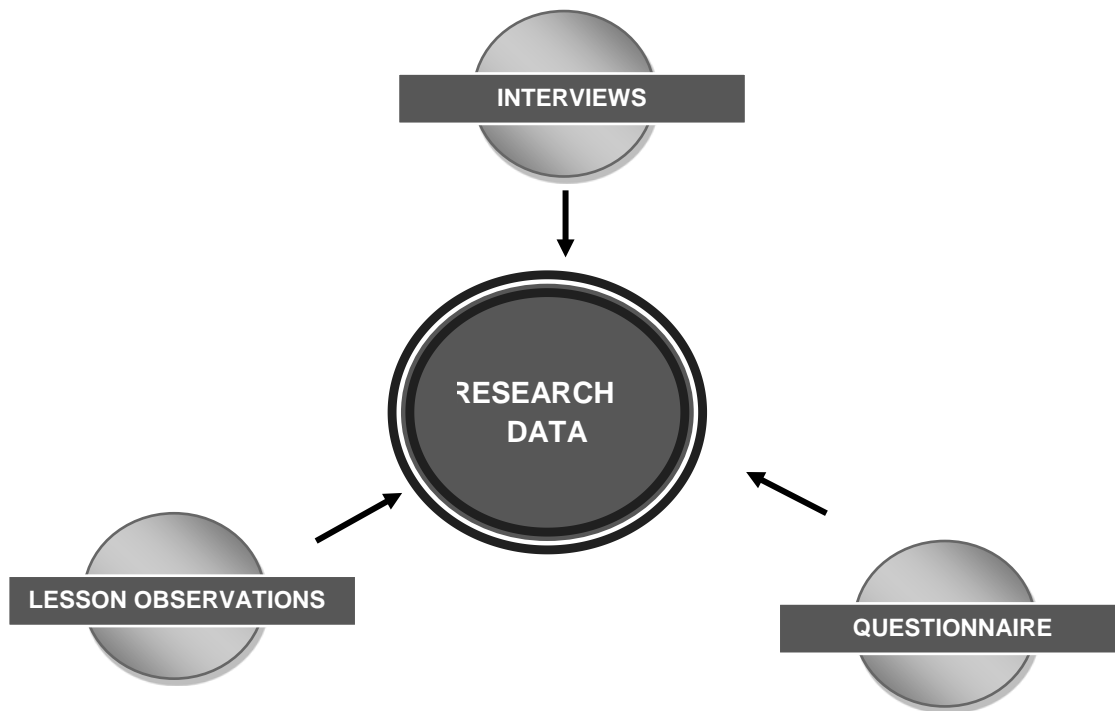


Figure 5.5: Generation of research data

5.5.2 Data generation

To gather sufficient data to effectively address the research questions asked, a combination of research methods that are shown in Figure 5.5 above were used.

5.5.2.1 Questionnaires

1) The teacher profiling questionnaire

The teachers worked on the questionnaire individually, as they were not permitted to discuss it with their colleagues. Each of the 48 participating teachers provided their biographical details and descriptive information on the following aspects:

- pedagogical knowledge related to problem-solving;
- beliefs about problem-solving, and about problem-solving as a vehicle for learning;
- the ability to select suitable problems for use as a vehicle for learning;
- own problem-solving skills.

This questionnaire was the main source of the research's quantitative data.

- **Teacher professional development evaluation questionnaire**

After the teacher professional development of the 15 randomly selected teachers from the five schools had three half-day workshops to introduce them to PBL, the teachers were

given an open-ended evaluation questionnaire through which they could evaluate the impact of the professional development that they had undergone.

5.5.2.2 Semi-structured interview schedule

Fifteen teachers were interviewed to determine the problems that they had experienced in using problem-solving as a vehicle for learning, and the support that they required in using problem-solving as a vehicle for learning.

The interviews enabled face-to-face discussion to take place with the teachers. Wisker (2009) points out that, if one decides to use interviews, one has to decide on whether one will take notes (which is distracting) during, or whether one will tape (accurate but time consuming), the interview. The other alternative is to rely on one's memory as to being able to recall what has been said, or to ask the respondents to write down their answers.

Reliability and validity of the semi-structured interview

The sequence and wording of all the questions in a semi-structured interview have to be the same for each respondent, so that any difference in answers is the result of the differences among the respondents, and not a result of how the questions are posed (Harris & Brown, 2010). Reliability and validity is achieved by standardising the questions asked. Hence, in a semi-structured interview, reliability and validity is achieved not by repeating the same words in each question, but rather by equivalence in meaning. The equivalence of meaning achieves the reliability and validity of semi-structured interview, and facilitates comparability (Barriball & While, 1994). Reliability and validity of semi-structured interviews is also enhanced by their audiotaping, and by the taking of comprehensive notes, and the systematic transcription and analysis of the responses received (Campbell et al., 2013). In view of the above comments, the researcher tape-recorded the interviews. Although doing so was time-consuming, it was decided on as the best option, as it was an accurate means of capturing the respondents' responses, while also serving to enhance the reliability and validity of the proceedings, as has already been pointed out above. With the five respondents who objected to being tape-recorded, the researcher took notes.

The most practical way of achieving validity with structured interviews is to minimise bias as much as possible (Cohen, Manion & Morrison, 2011). In the current study, the researcher tried to minimise bias by using open-ended questions that were neutral in nature, and he did not deliberately set out to obtain answers that supported his views. For example, he asked the question:

What has been your experience with regard to your planning for and your implementing of problem-based learning?

During the interview, closed-ended questions were used to elicit answers about fixed facts. The researcher was, thus, able to manage the data obtained, and to quantify the responses received, quite easily. However, the problem with such questions is that they limit the responses that the interviewees can give, and they neither enable them to think deeply about the issue at stake, nor do they test their real feelings or values about a topic (Wisker, 2007).

Open-ended questions were also used, as the asking of such questions serves to elicit an almost endless number of responses. In so doing, the researcher obtained a very good idea of the variety of ideas, knowledge and feelings that the teachers had with regard to problem-solving as a vehicle for learning. The asking of open-ended questions also enabled the teachers to think and to talk for much longer periods that they might otherwise have been able to do, and to express their knowledge and views more freely than would else have been possible. The semi-structured interviews also helped shed light on the quantitative data that were collected by means of the questionnaire. (See Appendix 12 for the interview schedule, containing a mixture of closed- and open-ended questions).

5.5.2.3 Observation schedule (classroom observations)

Kawulich (2005, p. 1) and the Centers for Disease Control and Prevention (CDC, 2008, p. 1) define observation as a way of gathering data by means of watching behaviour, and events, or of noting physical characteristics in their natural setting. Observations can be overt (with everyone who is observed knowing that they are being observed), or covert (with none who is being observed knowing that they are being observed). The benefit of covert observation is that those who are being observed are more likely to behave naturally if they do not know that they are being observed. However, one typically needs to conduct overt observations, because of the ethical problems that exist in relation to one concealing one's observation. Observation is characterised by such actions as having an open, nonjudgmental attitude; being interested in learning about others; being aware of the propensity for feeling culture shock, and for making mistakes, the majority of which can be overcome; being a careful observer, and a good listener; and being open to the unexpected in what is learned.

An observation schedule (see Appendix 7) was used to identify the extent to which the teachers' beliefs about problem-solving manifested themselves in the classroom, and to identify the teacher's ability to design lessons that provided the learners with opportunities to learn through problem-solving.

5.6 REASONS FOR USING OBSERVATION FOR DATA COLLECTION

Observation methods are useful to researchers for a variety of reasons. Kawulich (2005, p. 2) points out that observations provide researchers with ways of: checking for the nonverbal expression of feelings; determining who interacts with whom; grasping how participants communicate with one another; and checking how much time is spent on various activities. Secondly, the use of observation methods increases the validity of a study, as observations may help the researcher gain a better understanding of the context and phenomenon under study.

The validity of observation is stronger if it is used together with additional strategies, such as interviewing, document analysis, surveys, questionnaires, or other more quantitative methods. Kawulich (2005) lists five reasons for including participant observation in research, of which all are likely to increase the study's validity:

- 1) It makes it possible to collect different types of data. Being on-site over a period of time familiarises researchers with the community, thereby facilitating their involvement in sensitive activities to which they generally would not be invited.
- 2) It reduces the incidence of 'reactivity', or of people acting in a certain way when they are aware of being observed.
- 3) It helps the researcher to develop questions that make sense in the native language, or which are culturally relevant.
- 4) It gives the researcher a better understanding of what is happening in the culture of the subjects, and it lends credence to one's interpretations of the observation. Participant observation also enables the researcher to collect both quantitative and qualitative data by means of surveys and interviews.
- 5) It is sometimes the only way by means of which to collect the right data for one's study.

In the current study, the researcher used observations for the advantages that such a method provides. Observer bias was minimised by the nature of the issues observed, which the observer recorded, as guided by the observation schedule. The 'Hawthorne effect' was not of major concern, because, even though the researcher was honest, he was neither too technical, nor overly detailed, in explaining to the participants what exactly he would be observing (Kawulich, 2005).

The researcher had spent many hours in the classroom of participating teachers, where he was allowed to observe their lessons in an atmosphere of mutual trust. The researcher's role

inside the class has, throughout the years of the project, developed from being one of being a passive observer, to one of being an active, participating observer, and, around the time of the study, a co-teacher. The researcher adhered to Chesterfield's (1997, p. 9) good observation practices by being sensitive, considerate and helpful whenever possible; by recognising the teacher as the expert in respect of what was taking place in the class; by interacting with the teacher, learners and other staff members; and by being less of an evaluator or judge, and more of a listener and confidant.

5.6.1 The processes of conducting observations

Kawulich (2005, p. 2), in focusing on the process of conducting observations, describes three types of processes:

- 1) The first is *descriptive observation*, in which one observes anything and everything, assuming total lack of knowledge about what is observed. The disadvantage of this type of conducting observations is that it can lead to the collection of details that are not necessarily relevant to the study.
- 2) The second type of observation, namely *focused observation*, involves the observation being supported by interviews, in terms of which the participants' insights guide the researcher's decisions about what to observe.
- 3) The third type of observation is *selective observation*, in which the researcher focuses on different types of activities to help delineate the differences in the activities.

The above processes considered, the researcher in the current study opted for *focused observation* of specific issues in the lessons being observed. An observation schedule was developed, in which the researcher made notes about the various elements to be recorded. The observations were systemised by means of the use of the observation schedule, which facilitated the recording of, and the focusing on, the teachers' beliefs about problem-solving, and how these manifested themselves in classrooms. Each teacher's ability to design lessons that provided their learners with opportunities to learn through problem-solving was also recorded and emphasised in this way.

5.6.2 Collecting useful observation data

Kawulich (2005) provides several strategies for conducting observations after one has gained entry into the study setting. Some of the strategies that the researcher adopted in the present instance were being unobtrusive in dress and actions; becoming familiar with the setting before beginning to collect the data; keeping the observations short at first, to keep those involved from becoming overwhelmed; and being honest, but neither too technical, nor

too detailed, in explaining to participants what he was doing; and actively observing details that were later recorded.

Observation findings are considered to be more trustworthy when a researcher can show that they spent a considerable amount of time in the study setting. In the present study, the prolonged interaction enabled the researcher to gain more opportunities to observe and participate in a variety of activities than he would otherwise have had (Kawulich, 2005). At the time of the research, the researcher had spent two years working with the teachers involved, hence it can be seen that a sufficient amount of time had been spent in the setting.

During the early months of the project, the researcher followed Chesterfield's (1997, p. 11) recommended observation strategies of being a non-participant observer, and of not being part of the central activity in the classroom. The researcher sat at a learner's desk which was usually located at the back of the class, and did not take part in what was happening in the classroom activity, but rather focused on the teacher-learner interaction. This relationship building process resulted in the researcher's acceptance by the teachers in their classrooms, as well as reducing the threat of being seen to be observed, hence lessening the impact of observer bias.

5.7 DATA ANALYSIS

The data were analysed both qualitatively and quantitatively in so far as they addressed the research questions. Theoretical coding was used to code the various data. The data were analysed into theoretical codes using ATLAS.ti. Thornberg and Charmaz (2014, p. 159) describe theoretical coding as:

The identification and use of appropriate theoretical codes to achieve an integrated theoretical frame work. Theoretical code consists of ideas and perspectives that the research imported to the research process as analytic tools and lenses from outside, from range of theories.

For example, the researcher used the theories of constructivism (see Section 2.6); MKT (see Section 2.7); the traditional and non-traditional perspectives on the teaching and learning of mathematics (see subsections 2.12.2 and 2.12.3); and the characteristics of the problems that were set for problem-solving (see subSection 3.5.5) to create theoretical codes (i.e. analytic tools and lenses) with which to analyse the teachers' responses to the various research questions.

The qualitative data were analysed, using a thematic approach in terms of which the data were categorised according to themes (Ormston, Spencer, Barnard & Snape, 2013). The

analysis of qualitative data took the form of the systematic process of coding, categorising and interpreting the data to provide explanations in relation to the research questions asked (McMillan & Schumacher, 2006, p. 364). Figure 5.6 below indicates the general process of data analysis.

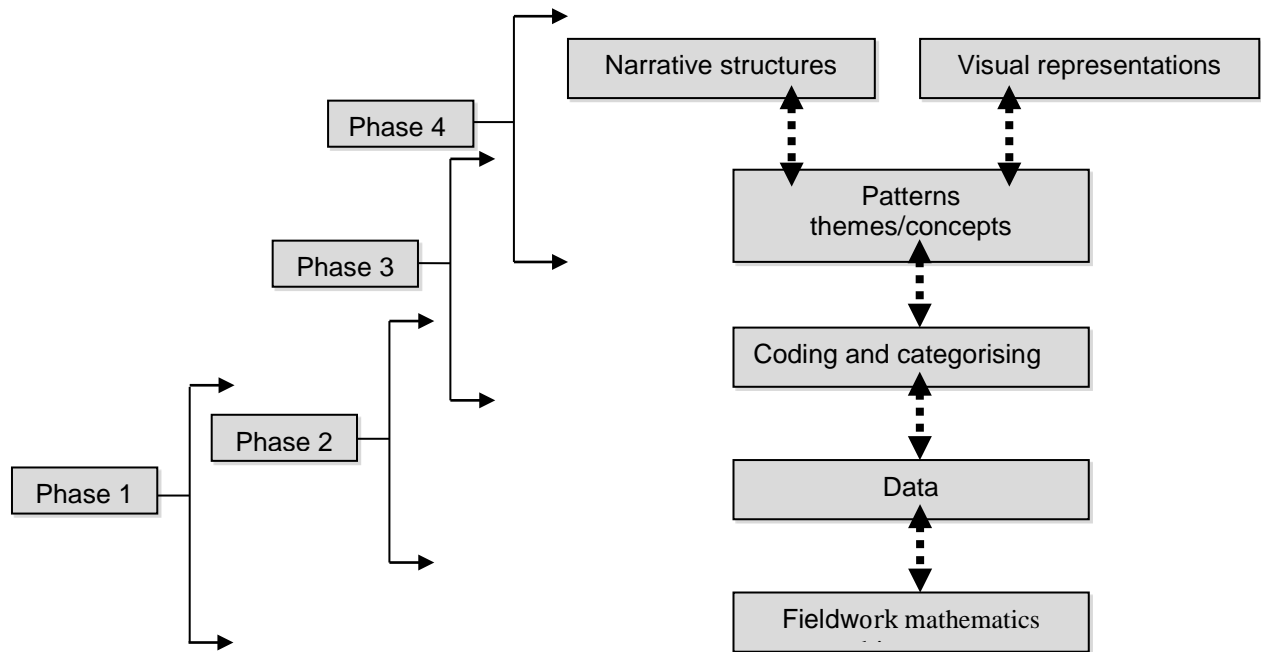


Figure 5.6: General process of data analysis (adapted from McMillan & Schumacher, 2006, p. 365)

The data analysis process had four iterative phases because, as the researcher moved from phase to phase, he constantly returned to double-check and refine his analysis and interpretations (McMillan & Schumacher, 2006, p. 364). The network that was generated by ATLAS.ti allowed for comprehensive and transparent data analysis (Ormston et al., 2013).

5.8 SUMMARY OF THE RESEARCH METHODOLOGY PROCESS

The summary of the research methodology process is presented in Figure 5.7 below. After the questionnaire was administered, professional development was organised for a sample of the respondents, based on their needs that were identified during the questionnaire data analysis.

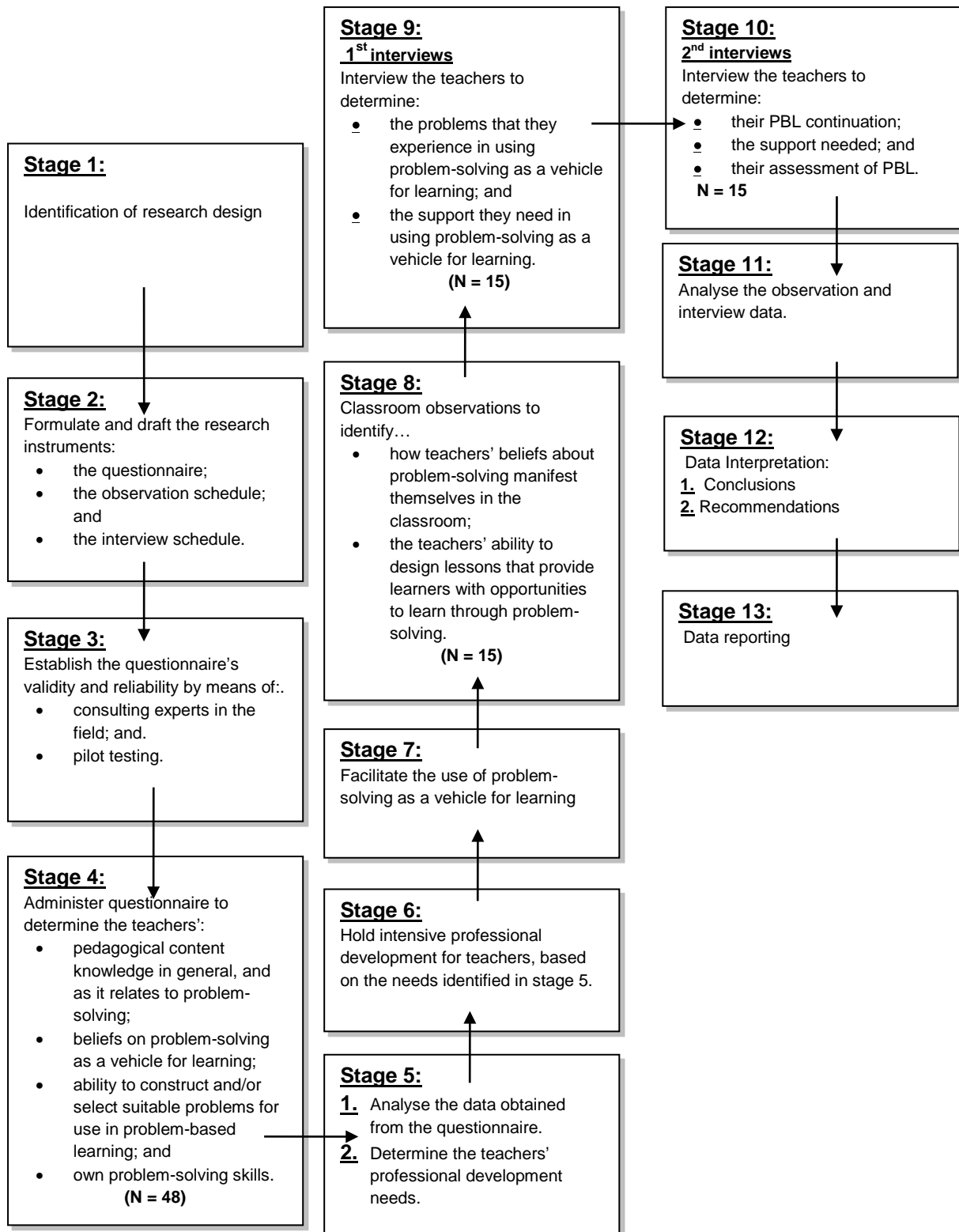


Figure 5.7: Summary of the research methodology process

After their professional development, the teachers were expected to use problem-solving as a vehicle for learning. They were also observed using PBL to collect data to determine how teachers' beliefs about problem-solving manifest themselves in the classroom, as well as to

determine their ability to design lessons that provided learners with opportunities to learn through problem-solving.

The stratified random sample of 15 teachers was also interviewed using the semi-structured interview schedule, so as to collect sufficient data to determine what problems they experienced, and the support that they required in using problem-solving as a vehicle for learning.

5.9 ADHERENCE TO ETHICAL STANDARDS

Whereas subsection 1.6.1 focused earlier on the ethical considerations of research, this section briefly describes how the ethical considerations concerned were adhered to during the study. Before the data collection started, permission was obtained from the WCED, the school principals, and the participating teachers (see Appendix 3). Eiselen and Uys (2005, p. 4) point out that the rights of respondents as human beings should be respected at all times. The researcher adhered to the rights standards, as are outlined below. (See Appendix 2 for the information and consent sheet used in this regard.)

- The teachers who participated in the research were made aware of what was required of them in relation thereto. Each individual teacher was informed that the decision to take part in a survey by completing the questionnaire was their own choice, and that not participating in it would not affect their receiving support from the project.
- Each teacher was made aware that, should they wish to withdraw from the study at any time, and not provide their reasons for such withdrawal, that they would be able to do so freely, without fear of being victimised regarding their decision not to participate in the project further.
- At no time did the researcher coerce any teacher into providing information, especially information that might be perceived as sensitive or incriminating.
- Each individual teacher was given the assurance that their responses would remain anonymous, and that the information that they provided would be treated as confidential at all times.

The above rights were adhered to not only to protect the human rights and welfare of the teachers, but also to minimise the risk of physical and mental discomfort, harm and/or danger from the research procedures (Canterbury Christ Church University, 2006).

5.10 CONCLUSION

This chapter has focused on the research methodology, detailing the research design, the instruments that were used in the research, and the process of establishing the validity and reliability of the research questionnaire. The chapter also considered the research's data generation mechanism, and the data analysis process.

The next chapter focuses on the presentation of the research data that were collected from the research questionnaire, and on the open-ended questionnaire that was administered after the professional development had taken place. It covers the time span of the study from when the observations were made in relation to PBL during the teachers' lessons, the period of, post, staff development in relation to PBL, and the first and second set of interviews that were held with the teachers thereafter.

CHAPTER 6: DATA PRESENTATION, ANALYSIS AND DISCUSSION

“Data analysis is the process of developing answers to questions through the examination and interpretation of data. The basic steps in the analytic process consist of identifying issues, determining the availability of suitable data, deciding on which methods are appropriate for answering the questions of interest, applying the methods and evaluating, summarizing and communicating the results.”

(Statistics Canada, 2009 p. 1)

6.1 INTRODUCTION

The purpose of data presentation and analysis in a study is to answer the research questions set, and to determine the different trends and relationships among the variables. This chapter presents the analysis of the gathered data, together with the researcher's interpretation of the results that were obtained from the completed questionnaires, the lesson observations, and the teacher interviews. The data are presented, mostly through tables and figures, in the following seven sections:

- Section 1: Data from the questionnaire (p. 157).
- Section 2: Data from the workshop evaluation (p. 194).
- Section 3: Data from the lesson observations (p. 198).
- Section 4: Data from the teacher's reflections on the lesson (p. 209).
- Section 5: Data from the problems used by the teachers in their PBL lessons (p. 218).
- Section 6: Data from the first teacher interviews (p. 218).
- Section 7: Data from the second teacher interviews (p.225).

6.2 SECTION 1: DATA FROM THE QUESTIONNAIRE

The questionnaire had nine Subsections, namely:

1. Mind map on problem-solving
2. Approach to the teaching and learning of mathematics
3. Approach to problem-solving
4. Self-evaluation
5. Approach to problem-solving lessons

6. Teachers' beliefs about the teaching and learning of mathematics
7. Teachers' problem-solving skills
8. Selection of suitable teaching/learning materials
9. Demographic data, including access to key documents on problem-solving, the problem-solving workshop, and the attendants' biographical data.

The data in this chapter are presented and discussed according to the nine sections listed above. Each section describes a set of related measures that were designed to be assessed and analysed separately (Clark & Watson, 1995).

Reliability of the questionnaire

Calculation of Cronbach's alpha: Cronbach's alpha is a measure of internal consistency (i.e. reliability). It measures how closely related a set of items is as a group. It is most commonly used when a questionnaire contains multiple Likert-type questions that form a scale, and when one wishes to determine whether the scale used is reliable. The alpha is expressed as a number between 0 and 1. Cronbach's alpha, which describes the extent to which all items in a test measure the same concept or construct, should be determined before a test can be employed for research, so as to ensure the validity of the test instrument concerned (Tavakol & Dennick, 2011).

In the current study, the questionnaire contained more than one concept or construct. Secondly, both qualitative and quantitative data were gathered. Therefore, it did not make sense to report the alpha for the questionnaire as a whole, as it was inevitable that most of the questions would inevitably inflate the value of the alpha (Tavakol & Dennick, 2011, p. 54). Thus, the alpha was only calculated for the sections of the questionnaire that elicited quantitative data, using a 5-point Likert scale.

6.2.1 Subsection 1: Mind map on problem-solving

This section presents the data that were obtained in relation to Question 1, which was a mind map about problem-solving. In the question, the respondents were required to answer the question by writing, in each bubble, what they thought about problem-solving (see Figure 6.1 below).

Q1: Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think:

- a) Problem-solving is.
- b) Problem-solving is not.
- c) Problem-solving requires of the teacher.
- d) Problem-solving requires of the learner.

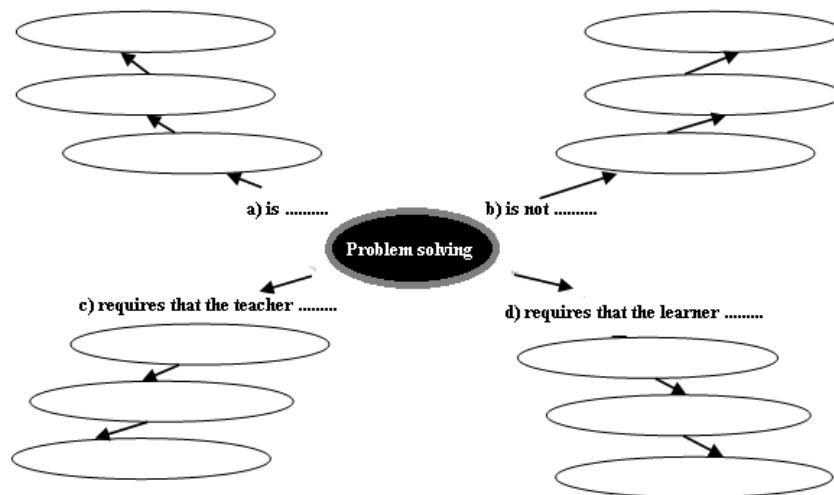


Figure 6.1: Mind map about problem-solving

The data from this section, which were all qualitative, were transferred onto a data column sheet, and then analysed using ATLAS.ti. Besides improving the efficiency of a qualitative data analysis, ATLAS.ti improves the transparency of data analysis, thereby enhancing the validity of the research involved (Zhang & Wildemuth, 2009).

6.2.1.1 Mind map bubble a): What problem-solving is

In Subsections 3.3.1 to 3.3.3, three views on what problem-solving is about emerged. The first view expressed, regarding teaching *about* problem-solving, was that problem-solving is about the process or strategies for problem-solving, in terms of which the focus is on teaching the learners how to problem solve (Subsection 3.3.1). In regard to teaching *for* problem-solving, the view that emerged was that problem-solving is about the development of the learners' problem-solving skills and strategies that are aimed at solving routine, or nonroutine, problems (see Subsection 3.3.2). The third view, regarding teaching *via* problem-solving, was that problem-solving is about using problems for developing mathematical concepts, skills and procedures. In such terms, learning is problem-simulated,

or it starts with a problem (see Subsection 3.3.3). The teachers' views on what problem-solving is were analysed according to the three views described above, as based on the relevant literature. The views that were coded 'ambiguous' were those that could not be conclusively categorised in any of the aforementioned categories. Table 6.1 shows the results of this analysis.

<p>Q1: Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think...</p> <p>Problem-solving is ...</p>	The percentage of teachers who thought problem-solving is...			
	about the problem-solving process, or strategies for problem-solving.	about the development of the learners' problem-solving skills and strategies.	about the use of problem-solving to develop mathematical concepts, skills and procedures.	Ambiguous views expressed on problem-solving.
	21%	56%	5%	18%
	(Teaching <i>about</i> problem-solving)	(Teaching <i>for</i> problem-solving)	(Teaching <i>via</i> problem-solving)	
N = 48 (Non-responses = 6)				

Table 6.1: Teachers' views on what problem-solving is

Of the 42 teachers who completed this section, only 5% of the teachers described problem-solving as a means of developing mathematical concepts, skills and procedures. Extracts 1 and 2 below are examples of such teachers' thoughts on the issue.

Extract 1: "A way of thinking. Practical. A way of learning a new concept."

Extract 2: "Most effective method of teaching mathematics. Creating logical thinking skills. Good in developing logical mind."

The majority (56%) of the teachers thought problem-solving was about the development of the learners' problem-solving skills and strategies.

Extract 3: “Discuss different ways to get to the answer. Children struggle to find answers on their own. Present problem in a story form.”

Extract 4: “About solving the problem but about how you get your answer...”

Of the teachers, 21% thought problem-solving was about the process or strategies for problem-solving.

Extract 5: “Helping to improve the skills to problem-solving. Practical. Improves mathematics skills.”

Of the teachers' views, 18% were too brief, or not clear enough, to be categorised into any of the three categories. The teachers concerned would have needed to have been asked follow-up questions to seek clarification of what they wrote. Extracts 6 and 7 below are examples of such views.

Extract 6: “Interesting, challenging, what you do.”

Extract 7: “They can think orally. Where the learners can communicate.”

Discussion and analysis: The data given above reveal that for the majority of the teachers, problem-solving is about the development of the learners' problem-solving skills and strategies. The fact that only 5% of the teachers' views referred to the use of problem-solving for the development of mathematical concepts, skills and procedures implies that only 5% of them were likely to have held a constructivist view of teaching and learning mathematics, in terms of which the learners are regarded as active participants in the learning process.

6.2.1.2 Mind map bubble b): What problem-solving is not

Only 30 (63%) of the 48 teachers who completed the questionnaire completed this section of question 1. The teachers' responses on what problem-solving is not were analysed using the same theoretical understanding and themes as were given in mind map bubbles (A). The teachers' responses were categorised into the three theoretical perspectives on problem-solving (see Table 6.2 below).

<p>Q1: Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think...</p> <p>Problem-solving is not...</p>	The percentage of teachers who thought problem-solving is not ...			
	about processes or strategies for problem-solving.	about the development of the learners' problem-solving skills and strategies.	about the use of problem-solving to develop mathematical concepts, skills and procedures.	Ambiguous views expressed on problem-solving.
	20%	27%	0%	53%
	(Teaching <u>about</u> problem-solving)	(Teaching <u>for</u> problem-solving)	(Teaching <u>via</u> problem-solving)	
N = 48 (Non-responses = 18)				

Table 6.2: Teachers' views on what problem-solving is not

None of the 30 teachers who completed this section described problem-solving as not "being used as a means to develop mathematical concepts, skills and procedures". A significant number (27%) of the teachers' responses focused on strategies or processes for problem-solving. Extract 8 below is an example of such a response.

Extract 8: "Is not about addition or subtraction. Drilling learners to know. A mental calculation."

The majority (53%) of the teachers' views were too brief, or not clear enough, to be conclusively categorised into any of the three categories. In these case, follow-up questions would have had to have been asked to seek clarification. Extracts 6 and 7 below are examples of such views.

Extract 9: "Boring."

Extract 10: "Always easy. Specific rules."

Discussion and analysis: It was difficult for the teachers to explain their views clearly about what problem-solving is NOT. Hence, most of the responses that were given in this section were difficult to analyse, as they did not contain sufficient meaning on which to base the classification.

6.2.1.3 Mind map bubble c): What problem-solving requires of the teacher

This part of the mind map required the teachers to outline the role of the teacher during problem-solving. In subsections 3.3.1 to 3.3.3 of the literature review, three main views emerged about the role of the teacher during problem-solving. In teaching via problem-solving, the teacher's main role is to use problem-solving to develop mathematical concepts, skills and procedures (see Subsection 3.3.3). In teaching about problem-solving, the role of the teacher is to teach the learners how to problem solve (see Subsection 3.3.1). In teaching for problem-solving, the teacher's main role is that of monitoring, facilitating problem-solving, and modelling problem-solving behaviour (Subsection 3.3.2).

The 42 teachers' responses that involved completing the phrase "*Problem-solving requires the teachers to...*" were coded using the three views that emerged from the literature review on the role of the teacher during problem-solving. Table 6.3 shows the results of this process.

<p>Q1: Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think...</p> <p>Problem-solving requires of the teacher...</p>	The percentage of teachers who thought problem-solving requires of the teacher ...			
	to use problem-solving to develop mathematical concepts, skills and procedures.	to teach the learners how to problem solve.	to monitor, facilitate problem-solving, and model problem-solving behaviour.	Ambiguous views expressed on the role of the teacher.
	5%	25%	56%	14%
	(Teaching <u>via</u> problem-solving)	(Teaching <u>about</u> problem-solving)	(Teaching <u>for</u> problem-solving)	
N = 48 (Non-responses = 6)				

Table 6.3: Teachers' views on what problem-solving requires of the teacher...

Table 6.3 above shows that the majority of the teachers (56%) viewed the role of the teacher as being that of monitoring, facilitating problem-solving, and showing and modelling problem-solving behaviour. Extracts 11 and 12 below are examples of such teachers' responses.

Extract 11: “Do group work. *Show different ways of getting to the answer.* Help the kids to understand concepts by explaining.”

Extract 12: “Analyse the problem. Ask questions such as ‘what if?’ to explain how to solve.”

The views expressed by the teachers whose responses are given above showed some level of the encouragement of learner participation in problem-solving. However, one notes that this was a behaviourist, and not a constructivist approach, to problem-solving, in which learner involvement occurs in situations where the learners construct their own understanding through problem-solving and group discussion. Rather, it is evidence of use of the traditional teaching style, which relies heavily on the teacher exposition of fundamental problem-solving behaviour, which is then consolidated by means of practice that takes place in groups. The teacher delivers the ready-made problem-solving strategies and skills to the learners by *showing and demonstrating how to use mathematics* to solve problems during problem-solving.

Some examples of extracts of traditional teaching style are:

Extract 11: “First the educator solves, then the learners solve their own.”

Extract 12: “Is able to show the children how to solve the problem”

Extract 13: “Introduce, demonstrate...teach...assess”

The above extracts reveal that the majority of the teachers (81%) were more inclined to see their role as directly transmitting strategies and skills for problem-solving, rather than that of supporting active learning. In terms of such teaching, the learners are given opportunities to practise the strategies and skills concerned through solving given problems. Extract 14 below shows the view that the learning of mathematics is teacher-stimulated, rather than problem-stimulated.

Extract 14: “To understand that he can teach, approach other concepts through problem-solving.”

The role of the teacher during problem-solving was viewed by the majority of the teachers in terms of their traditional role, being that of the teacher being a transmitter of knowledge, and one who leads the learners, gives instructions and defines the problem. This is done by

showing and demonstrating to the learners, and by means of solving problems, and explaining them to the learners, as is shown in the following two extracts:

Extract 11: “First the educator solves, then the learners solve their own.”

Extract 12: “...show the children how to solve the problem”.

These extracts show that, though the teachers facilitated problem-solving in their classes, it was not viewed as an essential vehicle for learning in the same way as it is in PBL. The teachers tended to have very teacher-centred views about problem-solving, as in the case of the teacher (extract 11) who said that problem-solving requires that “*first the educator solves then the learners solve their own*”.

6.2.1.4 Mind map bubble d): What problem-solving requires of the learner

In the literature review in subsections 3.3.1 to 3.3.3, the three distinct roles that the learners play in problem-solving emerged. In teaching about problem-solving, the focus is on the learner's ability to understand the problem, to design a problem-solving strategy, to implement the strategy, and to look back on, or to check on the correctness of their solution (Subsection 3.3.1). In teaching for problem-solving, the learner applies the acquired mathematical knowledge to the solving of routine or nonroutine problems (Subsection 3.3.2). In teaching via problem-solving, the learner constructs their own knowledge of mathematical concepts, processes and techniques. The teachers' responses in completing the phrase *problem-solving requires the learner to...* were analysed according to the learner roles mentioned. Table 6.4 below shows the results of this analysis.

Q1: Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think... Problem-solving requires of the learner ...	Percentage of teachers who thought problem-solving requires of the learner ...			
	to learn to use taught problem-solving techniques to solve routine or nonroutine problems.	to understand the problem, design a problem-solving strategy, implement the strategy	to construct own knowledge, understand mathematical concepts, processes and techniques	Views not clear.
	27%	51%	2%	20%
	(Teaching <u>about</u> problem-solving)	(Teaching <u>for</u> problem-solving)	(Teaching <u>via</u> problem-solving)	
N = 48 (Non-responses = 7)				

Table 6.4: Teachers' views on what problem-solving requires of the learner...

The majority of the teachers (51%) thought that problem-solving requires the learner to understand the problem, to design a problem-solving strategy, and to implement the strategy, which translates to teaching for problem-solving. Extracts 17 and 18 below are examples of such views.

Extract 17: “Use *methods* they already understand as well as new *methods* to find the solutions.”

Extract 18: “To identify the problem. Contribute alternative *solutions* and brainstorm. Select a *solution* to solve the problem.”

The focus of the teachers concerned was on the solution or method used, and not on the knowledge, or understanding, of the mathematical concepts. This is in contrast to the views of the teacher (in Extract 19) who considered that learners construct their own knowledge and understanding of mathematical concepts, processes and techniques.

Extract 19: “Think logical. Participate in *effective learning process*. To be logical thinkers to grasp quickly.”

In the above extract, the teacher’s views are that problem-solving is an effective learning process, in which the learner is an active participant. Problem-solving is not for the learner to use taught problem-solving techniques to solve routine or nonroutine problems, as was expressed by 27% of the teachers, and which is illustrated in extracts 20, 21 and 22 below.

Extract 20: “Understand the *rules, basic operations*. Understand the language.”

Extract 21: “Use different strategies to lead to correct answer”

Extract 22: “Must be able to answer the question. Is able to identify *the operations*. Understand *the basics*.”

The teachers’ responses above focus on the learner’s acquired mathematical knowledge respecting the solving of problems. Hence, there is a need for the learners to understand the rules, the basics and basic operations to apply in problem-solving.

6.2.1.5 Discussion, and factual and interpretative conclusions

The analysis of the data that were obtained in relation to the mind map on problem-solving reveals that, to the majority of the teachers in the study, problem-solving was about the development of the learners’ problem-solving skills and strategies, using their acquired

mathematical knowledge. Their views were that, during the problem-solving and knowledge application process, the role of the teacher is to show, explain and demonstrate the strategies for problem-solving. Problem-solving was also seen as using the learnt mathematics to solve problems, and, therefore, as a means of applying the known mathematics. Their view was that problem-solving is the cornerstone of mathematics. Hence, without the ability to solve problems, the usefulness and the power of mathematical ideas, knowledge and skills is limited. Unless learners can solve problems, the mathematical facts, concepts and procedures that they know are of little use. The teachers are, consequently, teaching mathematics for problem-solving.

6.2.2 Subsection 2: Approach to the teaching and learning of mathematics

Subsection 2 was made up of three questions. The responses to the questions in this section used a 6-point Likert scale; hence, Cronbach's alpha was calculated to determine the reliability of this set of questions as a scale of measure.

6.2.2.1 Scale reliability

Tables 6.5 and 6.6 below show the **reliability statistics** and the **item-total statistics**, respectively calculated using SPSS to obtain Cronbach's alpha, for the scale *Approach to the teaching and learning of mathematics*.

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of items
.656	.680	3

Table 6.5: Reliability statistics: approach to the teaching and learning of mathematics.

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item-total correlation	Squared multiple correlation	Cronbach's alpha, if item deleted
Q2	9.65	3.566	.562	.334	.418
Q3	9.35	5.614	.456	.239	.624
Q4	10.21	3.408	.461	.219	.601

Table 6.6: Item-total statistics: approach to problem-solving

The reliability statistics table presented above shows a Cronbach's alpha of 0.656 (0.7 to 1 decimal place). According to Hanneman (2006), the 'rule of thumb' is that the alpha should exceed 0.8. The literature reviewed in Chapter 5 also reveals that, in practice, scales with lower reliabilities are often used (and productively so). A cut-off point of 0.7 is considered,

which can realistically be acceptable, because of the diversity of the constructs being measured, and the other desirable properties, such as meaningful content coverage (Field, 2006; Hanneman, 2006; Santos, 1999; Schmitt, 1996). The set of questions had meaningful content related to teaching practices. It was because of such justification that this set of questions was kept. (See Subsection 5.4.2.2 for the detailed motivation to use lower thresholds of Cronbach's alpha).

6.2.2.2 Data presentation

In this subsection, an analysis of all the questions that sought to determine the teachers' views and practices in teaching mathematics in general is presented. The analysis of data in this subsection is guided by the literature review. In the literature review in Section 2.12, two opposing views were found. On one hand was the view that mathematics is learned by dispensing, or transmitting, knowledge to the learners. This is referred to as the direct transmission approach (Confrey, 1990, p. 107). The teachers who hold this view believe, among other things, that learners passively receive knowledge from the teacher. Their teaching reflects characteristics that imply that teaching mathematics involves showing and telling, with the teachers simply explaining and demonstrating. This teaching of mathematics is, in many ways, teacher-centred.

The other view is that the learners are active participants in the learning process, in terms of which the teacher facilitates the process of knowledge construction (Roesken & Toerner, 2011). This view is in line with the current views of constructivism that were discussed in Subsection 2.6.5. This learning process leads to relational understanding, as was debated in subSection 2.3.2. Table 3.3 presents data on the approach to teaching and learning mathematics, based on the SPSS tables that were generated for each question.

This year in your mathematics lessons, how often did you do the following? Mark X for each item.							
Q	Activity	Never	Less than once a month	1–3 times per month	1–3 times per week	3–4 times per week	Every day
Q2	Ask the learners to do pen-and-paper calculations, and to practise.	5%	0%	7%	19%	23%	47%
Q3	Demonstrate to the class a procedure on the chalkboard, and then let the learners practise.	0%	0%	2%	17%	37%	56%
Q4	Teach the learners mathematics by focusing on rules and procedures.	4%	7%	0%	17%	22%	39%
Q2 (N) = 48 Missing values = 5 Q3 (N) = 48 Missing values = 7 Q4 (N) = 48 Missing values = 2							

Table 6.7: Responses to questions on the approach to the teaching and learning of mathematics

6.2.2.3 Discussion of the data

Of the teachers, 43 responded to question 2, of whom 70% asked the learners to do pen-and-paper calculations, and to practise the procedure(s) more than 4 times a week, whereas 47% did so daily. Of the 41 teachers who responded to question 3, 93% stated that they taught mathematics by demonstrating the procedures on the chalkboard, and then letting the learners practise the procedures. The majority of the teachers did so daily. Of the 46 teachers who responded to question 4, 61% stated that they normally taught their learners mathematics by focusing on the rules and procedures. The majority of the teachers were also doing so daily.

6.2.2.4 Discussion, and factual and interpretative conclusions

In summary, the analysis of data presented in Table 3.3 revealed that the majority of the 48 teachers who took part in this study followed a traditional approach of direct transmission, or dispensing, of knowledge. Such an approach is characterised by focusing on rules and procedures, demonstrating to the class on the chalkboard, and then letting the learners practise through making pen-and-paper calculations. The majority of the teachers did so daily. To the Foundation Phase teachers concerned, the memorisation and mastery of basic facts and procedure provided primary evidence of learning mathematics. They express a belief that the affirmation of mathematics learning is shown through expert memorisation and the performance of procedures, rather than through the ability to explain one's

understanding of the material involved. It would appear that such teachers' lessons simply demonstrated how to conduct a procedure. Lessons like this do not encourage the development of understanding, because they do not ensure that learners will remember the procedure involved, or improve their adaptive reasoning powers (Gojak, 2012).

6.2.3 Subsection 3: Approach to problem-solving

The subsection *Approach to problem-solving* was made up of six questions, as is shown in Table 6.8 below. The responses to the questions in this section used a 4-point Likert scale. Hence, Cronbach's alpha was calculated as a scale of measure to determine the reliability of this set of questions.

6.2.3.1 Scale reliability

Tables 6.8 and 6.9 below show the reliability statistics and the item-total statistics tables for the scale *Approach to problem-solving*.

Cronbach's alpha	Cronbach's alpha based on standardised items	N. of items
.563	.570	6

Table 6.8: Reliability statistics: Approach to problem-solving

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item-total correlation	Squared multiple correlation	Cronbach's alpha, if item deleted
Q9	10.67	5.702	.225	.198	.555
Q10	11.03	5.078	.456	.378	.446
Q11	10.41	5.301	.320	.176	.510
Q12	11.10	5.463	.272	.336	.534
Q13	11.10	6.305	.275	.269	.535
Q14	11.08	5.389	.304	.270	.518

Table 6.9: Item-total statistics: Approach to problem-solving

The last column "Cronbach's alpha, if item deleted" in Table 6.9 above, which shows what the Cronbach's alpha would be if one got rid of a particular item, is very important. For example, at the top of this column, the number is .555. This means that the Cronbach's alpha of this scale would drop from .563 to .555 if one got rid of that particular item. Since the higher an alpha is the greater is its indication of reliability, it would not be advisable to get rid of the first item. In fact, if one looks down the "Cronbach's alpha if item deleted" column, one can see that no value is greater than the current alpha of the whole scale: .563. This

means that one does not need to get rid of any items. Therefore, none of the questions was removed.

As a result of the above, this scale had a Cronbach's alpha of 0.563 (0.6 to one decimal place). The level of internal consistency for the scale was low, because a Cronbach's alpha of 0.7 is considered to be an acceptable reliability coefficient in the social sciences. However, lower thresholds are sometimes used (Santos, 1999). The items for this scale were maintained due to the reasons that were earlier discussed in Subsection 5.4.2. The reasons were that "when a measure has other desirable properties, such as meaningful content coverage of some domain...low reliability may not be a major impediment to its use" (Schmit, 1996, p. 352). It also fell within the minimally acceptable range 0.6 to 0.64. (DeVellis, 2003).

6.2.3.2 Data presentation

The responses to the six items in this scale were analysed using SPSS. Table 6.10 below shows the responses that were obtained in relation to how the teachers approached problem-solving.

To what extent do you agree or disagree with the following statements? Mark X to show whether you strongly agree, agree, disagree or strongly disagree.					
Q	Statement	Strongly agree	Agree	Disagree	Strongly disagree
Q9	Problem-solving is about word sums.	13%	37%	43%	7%
Q10	Teaching should be built up around problems with clear, correct answers, and around ideas that most students can quickly grasp.	20%	55%	18%	7%
Q11	A quiet classroom is generally required for effective learning to take place.	9%	27%	51%	13%
Q12	Learners should learn and master basic number facts before they do problem-solving.	35%	30%	33%	2%
Q13	How much students learn depends on how much background knowledge they have: that is why the teaching basic facts is necessary.	16%	68%	16%	0
Q14	Effective/Good teachers demonstrate the correct way of solving a problem.	33%	47%	26%	4%
Q9 N = (48) Missing values = 2 Q10 N = (48) Missing values = 4 Q11 N = (48) Missing values = 3 Q12 N = (48) Missing values = 2 Q13 N = (48) Missing values = 5 Q14 N = (48) Missing values = 5					

Table 6.10: Responses obtained in response to questions on the approach to problem-solving

6.2.3.3 Discussion of data

Of the teachers, 46 responded to question 9, with 50% (23) stating that they were of the opinion that problem-solving is about word sums. Of the 44 teachers who responded to question 10, 33 (75%) were of the opinion that teaching should be built up around problems with clear, correct answers, and around ideas that most learners can quickly grasp. Of the 45 teachers who responded to question 11, 36% thought that a quiet classroom is generally required for effective learning. Out of the 46 teachers who responded to question 12, 65% (30) were of the opinion that the learners should master basic number facts before they attempt problem-solving. Of the 43 teachers who responded to question 13, 84% thought that much of what the learners learn depends on how much background knowledge they possess. That is why teaching basic facts was so important for the teachers involved. Lastly,

72% of the 47 teachers who responded to question 14 thought that effective/good teachers demonstrate the correct way to solve a problem.

6.2.3.4 Discussion, and factual and interpretative conclusions

In Subsection 2.12.1, several traditional teachers' views about the teaching and learning of mathematics were presented. The main traditional view was that the teacher's role was to transmit to the learner those mathematical skills, facts, and procedures deemed necessary for the learners' successful performance in mathematics. Therefore, the teachers demonstrated these skills, facts and procedures, because they regarded themselves as the instruments by which mathematical knowledge is communicated. In terms of such thinking, the learners were expected to quietly and dutifully receive the mathematical skills, facts, and procedures from them.

An analysis of the data presented in Table 6.10 shows the presence of certain traditional features in the majority of the teachers' views, which was exhibited by them agreeing or strongly agreeing with the following statements:

- Effective/Good teachers demonstrate the correct way to solve a problem (80%).
- Learners should learn and master basic number facts before they do problem-solving (65%).
- Teaching should be built around problems with clear, correct answers, and around ideas that most learners can quickly grasp (75%).

Considering the data presented in this section, one can see that, on the one hand, there is the presence of the traditional behaviourist view, which views the teacher's role as that of transmitting knowledge and of demonstrating correct ways of solving problems. On the other hand, there is the constructivist view that sees the teacher's role as being that of facilitating active learning by the learners. One, consequently, concludes that the constructivist view of teaching is generally less prevalent among the teachers surveyed, compared to the former direct transmission view.

6.2.4 Subsection 4: Self-rating

This subsection of the questionnaire sought to provide information about how the teachers rated themselves in four areas by responding to the four questions that are shown in Table 6.13 below. The responses to questions in this section used a 4-point Likert scale, hence Cronbach's alpha was calculated to determine the reliability of this set of questions in relation to a scale of measure.

6.2.4.1 Scale reliability

Tables 6.11 and 6.12 show the reliability statistics and the item-total statistics tables, respectively. This subscale had a Cronbach's alpha of 0.8 (rounded off to 1 decimal place), indicating high reliability (Tan, 2009, p. 102).

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of items
.801	.809	4

Table 6.11: Reliability statistics: Self-rating

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item-total correlation	Squared multiple correlation	Cronbach's alpha, if item deleted
Q5	10.44	1.798	.396	.291	.863
Q6	10.13	1.527	.766	.785	.676
Q7	10.11	1.737	.561	.487	.775
Q8	10.18	1.513	.788	.752	.665

Table 6.12: Item-total statistics: Self-rating

Table 6.12 above presents the "Cronbach's alpha, if item deleted" in the final column. This column shows the value that the Cronbach's alpha would have if a particular item were to be deleted from the scale. The removal of question 6 would result in a higher Cronbach's alpha of 0.863. The question was, however, retained, as it provided a different dimension for self-rating. Maintaining the question meant a wider content coverage for this scale, while still upholding a high-level Cronbach's alpha.

6.2.4.2 Data presentation

How do you rate yourself in the following areas? Mark X for each statement.					
Q	Statement	Very poor	Poor	Fair	Very good
Q5	Knowledge and the ability to use problem-solving as a vehicle for learning mathematics	2%	2%	72%	24%
Q6	Knowledge of the teaching of mathematics at Foundation Phase level	0%	0%	54%	46%
Q7	The ability to understand learners and their learning needs	0%	0%	50%	50%
Q8	Motivation to teach mathematics at Foundation Phase level	0%	0%	55%	45%
Q5 N = 48 Missing values = 2 Q6 N = 48 Missing values = 0 Q7 N = 48 Missing values = 0 Q8 N = 48 Missing values = 1					

Table 6.13: Self-rating

6.2.4.3 Discussion of data

Table 6.13 shows that 50% or more of the teachers rated themselves as fair in each of the four areas concerned. The fact that the majority of the teachers rated themselves fair, including in terms of the motivation to teach mathematics in the Foundation Phase, indicates a lack of high self-rating among the teachers involved.

6.2.4.4 Discussion, and factual and interpretative conclusions

Even though the teachers in this study expressed differing views about the teaching and learning of mathematics, they tended to be confident about their own effectiveness. The majority of the teachers tended to have confidence in their own teaching, regardless of the beliefs that they held.

Of the teachers, 24% rated their knowledge for using problem-solving as a vehicle for learning as being very good. It should be noted that such use is not in the constructivist context, but it is in the direct transmission context, which centres on knowledge dispensing, as was revealed in subsections 2 and 3.

6.2.5 Subsection 5: Problem-solving lessons

This section of the questionnaire sought to provide information about the teachers' problem-solving lessons. The teachers, on being presented with two classroom scenarios, were asked how they identified with each scenario. The responses to the questions in this section

used the 5-point Likert scale; hence, Cronbach's alpha was calculated to determine the reliability of this set of questions on a scale of measure.

6.2.5.1 Scale reliability

Tables 6.14 and Table 6.15 below show the reliability statistics and the item-total statistics tables, respectively, on the scale *the problem-solving lessons*.

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of items
.469	.478	2

Table 6.14: Reliability statistics: the problem-solving lessons

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item-total correlation	Squared multiple correlation	Cronbach's alpha if item deleted
Q18	2.28	.726	.314	.099	–
Q19	2.28	.465	.314	.099	–

Table 6.15: Item-total statistics: the problem-solving lessons

The above tables show that this scale had a Cronbach's alpha of 0.5 (rounded off to 1 decimal place), which is considered weak. The scale was, however, used in the current research because of the diversity of the construct being measured (Field, 2006), and because it allows for meaningful content coverage (Schmitt, 1996, p. 352).

6.2.5.2 Data presentation

Table 6.16 presented below shows the teachers' responses to the two questions on the teaching and learning scenarios that related to problem-solving lessons.

Scenario Q18: Problem-solving with Mr Adams

Mr Adams is a Grade 3 teacher. Here he explains to his colleague how he handles problem-solving with his class.

Monday 4 June 2012
Problem for today
Joe has 5 marbles. His brother James has 6 more marbles. How many marbles do they have altogether?

I present a problem to the class. We solve the problem together as a class. I go over the problem-solving process, explaining it in detail. Then I present similar problems for the learners to solve.

Scenario Q19: Problem-solving with Mr Adams

Mr Adams' Grade 3 class is working on the problem on the chalkboard. He moves around the groups, and, on finding that most of the groups are struggling with the problem, says the following.

Monday 18 June 2012
Problem for today
What will the 4th pattern be?

This is how to solve the problem. Look at how many triangles are in the 1st, 2nd and 3rd patterns to find the pattern. Use this pattern to work out how many triangles will be in the 4th pattern.

	Always	Most of the time	Sometimes	Rarely	Never
Study the above scenerio. How often do you handle problem-solving in the same way in which Mr Adams handles problem-solving with his class?	11%	53%	34%	2%	0%
Study the above scenerio. How often do you, like Mr Adams, have to show or tell your learners how to solve a problem when they find it difficult to do so?	21%	34%	40%	4%	0%

Scenario Q18 N = 48 Missing values = 1

Scenario Q19 N = 48 Missing values = 1

Table 6.16: The problem-solving lessons

6.2.5.3 Discussion of data

Forty-seven (47) teachers responded to question 18, with 64% indicating that they demonstrated the process of problem-solving, with 11% asserting that they always used the method involved, and with 53% saying that they did so most of the time. Thereafter, they gave their learners similar problems to solve. Only 2% said that they rarely used the method

shown. Of the 47 teachers who responded to question 19, 53% stated that they showed, or told, their learners how to solve the problem when the learners had difficulty with solving the problem. Only 4% rarely did so.

6.2.5.4 Discussion, and factual and interpretative conclusions

An analysis of the teachers' responses shows the following:

- The majority (64%) of the teachers presented a problem to the class, with the whole class solving the problem together as a class. The teachers then went over the problem-solving process, explaining it in detail to the class, and then presenting similar problems for the learners to solve.
- A significant number of the teachers (53%) did not 'let go', and allowed the learners an opportunity to figure out, or to struggle with arriving at, the solution to the problem. Instead, when learners encountered difficulties in solving the problems, they showed, or told, them how to solve the problem. Of the teachers, 40% reported that they sometimes acted so, whereas only 4% said that they rarely acted so.

The above information reveals that the majority of the teachers' problem-solving lessons emphasised the problem-solving strategies, and presented the learners with similar problems to solve, in order that they might practise the problem-solving strategies concerned. The teachers were quick to 'show and tell' the learners how to solve a problem when they had trouble with solving it. Tyminski (2010, p. 296) calls the feeling related to giving in to the urge to tell the learners exactly what to do 'teacher lust'. Data from the interviews (as revealed in the two extracts from the 1st interviews) also served to confirm the presence of teacher lust in some of the participants in the study.

Interviewer: What has been your experience with regard the temptation to show and tell the learners what to do?

Teacher A: Yes, you do sometimes get the temptation when you see your learners. They don't actually get engaged quickly in what they have to investigate, especially when you don't give them their chance... to explore themselves on [i.e. in] their different ways. If you didn't give them time, then you end up being tempted.

Interviewer: What has been your experience with regard to the temptation to show and tell the learners what to do?

Teacher B: The temptation to show and tell is still...a big problem.

Based on the above data, one can conclude that the majority of the teachers participating in the study were creating learning opportunities, but that they then snatched them away by showing the learners how to solve the problem. The teachers did not allow sufficient time for the learners to grapple with ideas and problems, and to engage in critical thinking, thus denying their learners opportunities for true learning.

6.2.6 Subsection 6: Teachers' beliefs

This subsection of the questionnaire sought to provide information about the teachers' beliefs, elicited in response to the three questions shown in Table 6.17 below. The responses to the questions in this section used a 5-point Likert scale; hence, Cronbach's alpha was calculated to determine the reliability of the set of questions.

Cronbach's alpha	Cronbach's alpha based on standardised items	No. of items
.474	.495	3

Table 6.17: Reliability statistics: teacher beliefs

Q	Scale mean if item deleted	Scale variance if item deleted	Corrected item-total correlation	Squared multiple correlation	Cronbach's alpha, if item deleted
Q15	4.91	2.570	.297	.117	.418
Q16	4.09	1.548	.376	.163	.212
Q17	4.65	1.743	.261	.072	.457

Table 6.18: Item-total statistics: Teacher beliefs

The above tables show that this scale had a Cronbach's alpha of 0.5 (rounded off to 1 decimal place), which is considered weak. The scale was, however, used in the current research because of the diversity of the construct being measured (Field, 2006), and because it had meaningful content coverage (Schmitt, 1996, p. 352).

The research evidence indicates that "teachers' beliefs color and influence their teaching practices, how they believe content should be taught, and how they think students learn" (Harwood et al., 2006, p. 69). Table 6.19 below presents the data on the teachers' beliefs about the teaching and learning of mathematics.

To what extent do you, as a teacher, believe that...? Mark X to show the extent of your belief					
Q	Belief	Strongly do not believe	Do not believe	Believe	Strongly believe
Q15	"Many learners are just not able to learn mathematics."	23%	64%	13%	0%
Q16	"Teachers must return to the basics of emphasising the mastery and memorisation of facts and skills."	13%	25%	37%	25%
Q17	"Learners learn mathematics best when they sit, listening carefully, and watching you, the teacher, demonstrate and explain the mathematics, and then practise what they have seen and heard."	30%	21%	34%	15%
Q15 N = 48 Missing values = 1 Q16 N = 48 Missing values = 0 Q17 N = 48 Missing values = 1					

Table 6.19: Teachers' beliefs

Of the teachers who responded to the questions in this subsection:

- 1) Forty-nine per cent (49%) expressed a belief that learners learn mathematics best when they sit, listening carefully, and watching the teacher demonstrate and explain, so that, thereafter, they can practise what they have seen and heard.
- 2) Sixty-two per cent (62%) expressed a belief that teachers must return to the basics of emphasising the mastery and memorisation of facts and skills.
- 3) Thirteen per cent (13%) expressed a belief that many learners are just not able to learn mathematics, whereas the majority of the teachers (41; 87%) expressed a belief that all learners are able to learn mathematics. Teachers who do not believe that most learners are capable of learning mathematics tend to write off many such learners at the first signs of failure.

6.2.6.1 Discussion, and factual and interpretative conclusions

The teachers in the current study expressed a belief that mathematics is mainly about the mastery and memorisation of basic facts and skills. This deeply held belief manifested in 62% of the teachers desiring to return to the basics of emphasising the mastery and

memorisation of facts and skills. This belief explains why a significant percentage of the teachers (49%) involved believed that the best way of teaching mathematics is by means of demonstrating and explaining, and the best way for learners to learn mathematics is by listening carefully, and then practising what they have seen and heard.

The fact that the above-mentioned teachers tended to hold traditional beliefs about the nature of mathematics has the potential for perpetuating mathematics teaching in terms of a more traditional, direct transmission approach. This also explains why the teachers were continuing to use traditional methods that resulted in few learners achieving success in mathematics, even though they expressed a belief that most learners are able to learn mathematics (Raymond, 1997).

6.2.6.2 Fairly low Cronbach's alpha in subsections 6.2.2.1, 6.2.3.1, 6.2.5.1 and 6.2.6.1

Four of the nine subsection in the questionnaire were aimed at gathering quantitative data focused on a different concept or construct. Tavakol and Dennick (2011, p. 54) point out:

[I]f a test has more than one concept or construct, it may not make sense to report alpha for the test as a whole as the larger number of questions will inevitable inflate the value of alpha. In principle therefore, alpha should be calculated for each of the concepts rather than for the entire test or scale.

Hence, Cronbach's alpha was calculated for each of the four subsections concerned.

Cronbach's alpha estimates for the four subsections ranged between 0.47 and 0.80. The internal consistency of the outcome measure, therefore, ranged from low to high, with three of the four subsections falling below the benchmark of 0.70, which usually determines acceptable reliability. The below 0.70 subsections consisted of: (a) the approach to the teaching and learning of mathematics, consisting of three items ($\alpha' = 0.66$); (b) the approach to problem-solving, consisting of six items ($\alpha' = 0.56$); and (c) the problem-solving lessons, consisting of two items ($\alpha' = 0.47$).

Though the Cronbach's alpha of the above-mentioned scales are considered 'low', 'questionable', or 'moderate', according to Cronbach's alpha rules of thumb, the scales were used in the current research study, because of the diversity of the construct that was being measured (Field, 2006), and because of their other desirable properties, such as meaningful content coverage (Schmitt, 1996). It is common, for such reasons, to have a lenient cut-off (Maizura, Masilamani & Aris, 2009) in social sciences research. For these reasons (i.e. diversity of construct being measured, and meaningful content coverage) attaining a fairly low Cronbach's alpha did not necessarily compromise the credibility of the study. Such a low

Cronbach's alpha has been used before, with examples of such instances including the following:

- Maizura et al. (2009), their study, using scales, decision latitude, and psychological job demands, had a low Cronbach's alpha of 0.64 on the psychological job demands scale.
- Mokkink et al.'s (2011) study had a low Cronbach's alpha on the group factors mental of 0.56, and on the bulbar of 0.48.
- Gregorich et al. (1990) had reliability scales ranging from 0.47 to 0.67 in terms of Cronbach's alpha.

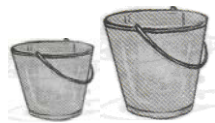
The small number of items in each subsection might have resulted in the relatively low Cronbach's alpha. Cronbach's alpha estimation of reliability has been shown to increase with scale length, meaning with the number of items on the scale (Swales & McIntyre-Bhatty, 2002; Voss, Stem & Fotopoulos, 2000). In the current study, the researcher had to keep the number of items per subsection to a minimum, so as to keep the questionnaire short. If the original number of items had been retained, the questionnaire would have been too long, and the teachers concerned might have resisted completing it in full.

6.2.7 Subsection 7: Teachers' problem-solving skills

This section presents the qualitative findings on the teachers' own problem-solving skills.

Q20 Solve the following problem in the space provided in any way that you see fit, and show how you reached the answer.

You are standing on the bank of a river with two pails.



One pail holds exactly 3 litres of water, and the other holds exactly 5 litres. The pails are not marked for measurement in any other way. How can you carry exactly 4 litres of water away from the river?

The teachers' solutions to the above problem were analysed using ATLAS.ti to determine their problem-solving skills. The coding of the solutions was done using Szetela and Nicol's (1992, p. 42) analytic scale for problem-solving.

Category of responses to the problem in Q20	Percentage of teachers
No attempt made	31%
Completely misinterpreted the problem	33%
Misinterpreted major parts of the problem	15%
Misinterpreted minor parts of the problem	6%
Partially correct the procedure, but with major fault	15%
Correct solution	0%
(N) = 48	100%

Table 6.20: Analysis of the responses to Question 20**6.2.7.1 Discussion of data**

Table 6.20 above shows that 31% of the teachers did not respond to the question asked, whereas 33% of them completely misinterpreted the problem. Hence, they came up with a totally inappropriate plan, resulting in an incorrect solution to the problem. The teachers concerned tried to understand the problem and solution in terms of a rules and routine problem mode. Examples of the responses of such teachers are shown in the following extracts:

Extract 23: “ $1\frac{1}{2} + 2\frac{1}{4} = 4$ litres”

Extract 24: “Carry $\frac{1}{2}$ of each of the pails $1\frac{1}{2} + 2\frac{1}{2} = 4$ l”

Extract 25: “ $\frac{1}{2}$ of 3l = $1\frac{1}{2}$. $\frac{1}{2}$ of 5l = $2\frac{1}{2}$. Add $1\frac{1}{2} + 2\frac{1}{2} = 4$ ”

Of the teachers, 15% understood the problem, but misinterpreted major parts of the problem. The majority of the teachers misinterpreted the fact that the pails were not marked, hence one could not pour out 1 litre, a third of a litre, or $\frac{1}{2}$ a litre of water from the pail. Examples of the responses that were given follow:

Extract 26: “Fill the 5 litre pail with water, then pour water to fill the 3 litre and then pour out $\frac{1}{2}$ of the remainder out and pour back the 3 litre into the 5 litre pail that would make 4 litres.”

Extract 27: “I will fill in the 3 litre bucket with water and pour the water into the 5 litre bucket. Then I will fill in the 3 litre bucket $\frac{1}{4}$ of water and pour it into the water in the 5 litre bucket Remove 3 litres from 5 that will make 2

litres and remove 1 litre from that pail of 3 litres the total is 4 litres.”

Of the teachers, 6% understood the problem, but misinterpreted minor parts of the problem. Hence, their problem-solving procedures were partially correct, although they contained minor faults. Examples of such responses follow:

Extract 28: “Remove 3 litres from 5 that will make 2 litres and remove 1 litre from that pail of 3 litres the total is 4 litres.”

Extract 29: “Fill the 3 litre pail with water from the 5 litre pail (2 litre left in the 5 litre pail. Estimate the 1 litre left in the 5 litre pail and pour out. Pour 3 litre pail of water into the 5 litre pail.”

6.2.7.2 *Discussion, and factual and interpretative conclusions*

The analysis of data in Table 6.21 below reveals that all the teachers had difficulty with solving this particular problem.

6.2.8 Subsection 8: Selection of suitable teaching/learning materials

This subsection presents and discusses the data that were obtained from the section of the questionnaire investigating the teachers’ selection of suitable learning/teaching materials. The teachers were presented with a scenario, in relation to which they had to choose one of the four cards (as are shown in Table 6.21 below) to use in a lesson. The lesson was meant to help the learners recognise and name triangles, and to discuss whether their sides were straight or curved. The teachers, after selecting one card, had to give reasons for their selection.

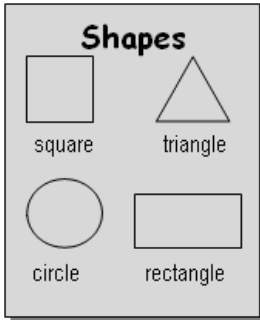
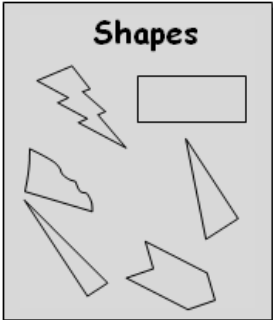
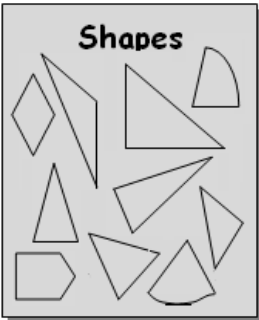
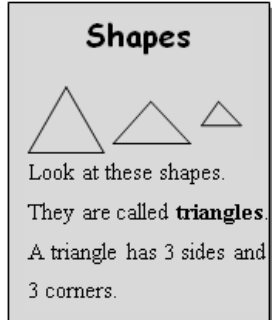
Q21	<p>Ms Miller wants her students to be able to recognise and name triangles, as well as to discuss whether their sides are straight or curved. To help them, she wants to give them some shapes that they can use to test their ability. She goes to the store to look for a visual aid to help with the lesson. Which of the following aids is most likely to help the students improve their ability to recognise and name triangles? (Circle ONE answer.)</p>
<div style="display: flex; justify-content: space-around; align-items: flex-end;"> <div style="text-align: center;">  <p>A</p> </div> <div style="text-align: center;">  <p>B</p> </div> <div style="text-align: center;">  <p>C</p> </div> <div style="text-align: center;">  <p>D</p> </div> </div>	
Q22	Please explain why you chose this activity.

Table 6.21: Selection of visual aid materials

The responses to this section were analysed using ATLAS.ti. The findings obtained are given below.

Choice of activity	Percentage of teachers
1. Visual aid A	23%
2. Visual aid B	0%
3. Visual aid C	27%
4. Visual aid D	31%
5. No response	19%
Total (N) = 48	100%

Table 6.22: Choice of visual aid

Activity A: Of the teachers, 23% selected visual aid A. Of this percentage, 55% did not explain why they had selected the activity. For such teachers, learning was achieved by means of their reliance on the memorisation of facts and definitions, and on the reproduction of familiar material, as is illustrated in extracts 30 and 31 below.

Extract 30: “You can see the shapes and the name. You don’t need triangles of different shapes, you just need the one. This

will also help to show different sides (round or straight)[,]
the amount of side and in which way they can move.”

Extract 31: “It gives all the definitions of a triangle; learners can be able to differentiate the shapes, show how a triangle is different from other shapes.”

The teachers concerned chose visual aid A because the use of this visual aid achieves correct definitions and mastery of facts about a triangle, an approach that has been repeatedly criticised (Hennings et al., 2012), as the approach does not allow learners to build up their own knowledge.

Activity C: Of the teachers concerned, 27% chose visual aid C. The aid was the most appropriate of the four visual aids, as it contained triangles that were presented in varying forms, as well as containing non-examples. Activity C could, therefore, be used to gain a conceptual understanding of the triangle, rather than to memorise and master the rules and facts about a triangle. Visual aid B would have been the second best aid to choose, as it also contained non-examples, so that the learners would have had to justify why some of the shapes were triangles, whereas others were not. Visual aid B, however, contained few varying forms of a triangle.

However, the reasons forwarded by 12 of the teachers contained elements of instruction that focused exclusively on acquiring knowledge through dependency on the dispensing of facts, rules and definitions. Extracts 32 and 33 below are examples of such reasons.

Extract 32: “Because they *should know* that [a] triangle can come up in different shapes and sizes. They can [be able to] recognise that a triangle has 3 sides and 3 corners and they are able to identify any shape that is not a triangle.”

Extract 33: “C *shows* there are different kinds of and sizes of triangles. It also *shows that* lines always have to be straight to show triangular shapes.”

The wording such as “*should know*” and “*shows that*” suggests knowledge dispensation, rather than conceptual understanding. In addition, the use of this visual aid in instruction centres on the supplying of facts, rules and definitions. The above examples reveal that teachers can use the materials designed for a particular pedagogy (in this case, a constructivist pedagogy) for a different pedagogy (in this case, for the direct transmission of

knowledge). Therefore, exposing teachers to instructional material that is designed for use in constructivist pedagogy does not guarantee that the material will be used in the intended manner in the classroom setting.

Activity D – Of the teachers, 31% chose visual aid D. The reason for them doing so was that the activity concerned supported their approach to the teaching of mathematics, and their belief in what the learning of mathematics entails, as is demonstrated by the following extracts:

Extract 34: “I chose D because the picture has 3 different sizes. It also *states the name triangle and the sides and corners*. Also it *tells you* (ordinary life) what the shape of a triangle is.”

Extract 35: “Because here the focus is to *let them know* exactly the triangle, so the activity *explains clearly* about the triangle they see the real triangles and examples.”

The above extracts show that the major reason for choosing activity D was that it “*explains clearly*” with examples (representing the direct transmission approach to the teaching and learning of mathematics). The teachers would use activity D to explain, describe and name triangles. The belief that mathematics is about the mastery of facts made activity D ideal for the teachers concerned, as it *stated* the facts, and the name and number of the sides and corners (Ball & Stylianides, 2008).

6.2.8.1 Discussion, and factual and interpretative conclusions

In Subsection 2.10.2, it was established that it is difficult for teachers who believe that learners learn from being told and shown what to do to choose material that is learner-centred. This explains why the majority of the teachers in this study selected activity D. Doing so was in line with their beliefs regarding the teaching/learning of mathematics. Yet, in Subsection 2.12, it was established that traditional mathematics instruction (which focuses almost exclusively on instructional tasks that are aimed at achieving the correct answers through a reliance on the memorisation of facts, rules, formulas, definitions, and the use of algorithms) has been repeatedly criticised in the United States (Hennings et al., 2012). The teachers involved expressed a belief that learners learn from being told and shown how and what to do. They also believed that it is the teacher’s responsibility to show and demonstrate what to do. Hence, such teachers would have found activities B and C unsuitable, because they would have found the activities concerned frustrating, as they did not provide the

teacher with guidance on what do with the learners, and the steps to follow (Choppin, 2011; Remillard, 1999).

6.2.9 Subsection 9: Demographic data

This subsection presents and discusses the demographic data that were gathered about the teachers. The demographic data were split up into two sections; firstly, the access that was obtained to the key documents on problem-solving, and on the problem-solving workshop attendance; and secondly, personal details.

6.2.9.1 *Access to key documents on problem-solving and problem-solving workshop attendance*

The questions that are shown in Table 6.23 below were used to elicit data on the teachers' access to key documents related to problem-solving. The documents were the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011, and the CAPS documents for the Foundation Phase. This section of the questionnaire was aimed at finding out whether the teachers had previously attended any workshops on problem-solving.

	Access to key documents on problem-solving
Q23	Have you seen the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document at your school?
Q24	Have you read the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document for mathematics?
Q25	Have you implemented the requirements of the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document for mathematics in your planning?
Q26	Have you seen the CAPS document at your school?
Q27	Are you familiar with the requirements of these documents with regard to problem-solving?
Problem-solving workshop attendance	
Q28	Have you participated in a professional development session that focused on using problem-solving as a vehicle for teaching mathematics at the Foundation Phase?
Q29	If YES, who was the organiser of the session?
Q30	How long ago did this professional development take place?

Table 6.23: Access to key documents on problem-solving, and on problem-solving workshop attendance

Analysis of the data obtained showed:

- Under half of the teachers (43%) had not seen the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document at their schools, whereas only 53% of them had read it. This important document provides guidelines regarding a campaign that was designed to improve the teaching of mathematics. As many as 37% of the teachers admitted that they had not implemented the requirements of this document.
- Many more teachers had accessed the CAPS document than had accessed the Foundations for Learning Campaign 2008–2011 document. Of the teachers concerned, 98% reported that they had seen the CAPS document, although 12% of them were not familiar with the contents of the document. The reason for teachers having had better access to the CAPS document than to the Foundations for

Learning Campaign document might have been because the former document was given much more publicity than was the latter, due to all the teachers having attended a CAPS induction workshop.

- The majority of the teachers (64%) had not attended the professional development workshops on problem-solving as a vehicle for teaching mathematics in the Foundation Phase, which were organised by the WCED.

6.2.9.2 Personal details

This section elicited data on the teachers' gender, age, qualification, teaching experience, and grades taught. The questions that are shown in Table 6.24 below were posed to the teachers in this regard.

Personal details	
Q31	What is your gender?
Q32	Please indicate your age below.
Q33	Please indicate your professional qualification.
Q34	Please indicate how many years of teaching you have altogether, including this year.
Q35	Indicate how many years of teaching you have had in the Foundation Phase, including this year.
Q36	Grade currently teaching

Table 6.24: Personal details of the teachers concerned in this study

An analysis of the data on these questions showed that the entire population of teachers in the Foundation Phase in the five schools involved in this study were of the female gender, with their average age being 43 years. Secondly, 67% of the teachers held an appropriate teaching qualification for the phase, which is a primary school diploma. They also had an average of 15 years teaching experience, most of which had occurred in the Foundation Phase.

6.2.10 Summary findings of Section 1: data from the questionnaire

The data that were presented and analysed in Section 1 were generated from the questionnaire, which was distributed to all of the 48 Foundation Phase teachers in the project schools. Table 6.25 below summarises the findings from this section of the questionnaire.

Area of research focus	Summary of findings
1. Teacher's own views of what... problem-solving is; problem-solving is not; problem-solving requires of the teacher; and problem-solving requires of the learner (N = (48)).	<p>The responses to the mind map on problem-solving revealed that the majority of the teachers viewed problem-solving as being about using learnt mathematics to solve problems. Problem-solving was viewed as an application of known mathematics: the focus for the teachers was on applying the acquired mathematical knowledge to solve routine or nonroutine problems.</p> <p>Only 6% of the teachers indicated that problem-solving was a tool for the learning and teaching of mathematics</p>
2. Teachers' approach to the teaching and learning of mathematics (N = (48))	<p>The majority of the teachers indicated that they used the traditional approach (i.e. direct transmission) to the teaching and learning of mathematics. This was characterised by :</p> <ul style="list-style-type: none"> • the teaching of mathematics by means of focusing on rules and procedures; • demonstrating to the class on the chalkboard, and then letting the learners practise; and • asking the learners to do pen-and-paper calculations. <p>The majority of the teachers did the above-mentioned actions daily. To these teachers, the memorisation and mastery of basic facts and procedures provided primary evidence of learning. The learning was evidenced more through the expert memorisation and performance of procedures, rather than through the ability to explain understanding and through reasoning mathematically.</p>
3. Teachers' approach to problem-solving (N = (48))	<p>The data analysis revealed that the majority of teachers participated in teaching for problem-solving, and that they did not advocate the approach of teaching via problem-solving. This confirmed the teachers' traditional view of mathematics learning (as observed above), in accordance with which the teacher teaches, and the belief is held that the child demonstrates understanding by assimilating more or less what has been taught. This is evident in the teachers agreeing or strongly agreeing with the following:</p> <ul style="list-style-type: none"> ▪ Effective/Good teachers demonstrate the correct way in which to solve a problem. ▪ The learners should learn, and master, the basic number facts before they do problem-solving. ▪ Teaching should be built up around problems with clear, correct answers, and around ideas that most learners can quickly grasp.
4. Teachers' self-rating (N = (48))	<p>The majority of the teachers rated themselves as fair in each of the four areas covered, including the motivation to teach mathematics in the Foundation Phase. This indicated a rather low inner self-rating among the teachers.</p>

Table 6.25: Summary of findings from Section 1: data from the questionnaire

Area of research focus	Summary of findings
5. The problem-solving lessons (N = (48))	<ul style="list-style-type: none"> • The teachers' problem-solving lessons were characterised by an emphasis on the problem-solving process, and on the presentation of similar problems to solve for practice. • The teachers were quick to show and tell their learners how to solve a problem when the learners had difficulties with problem-solving. • In so doing, the majority of the teachers created learning opportunities (by posing problems to their learners), which they, however, soon snatched away, by showing the learners how to solve the problems at the earliest signs of difficulty exhibited by the learners. • In so doing, the teachers did not allow for sufficient time for the learners to grapple with the problem, and to engage in critical thinking. This detracted from the vital opportunity for true learning that was immediately available to the learners concerned.
6. Teachers' problem-solving skills (N = (48))	All the teachers who participated in this research dealt poorly with the mathematical problem that was set them. They failed to correctly interpret the problem, and devised inappropriate plans for dealing with it, hence coming up with wrong solutions to it.
7. Teachers' beliefs about the learning of mathematics(N = (48))	<p>The majority of the teachers held the following two positive beliefs about the learning of mathematics:</p> <ul style="list-style-type: none"> ▪ Many children are capable of learning mathematics. ▪ Learners are not passive receipts of knowledge. <p>The teachers were, however, of the belief that there was a need to return to the basics of emphasising the mastery and memorisation of facts and skills.</p>
8. Teachers' selection of suitable teaching/learning materials (N = (48))	The majority of the teachers' selections of learning materials focused almost exclusively on instructional material that was aimed at achieving correct answers through depending on the memorisation of facts, rules and definitions. The selection confirmed the teachers' traditional approach and beliefs about what mathematics is, and about how it should be taught.
9. Demographic data: access to key documents on problem-solving (N = (48))	<ul style="list-style-type: none"> • A significant number of the teachers had not accessed the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document. As many as 37% admitted to not having implemented the requirements of this important document. • The majority of the teachers (64%) had not attended a workshop on problem-solving as a vehicle for learning. • Up to 98% of the teachers had seen the relevant CAPS documentation, although only 12% of them were familiar with the contents thereof. • The participants were all women, with an appropriate teaching qualification for the phase, which was a primary school diploma. They had an average of 15 years teaching experience among them, most of which had been spent teaching the Foundation Phase.

Table 6.26: Summary of findings from Section 1: data from the questionnaire

6.2.11 Discussion, and factual and interpretative conclusions

This section of the study looked at several important features that shape effective teaching and learning. It focused on the teachers' knowledge of problem-solving, on the approaches to the teaching and learning of mathematics, and on problem-solving lessons, as well as on the teachers' beliefs about the teaching /learning of mathematics, on their problem-solving skills, and on their selection of teaching/learning materials.

Two views regarding the teaching of mathematics were explored. On the one hand was the traditional view of teaching mathematics (i.e. direct transmission), in which the teacher's role is that of transmitting knowledge, and of providing the correct solutions. On the other hand lay the constructivist view, in terms of which the teacher's role was that of facilitating active learning by the learners, who sought out the solutions for themselves. The constructivist view of teaching was less prevalent than was the direct transmission approach, making the latter the more dominant view among the teachers in the study.

The inclination towards direct transmission manifested itself in the teachers' approach to problem-solving, in terms of which, as the transmitters of knowledge, they saw their role as being that of demonstrating the procedures and strategies for problem-solving. In relation to the selection and use of teaching and learning materials, they tended to select materials that best showed or explained the mathematics concerned.

The above is a cause for concern, as the current preferred trend has moved beyond the view of teaching as the delivery of information, to teaching as the creation of opportunities in which learners formulate their own knowledge, as they grapple with and solve complex problems that require a significant amount of effort. The outcome of this process is the relational understanding of mathematics.

This study's findings contradict the findings of the Organization for Economic Co-operation and Development's (OECD's) (2009) TALIS, which was carried out in 23 European countries. The TALIS found that teachers tended to be more inclined to see their role as supporting active learning, rather than directly transmitting information.

The outcome of directly transmitting information is the creation of a learning and teaching environment that does not promote effective teaching and learning. It was on the basis of these findings that this group of teachers was introduced to PBL, which is a constructivist approach, in order to create a learning and teaching environment that is conducive to the shaping of effective teaching and learning.

6.3 SECTION 2: DATA FROM THE TEACHER PROFESSIONAL DEVELOPMENT EVALUATION

The analysis of teacher change (or growth), resulting from the teacher professional development that was offered in this study, should focus on the teachers' construction of a variety of knowledge types (i.e. content knowledge, pedagogical knowledge, and PCK) in response to their participation in the experiences provided by professional development (Clarke & Hollingsworth, 2002). The 15 randomly selected teachers from the five schools that were involved in the study attended three half-day workshops to introduce them to PBL. This teacher professional development on PBL covered the motivation for PBL, the nature of PBL, the PBL process, and the steps to be taken in PBL, as well as many more issues that were meant to equip the teachers with sufficient implementation knowledge of PBL (see Appendix 4 for the detailed programme involved). Therefore, after the three half-day teacher professional development sessions on PBL, the teachers were given an open-ended evaluation exercise that asked them the following questions:

- 1) How has the teacher professional development been helpful to you?
- 2) Will you be able to apply PBL in your class?
- 3) What further assistance will you require to be able to apply PBL?
- 4) Do you have any other comments to make?

The teachers' responses to these questions were analysed using ATLAS.ti. The findings that are contained in the following subsections emerged from the analysis.

6.3.1 Participant benefits from the teacher professional development

The analysis of the responses to question 1 revealed that the teachers had benefited from the teacher professional development in mainly two ways (see Table 6.26 below).

	The nature of the benefit experienced, expressed in terms of the percentage of teachers to have benefited from the development	
	Pedagogical knowledge	New perceptions about problem-solving
i) How has the teacher professional development been helpful to you?	67%	33%
N = 15		

Table 6.27: Benefits gained by the teachers from the teacher professional development

Pedagogical knowledge – The majority (67%) of the teachers indicated that they had benefited from the development in terms of the enhancement of their pedagogical knowledge related to problem-solving. Extracts 36 and 37 below are examples of such indications.

Extract 36: “It has been helpful and I’ve learnt that posing a problem to learners is the starting point in learning rather than asking or telling them how to solve a problem.”

Extract 37: “I learnt to have confidence in the learners and [to] allow them to [engage in] critical thinking and creativity. It has been an eye opener from being a teacher to a facilitator.”

New perception about problem-solving – Of the participants, 33% indicated that they had developed a new perception about problem-solving. Extracts 38 and 39 below are examples of the teacher responses that indicated the acquisition of this new perception.

Extract 38: “The teacher professional development was very helpful because I am looking with new eyes at PBL. I like the idea of starting with a problem.”

Extract 39: “It helped me a lot and gave me foresight in different ways of solving problems. As well as the importance of problem-solving.”

In general, the participants came out of the teacher professional development with a new perception of problem-solving.

6.3.2 The ability to apply PBL in class

After the professional development, all the participants indicated that they would be able to implement PBL, as is shown in Table 6.27 below.

	YES	NO
ii) Will you be able to apply PBL in your class?	100%	0%
N = (15)		

Table 6.28: Ability to implement PBL

While this section required just a 'yes' or 'no' answer to indicate to the facilitator whether there was a need for the further empowerment of the teachers in order that they might be able to implement PBL, 40% qualified their 'yes' with an additional comment. Extracts 40, 41 and 42 below are examples of the comments received:

Extract 40: "It helped me a lot. I want to go back and help my other fellow teachers and learners."

Extract 41: "I will implement it with my class as this approach stimulates the [i.e. a] high thinking level to [i.e. among] learners (critical thinking)."

Extract 42: "Of course yes it is a very effective method."

6.3.3 The nature of assistance required by the teachers in future

The teacher professional development participants were asked to state what type of assistance they might need in future, as they tried to implement PBL in their classes. This question was designed to find out any information gaps that the teachers perceived in themselves as they prepared to implement PBL. The analysis of the teachers' responses to question 3 revealed that the teachers would need assistance in the three areas indicated in Table 6.28 below.

	Nature of assistance required, expressed in the percentage of teachers requiring it		
iii) What further assistance will you require to be able to apply PBL?	Construction of PBL problems	Follow-up in schools and demonstration lessons	Material resources
	20%	60%	20%
N = (15)			

Table 6.29: Nature of assistance required

The majority of the teachers (60%) requested follow-up visits by the facilitator. Extracts 43 and 44 are examples of the type of requests that were received in relation to this issue.

Extract 43: “As I will start to implement, I might get stuck along the way then I will call for help.”

Extract 44: “I will real [i.e. would really] like to be helped in future as a follow up development teacher.”

The above extracts show that the teachers felt that they required concrete and practical assistance that would link to the day-to-day operations in their classrooms. The desire for in-class support is further illustrated by those who specifically requested for demonstration lessons as can be seen in extracts 45 and 46 below.

Extract 45: “Demonstrate in my class.”

Extract 46: “I would very much appreciate if the facilitator can come and do a practical lesson in my class based on PBL.”

The teachers concerned needed to be shown how to carry out PBL, as they expressed a belief that they learned better through demonstration. The need to be shown how to facilitate PBL was entrenched in their belief that learning is about being shown how something is done, followed by practice. It demonstrates that the teachers involved would not be likely to become committed to a new instructional approach or innovation until they had seen it work in their classrooms, with their own learners.

Of the teacher professional development participants, 20% requested help in the construction of problems for use during their PBL lessons. Extract 47 below is an example of such a request:

Extract 47: “I would like more assistance in the questions to start.”

The need for assistance in the construction of problems for use in PBL was also observed by the facilitator during follow-up visits and lesson observations. Section 3 of the data presentation of this study discusses these observations in detail.

Of the participants, 20% also requested assistance with obtaining sufficient resources to facilitate the implementation of PBL. Extract 48 below is an example of such a request.

Extract 48: “We need more mathematics materials in our school.
We want to improve our school results in the
systemic evaluation.”

6.3.4 Discussion, and factual and interpretative conclusions

The data presented in this section have indicated that the professional development objective of introducing the teachers to PBL, and of empowering them to implement PBL in their schools, had been achieved. The request for demonstration lessons is interpreted as a manifestation of the teachers’ beliefs in what the process of learning entails. For the teachers concerned, learning occurred best when one was shown ‘how’ rather than when one understands ‘how’ and ‘why’.

6.4 SECTION 3: DATA FROM THE LESSON OBSERVATIONS

This section gives a brief report on the data that were collected from observing the lessons that were conducted by 14 of the participating teachers, and it discusses some of weaknesses and strengths that were identified among the teachers in regard to the implementation of PBL in their classes. From the original study sample of 48 Foundation Phase teachers, 15 teachers were selected, using stratified random sampling, as was discussed in Chapter 4. This sample of teachers, after having undergone professional development in relation to PBL, were later visited and observed implementing PBL in their mathematics lessons. Although the sample concerned was 15 in number, the researcher observed 14 teachers, because one of the teachers had left the school concerned by the time that the study observations were conducted.

The objectives of the observations were twofold: to identify how the teachers’ beliefs about the teaching and learning of mathematics manifested themselves in the teachers’ problem-based lessons in the classroom; and to identify the teachers’ ability to design lessons that provided the learners with opportunities to learn through problem-solving.

6.4.1 The role of the teacher

The observations undertaken focused on the role of the teacher, so as to determine whether the teachers had assumed their new role as facilitators in their PBL lessons, or whether they had stuck to the traditional role of being the sole source and transmitter of knowledge. In the literature review in subsections 3.5.4 and 3.9.5, it was established that the PBL teacher, as a facilitator, abandons direct instruction, allowing the learners to assume greater responsibility

for their own learning than they would otherwise have had. The PBL teacher is not expected to provide information directly. Instead, as a facilitator, the PBL teacher helps the learners to become more self-directed, motivated and collaborative critical thinkers. The PBL teacher does this by observing the learners' learning activities, by diagnosing issues faced by the learners, and by intervening at the appropriate times. In Section 3.8, it was established that teachers are often reluctant to relinquish control of the learning process, so that PBL is implemented in a way that keeps the teacher in charge of what is learned, although the lesson contents are packaged into cases and small group discussions.




Hence, in analysing the role of the teachers in their PBL lessons, their role was divided into the two categories described above, namely that of i) *the traditional knowledge transmitter*, who is reluctant to relinquish control of the learning process, and ii) *the facilitator*, who abandons direct instruction, and who allows the learners to assume greater responsibility for their own learning than they might otherwise have. The classification of the teachers observed is described in the following subsections.

6.4.2 The traditional knowledge transmitter

Of the 14 teachers who were observed, 28% found it difficult to let go of their traditional functions as teachers. These teachers had their lessons modelled on the traditional model, in terms of which they were at the centre of teaching and learning. The teachers, while demonstrating, or explained mathematical issues or concepts, stood facing their class.

The Example of Ms A (see figures 6.2 and 6.3 below)




Ms A stood before her Grade 3 class showing them objects of different mass. Using the question and answer method, she asked them such simple questions as, "What do we measure?" Ms A then showed the learners empty food boxes, focusing on the mass written on the boxes, and asked, "What is the mass of this packet?" It was a very teacher-driven lesson.

Figure 6.2: Showing and telling	Figure 6.3: Question and answer	
 <p data-bbox="316 589 448 622">Figure 6.2</p>	 <p data-bbox="722 589 871 622">Figure 6.3a</p>	 <p data-bbox="1137 571 1286 604">Figure 6.3b</p>
<p data-bbox="188 658 576 840">Ms A <i>shows</i> her class an empty packet of maize meal, and an empty packet of flour. She <i>tells</i> them, "This packet of flower is 1kg and this packet of maize meal is 2kg."</p>	<p data-bbox="603 658 997 750">Ms A <i>asks</i> the class, "Which is heavier – the packet of flour, or the packet of maize meal?"</p>	<p data-bbox="1024 658 1399 719">A learner raises his hand to <i>answer</i> the question.</p>

In this lesson, the learners' involvement was restricted to listening to, and answering, the questions posed by the teacher. The learners were treated as listeners most of the time. The questions were mainly of the order, "What is the mass of this...?" and "Which is heavier a...or a..." The teacher's questions did not elicit, engage, and challenge the learners' thinking beyond the knowledge level at which they already were when the lesson began.

The Example of Ms B (see figures 6.4a–b and 6.5 below)

Ms B spent a significant part of the lesson showing and demonstrating to her learners the different instruments that were used to measure a metre.

Figure 6.4: Show and tell		Figure 6.5: Demonstrate
		
Figure 6.4a	Figure 6.4b	Figure 6.5
Ms B <i>shows</i> her class a metre-long ruler and <i>tells</i> them that, “This is a metre ruler. We use it to measure length.”	Ms B <i>shows</i> her class a tape-measure and <i>tells</i> them that it is called a tape-measure, and that it is used to measure length.	Ms B <i>demonstrates</i> to her class how to use a tape-measure to measure the waist.

Ms B’s class sat, listened and observed as she showed them a metre ruler, a measuring wheel, and a tape-measure. Demonstrating how to use each of the measuring instruments in turn, she occasionally asked the learners questions about what she was showing them, to check that they understood the content of the lesson.

The example of Ms C (see Figure 6.6) below)

Ms C gave each group a problem to solve. She is shown in the series of pictures below, playing the key function in the chalkboard work.

A person is seen from the back, wearing a white long-sleeved shirt, writing on a chalkboard. A large blue starburst graphic is placed over the person's face. The chalkboard contains handwritten text in a stylized font. On the left side, the text reads: 'itsalekile', 'grimfa', 'gsacki', 'uhlu', and 'i'. On the right side, the text reads: 'nts', 'nclw', 'ncw', 'aw', 'nw', 'R15', and 'p13'. The person's right hand is visible, holding a piece of chalk and writing on the board.

itsalekrie
grimfa
gcardh
nts
ndlw
ncw
aw
nw
R15
R13
R12


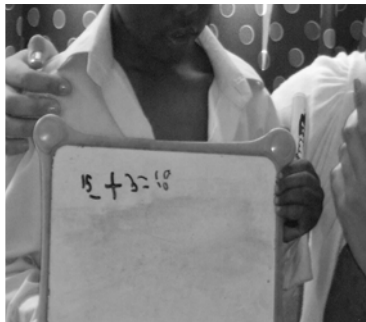
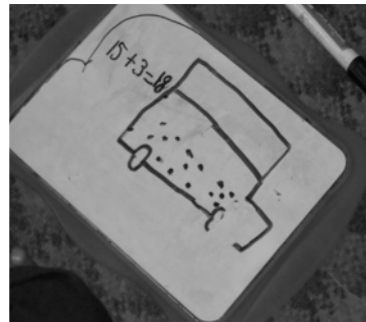
itzilekile
 ginxifa
 gacanda nts
 unluthi ndlw
 idinyuza ncw
 anekle ngw
 nkW
 ngw
 ncw
 nxw
 nxw
 nxw

1
 R15
 R13
 R12 10
 R30
 R10

Ms C demonstrates the addition process with the class, explaining that $R5 + R3 + R2$ gives $R10$, so that one needs to write down 0 and to carry 1.

In her lesson, Ms D took half of the class to sit on a carpet at the front of the classroom, where she did counting in 1s, and then in 2s up to 100. She then narrated a problem, on which she asked the learners to work.

Figure 6.7: Working with individuals, and with the group on the carpet




 <p>Figure 6.7a</p>	 <p>Figure 6.7b</p>	 <p>Figure 6.7c</p>
<p>Ms D gives the group on the carpet at the front of the classroom a problem, on which she asks learners to work individually.</p>	<p>Ms D discusses the problem with the individual learners. Here, she is seen discussing the problem with a learner whose solution to the problem was $15 + 3 = 10$.</p>	<p>A correct response from a learner is seen. However, the learner's response was not shared with the rest of the class.</p>

In this lesson, the learners worked individually on the problem, and the teacher discussed the solution to the problem with each child. Though the lesson was centred on a problem, the lesson was not problem-based, as there was no allowance for the learners to discuss the problem, and to work on it collaboratively.

The example of Ms E (see Figure 6.8a–c below)

Ms E drew rough sketches of 5c, 10c, 20c and 50c coins. She then asked the class to arrange the coins in order of size, starting with the largest. The learners took turns to come to the front of the class, and make drawings of the coins, in an attempt to put them in order of size (see the series of pictures below).

Figure 6.8: Chalkboard work, and class discussions

		
<p align="center">Figure 6.8a</p>	<p align="center">Figure 6.8b</p>	<p align="center">Figure 6.8c</p>
<p>On the chalkboard, Ms E has drawn one big circle to represent 5c, a small circle to represent 10c, another small circle to represent 20c, and a big circle representing 50c.</p>	<p>Ms E has the learners take turns to come up the board to arrange the 'coins' in order of size, starting with the biggest coin.</p>	<p>The teacher uses the chalkboard work to discuss the order of the coins.</p>

The learners were mostly treated like listeners, being asked questions only here and there. The learners' involvement was restricted to listening to, and to answering, the questions that were posed by the teacher. The teacher's questions did not elicit, engage and challenge learner's thinking beyond their knowledge level at the start of the lesson.

6.4.3 Discussion, and factual and interpretative conclusions







A common feature among the above-mentioned teachers was that the learner participation or involvement levels were very low; hence, the learners' learning opportunities were few. The transition from the teacher as knowledge transmitter to that of the teacher as a facilitator was a challenge for the teachers concerned. The teachers continued to view their role as being one of predominantly transmitting knowledge or facts. For the desired role as facilitators, these teachers needed to learn how to ask questions that elicit, engage and challenge the learner's thinking beyond their present knowledge level.

6.4.4 The facilitator

Nine of the 14 teachers who were observed (64%) had assumed their new role of a facilitator during the PBL lessons. The teachers made a role shift, from one of control of what and how the learners learned to one of the facilitation of learning among the learners.

Any place but in the front of the class: The above-mentioned teachers were neither seen to position themselves at the chalkboard, nor at the front of the classroom. Instead, they were to be seen with the learners in their groups. After introducing a problem that they had written up on the chalkboard or on work cards, they issued pieces of paper and markers to

each group, and then they asked each group to work on the problem. After doing so, they moved around from group to group, ensuring that the learners understood the problem concerned.




Figure 6.9: Teacher any place, but in the front of the class		
 <p>Figure 6.9a</p>	 <p>Figure 6.9b</p>	 <p>Figure 6.9c</p>
<p>A Grade 3 teacher is seen discussing an issue with a group.</p>	<p>This Grade 1 teacher presented the problem to the class orally. She then went to each group in turn to ensure that the learners understood the problem concerned.</p>	<p>This Grade 2 teacher gave each group a work card with a problem to solve. She is seen here discussing it with one of the groups.</p>
 <p>Figure 6.9d</p>	 <p>Figure 6.9e</p>	 <p>Figure 6.9f</p>
<p>This Grade 3 teacher is seen discussing an issue with a group.</p>	<p>After writing up the problem on the chalkboard, this Grade 1 teacher is seen here moving around the class, to ensure that the groups are working on the problem.</p>	<p>This Grade 3 teacher gave each group a work card with a problem to solve. She is seen here keeping a close watch on how the groups are doing with their problem-solving.</p>

The above-mentioned teachers did not practise the traditional methods of knowledge transmission. Instead, they spent class time with the groups, guiding, facilitating and monitoring the learners. In this way, they were able to support their active and self-directed thinking and learning.

Provision of the educational materials and guidance that facilitate learning: One of the roles of the teacher in PBL is to provide the appropriate materials and guidance that facilitate

learning. To be able to supply meaningful learning materials in PBL, the teacher must anticipate what materials the learners will need to be able to solve the problem.

Figure 6.10: Provision of the educational materials that facilitate learning



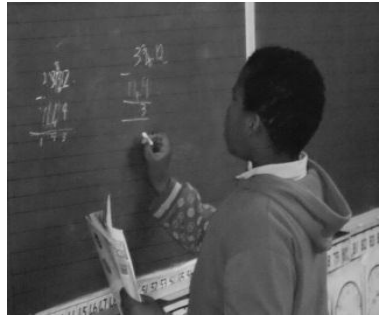

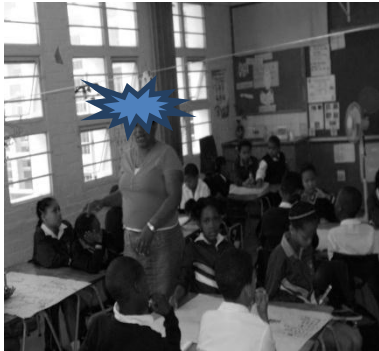

<p>Provision of Grade 2 learners with real notes and coins</p>  <p>Figure 6.10a</p>	<p>Provision of Grade 3 learners with an abacus</p>  <p>Figure 6.10b</p>	<p>Provision of Grade 2 learners with blocks</p>  <p>Figure 6.10c</p>
<p>This group of Grade 2 learners seen here using the real notes and coins to work out the solution to a problem.</p>	<p>These Grade 3 learners are using the abacus to help them with their calculations as they solve a problem.</p>	<p>This group of Grade 2 learners is using the blocks to solve a problem.</p>

Providers of the educational materials that facilitate learning: Only 3 of the 14 (21%) observed teachers provided meaningful educational materials that facilitated learning. The rest of the teachers just supplied papers and pens that the learners to work out their answers.

6.4.5 The PBL lesson structure

In Subsection 3.9.6, the three-part lesson structure of *before*, *during* and *after* was discussed. This was used as the theoretical framework for analysing the teachers' PBL lesson structure. The majority (64%) of the teachers adhered to the PBL lesson structure, particularly in terms of the *before* and *during* stages. The teachers ensured that a problem was well understood by the learners, and they walked around the groups discussing the problem with the different learners.

Figure 6.11: The PBL lesson structure

BEFORE	DURING	AFTER
1. Preparation	2. Learners' work	3. Class discussion
<p>Ms H:</p> <ul style="list-style-type: none"> made certain that the problem was understood; and established clear expectations of the products.  <p>Figure 6.11a</p> <p>This Grade 2 teacher can be seen visiting the groups in turn to ensure that they understand the problem before attempting to solve it.</p>	<p>Ms H:</p> <ul style="list-style-type: none"> let go; and avoided stepping in front of the struggle.  <p>Figure 6.11b</p> <p>These learners have been left to discuss the solution to the problem in their groups during their problem-solving.</p>	<p>Ms H:</p> <ul style="list-style-type: none"> let the learners evaluate, or ask questions of, one another; and the teacher summarised the main part of the solution in relation to the problem.  <p>Figure 6.11c</p> <p>A group shares its solution with the rest of the class, using the chalkboard to demonstrate how they arrived at their answer. Afterwards, the teacher summarised the solutions to the problem.</p>
 <p>Figure 6.11d</p> <p>Ms G ensures that the groups understand the problem before working on it.</p>	 <p>Figure 6.11e</p> <p>Ms G leaves the learners to discuss the solution to the problem in their groups.</p>	 <p>Figure 6.11f</p> <p>The groups present their solutions to the rest of the class.</p>

6.4.5.1 Class discussion

Eleven out of 14 (78%) of the observed teachers omitted this part of the lesson, with them providing neither for classroom discussion, nor for the sharing of solutions.

6.4.6 Discussion, and factual and interpretative conclusions

The data presented in this section reveal that, for 36% of the teachers observed, relinquishing the traditional, and adapting to the facilitator, role was a challenge. The teachers' traditional beliefs about the teaching and learning of mathematics manifested in the teachers' inability to make the shift from the former to the latter role. Of the teachers, 64% had partially conformed to the designing of lessons that provided the learners with opportunities to learn through problem-solving. They ignored the last part of the lesson, which would have entailed providing sufficient time for the groups' reporting back to the class. The omitted stage is a crucial element of a PBL lesson, as it provides the learners with an opportunity to gain from seeing every learning experience that was provided in the context of the problem on which they worked.

Of the participants who took part in this study, 64% changed one or more aspects of their teaching practice. Whether the changes have since had an effect on learner performance was not explored in this study.

6.5 SECTION 4: REFLECTIONS ON THE DATA FROM THE TEACHERS' LESSON

In the literature review (see Chapter 3), it was established that teachers' reflection on their lessons is vital, as they generate knowledge that is grounded in practice. They should also be able to describe the extent of their improvement and personal effectiveness in facilitating the level of mathematics reform that should be being implemented in the classrooms (Ricks, 2011, p. 251). The literature review also established that the teachers' self-assessments increase their effectiveness. This influences the teachers' goal-setting and effort expenditure, and the changes in goals and effort; thereby contributing to improved instructional practice, resulting in higher levels of learner achievement (Bruce & Ross, 2007). Hence, lesson reflection is deemed to be a professional development process, in terms of which the teachers systematically examine their practice (Walsaw, 2010), in order that they might become more effective teachers. The process is one in which the teacher's effectiveness is best measured against the learners' outcomes.

At the end of each observed lesson, the teacher was asked to write down a brief reflection on her lesson, whereupon the reflection was analysed, together with the other teachers' reflections, using ATLAS.ti. Doing this was seen as critical, because the reflections indicated their views on the professional development process. Codes were assigned to each of the teacher's reflections in a process of open coding (using own assigned codes). After the open coding, the codes were organised into shared themes. The themes were based on the ideas

and perspectives of the literature review, namely *goal setting and achievement*, *effort* and *learner achievement*. These literature-based themes became analytic tools by means of which the researcher analysed the teachers' reflections. The process is summarised in Figure 6.12 below.

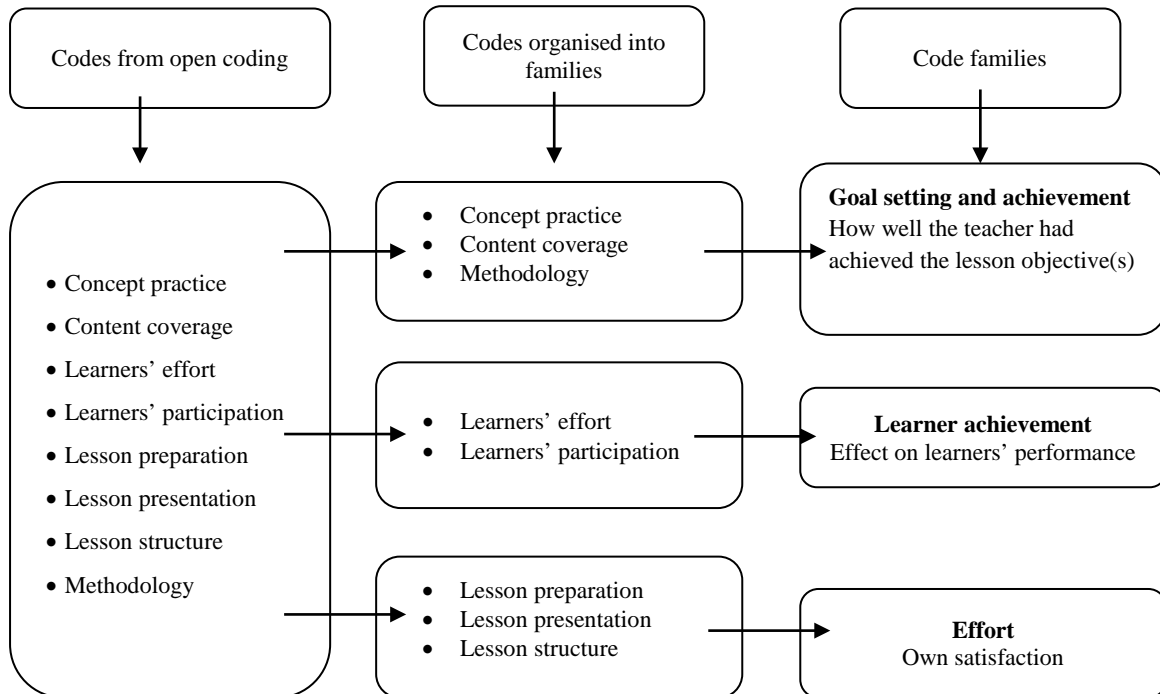


Figure 6.12: The coding process

The teachers' reflections on their lessons (see Appendix 11) were, thus, analysed, using the above coding procedure. The results are summarised in Table 6.30 below.

Nature of lesson evaluation expressed in percentage of teachers who evaluated themselves accordingly			
	Effect on learner performance	Achieved the lesson objective(s)	Level of own satisfaction with effort
Percentage of teachers	36%	36%	28%
N = 14			

Table 6.30: Reflections on the teachers' lessons

6.5.1 Effect on learner performance

The teachers' reflections, categorised as *effect on learner performance*, contained the teachers' reflections on how the lesson had impacted on the learners' mathematical achievements, or performance. The reflection's focus was on the learners' learning and

understanding of the mathematical processes concerned. Such a reflection could be said to be learner-centred. Only 36% of the teachers reflected on in which way their lesson had had an effect on the learners. The ATLAS.ti network views in Figure 6.13 below indicate these reflections.

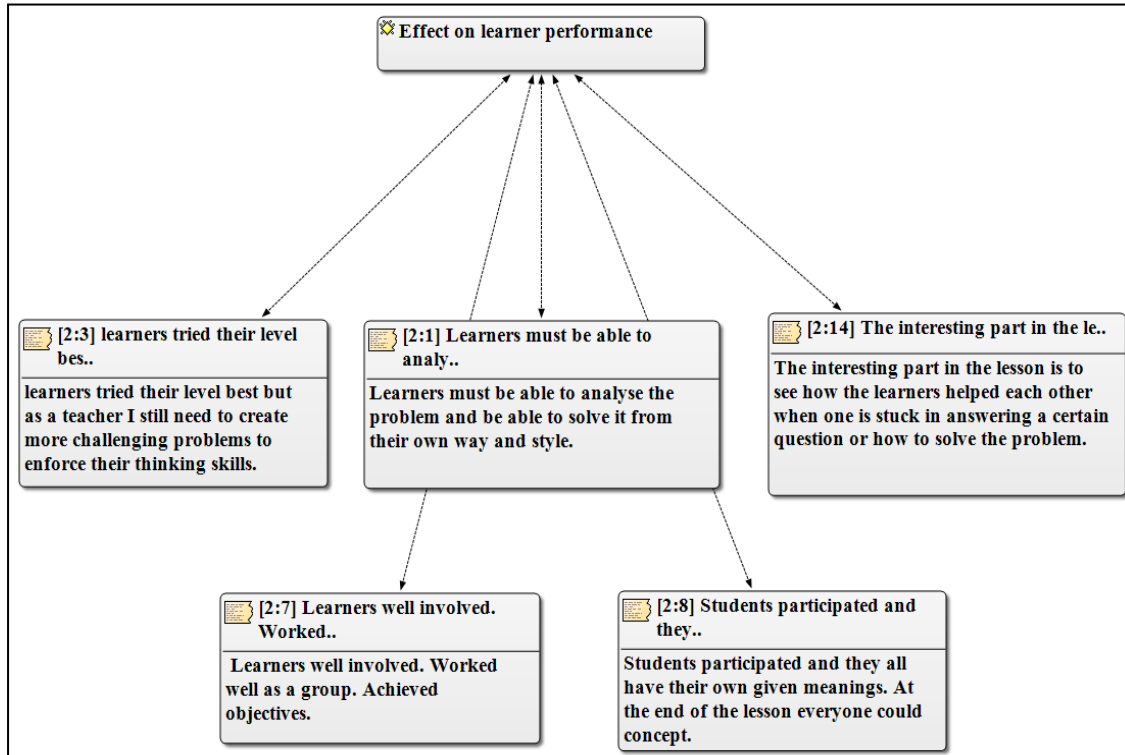


Figure 6.13: Effect of learner performance

Although the above-mentioned teachers commented on the learners' performances, the teachers' reflections were not very specific in terms of how the learning outcomes were achieved. Reflection [2:7] in Figure 6.13 above focused on the learners' involvement and participation in the lesson. The comment "Achieved objectives" is rather general.

6.5.2 Achievement of the lesson objectives

Of the observed teachers, 36% reflected on how well they had attained or achieved the lesson objective(s). The focus here was on themselves as teachers of the specific lessons, on how they accomplished what they had set to do in the lesson, and on how well they had achieved the lesson objective(s).

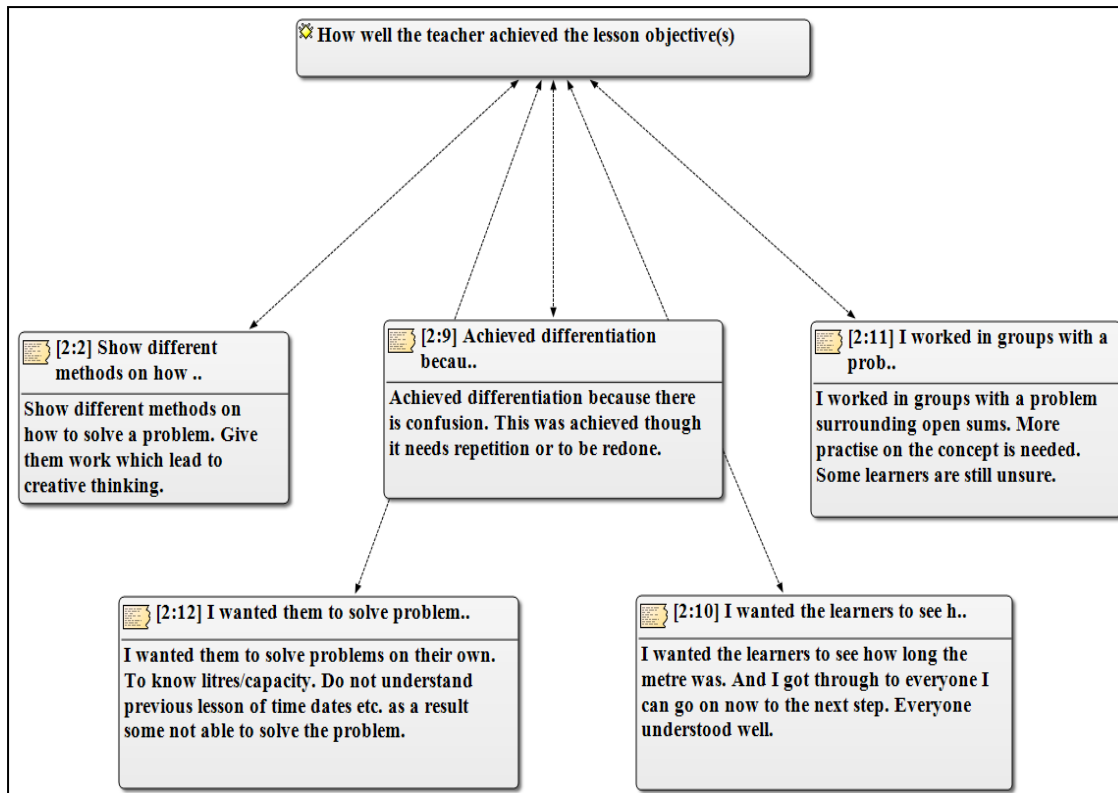


Figure 6.14: How well the teacher achieved the lesson objective(s)

The centre of focus in these reflections was the teacher, hence the use of the word “I” in the case of teachers [2:10], [2:11] and [2:12]. They reflected on how well they had achieved the lesson objective. One notes that the achievement of the objective is not based on the learners’ outcomes, but more on how the lesson went. In the above reflections, the achievement of the objective was measured in terms of what the teacher set to do and achieved. Examples of such measurement criteria were:

“Achieved differentiation...” [2:9]

“show different methods on how to solve a problem” [2:2]

“I wanted learners to see how long the metre was...” [2:10]

“I wanted them to solve problems on their own...” [2:12]

“I worked in groups with a problem...” [2:11]

The researcher’s greatest concern was that, with these reflections, as is observed in the discussion of the reflection on “effect on learner performance”, the focus was not on the learners themselves.

6.5.3 Level of own satisfaction with effort

In this category, 28% of the observed teachers reflected on how happy they were with their execution of the lesson (see Table 6.30) The teachers focused on how satisfied they were

about how the lesson had proceeded, or with the technicalities of lesson delivery, and not in terms of how the lesson had impacted on the learners. The ATLAS.ti network view that is shown in Figure 6.15 below shows their reflections.

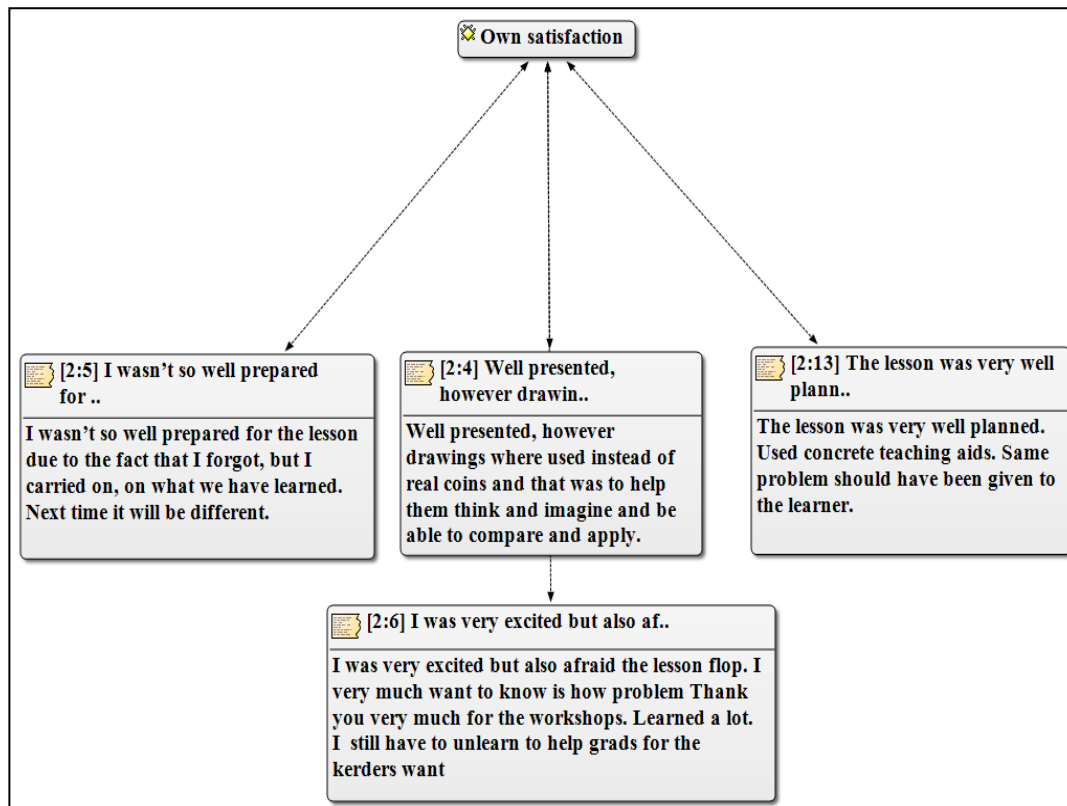


Figure 6.15: Level of satisfaction with own effort

The reflections concerned were once again teacher-centred, with the teachers reflecting on how satisfied they were with the way in which their lesson had proceeded. Their level of satisfaction was derived from how well the lesson had proceeded in terms of the planning [2:13], preparations [2:5], and presentation, [2:4] and [2:6]. In other words, the teachers' concerns were with how well they had been able to execute their lessons.

6.5.4 Discussion, and factual and interpretative conclusions

The analysis of the teachers' reflections on their PBL lessons revealed that the majority (64%) of the teachers' reflections indicated a preoccupation with instructional success, in terms of the technicalities of the preparation and presentation of the lesson, such as in the use of groups. The reflections were not supported by, or based on, evidence drawn from the learners' performance during the lessons taught. Little of the teachers' reflections were critical of how their teaching had impacted on the learners. The teachers were engrossed in reflecting on their teaching performances, with scant reference to how effective the performances were in relation to the learners. The teachers did not indicate whether or not the learners had achieved understanding of the material taught, and whether the objective(s)

for the lesson had been met. The reflections show that the teachers were not thinking more critically than before their development about their impact on, and their role in, crafting learning situations where the learners had access to quality mathematics learning experiences.

6.6 SECTION 5: DATA RELATING TO PROBLEMS USED BY THE TEACHERS IN THEIR PBL LESSONS

The literature review in Chapter 3 revealed that the problems concerned are considered one of the three key elements of PBL. Subsection 3.9.3 emphasises that effective problems are essential for PBL, as they initiate the learners' learning in PBL. In other words, poor problems lead to poor PBL. This section of the research presents data on the effectiveness of the PBL problems that were used by the teachers in the PBL lessons. The analysis is based on the current understanding of problem characteristics, as was discussed in subsections 3.9.3 and 3.9.4.

The literature review in Subsection 3.9.4 suggests that the effectiveness of problems can be defined in terms of six different functions. Function characteristics are the desired outcomes of a problem. The problems that were used by the teachers in the observed lessons were analysed using the six function characteristics that were presented in Subsection 3.9.4. An ATLAS.ti qualitative analysis of the problems, which involved coding them and producing network views for the various categories, was done. See Table 6.31 for the results of the final analysis.)

Function characteristics		
	Number of problems that met the criteria	Number of problems that did not meet the criteria
1. Leads to the intended learning issues	9 (75%)	3 (25%)
2. Promotes SDL	9 (75%)	3 (25%)
3. Stimulates critical reasoning	3 (25%)	9 (75%)
4. Stimulates collaboration	9 (75%)	3 (25%)
5. Promotes teamwork	9 (75%)	3 (25%)
6. Triggers interest	9 (75%)	3 (25%)

Table 6.31: Feature characteristics and function characteristics of problems used

6.6.1 Functional characteristic 1: Problems that lead to intended learning issues

A good PBL problem should lead learners to what, in Subsection 3.9.4, can be described as intended learning issues (Schmidt & Sockalingam, 2011), or, put simply, learning objectives. Problems with the functional characteristics of a PBL problem have solutions that are not so obvious that learners are able to solve them immediately. However, they have keywords that give the learners hints, or that even guide them, to another major keyword, and which, eventually, allow them to find the key concept and solution. Problems without this functional characteristic are so obvious that the learners would be able to solve them immediately. Three of the problems used by the teachers in their PBL lessons lacked functional characteristic 1. The problems concerned were as follow:

Tina walks 15 footsteps, and his brother [walks] 24 footsteps How many footsteps do they walk altogether?

Lizo has two 5c pieces. His sister has three 5c pieces. Write how much money this is altogether.

Jese has 342 stamps. She gave out 149 stamps. How many stamps does Jese now have?

The above problems were categorised as those that did not trigger learning issues. The three problems are too obvious in terms of what has to be done. They have a single correct answer, and they do not challenge the learners to explore multiple solutions.

6.6.2 Functional characteristic 2: Problems that trigger critical thinking/reasoning

Problems with the functional characteristic of a PBL problem are ones that actually make the learners think. Only three of the problems that were used by the teachers in their PBL lessons met this functional characteristic of a PBL problem. These were the following:

A taxi had 6 learners on board. It stopped to pick up learners. When the driver counted, there were 10 learners in the taxi. How many learners did the taxi pick up?

Andile has R70,00. Which items will she be able to buy for breakfast?

Carla earns R14,00 an hour to clean houses. She works from 8:00 a.m. to 2:00 p.m. How much money does she earn?

These problems have the potential to make the learners think. Their solution path is not obvious. It is difficult to figure out what the solution is. They have to generate multiple views about the problem, as well as how to solve it, thereby promoting critical reasoning. The problems present a complex situation that has no single clear-cut solution path. The use of complex problems (nonroutine problems), as is pointed out in Subsection 3.7.2, offers the learners learning experiences that develop high-order thinking skills, or critical reasoning.

6.6.3 Functional characteristic 3: Problems that trigger interest

Problems with the functional characteristic of a PBL problem are highly interactive and interesting. They are ones that concern the learners' everyday way of life. The majority (75%) of the problems that were used by the teachers had this characteristic. Below are two such problems that were used by two of the teachers observed.

Father has 12 apples. He gave these to 3 children. How many apples does each child get?

A taxi had 6 learners on board. It stopped to pick up learners. When the driver counted, they were now 10 learners in the taxi. How many learners did the taxi pick up?

The above problems are ones that concern the learners' everyday way of life, and that, thus, have the potential to trigger the learners' interest.

6.6.4 Functional characteristic 4: Problems that promote teamwork

Problems promoting teamwork are difficult to an extent. The problems challenge the learners to think hard, therefore calling for greater group discussion of the strategies to the solution, as they are highly unlikely to be able to solve the problem on their own. Table 6.30 shows that the majority (75%) of the problems that were used by the teachers had this functional characteristic. The problem below is an example of such a problem.

Carla earns R14.00 an hour to clean houses. She works from 8:00 a.m. to 2:00 p.m. How much money does she earn?

6.6.5 Functional characteristic 5: Problems that promote SDL

Problems fulfilling this functional characteristic of a PBL problem are those that activate the learners' minds and those on which they are highly unlikely to waste their time doing unrelated activities. As such problems are harder than are other problems, the learners are likely to keep referring to them as they work on the solution. The learners are not likely to be sidetracked from the key problem. The majority (75%) of the problems that were used by the teachers had this functional characteristic (see the example given below).

Mr Priza fills his car with petrol once per week. His tank holds 43 litres. How much petrol does he use in a month?

6.6.6 Functional characteristic 6: Problems that promote collaboration

A problem that promotes collaboration is one that is clear-cut and easy to understand, and which contains keywords that are crucial to the problem. It is one that would enable the learners to quickly start to research and brainstorm about the various concepts and ideas of the day's lesson. The majority (75%) of the problems that were used by the teachers had this functional characteristic. The problem shown below is an example of one such problem that was used by one of the teachers.

A grandmother wants to bake 4 cakes. Each cake will require 4 eggs. How many eggs must she buy from the shop?

6.6.7 Discussion, and factual and interpretative conclusions

The analysis of the problems that were used by the teachers in PBL shows that the majority of them were problems that lead to the intended learning issues or objectives, that promoted SDL, that promoted teamwork, and that triggered interest. However, the majority of the problems were not able to stimulate critical reasoning. Hence, the majority of teachers (75%) did not offer their learners learning experiences that developed their high-order thinking skills, or their critical reasoning. The quality of the problems used by these teachers suggests that a profitable approach to adopt to staff development would be to offer the teachers a workshop focusing on constructing problems that optimally promoted the development of higher order thinking skills and critical reasoning.

6.7 SECTION 6: DATA FROM THE 1ST TEACHER INTERVIEWS

To gain insight into the problems and type of support that the teachers needed to be able to carry out PBL, the 14 teachers who had been introduced to PBL were each interviewed after the lesson observation. During these semi-structured interviews, the teachers were asked to describe their experiences regarding the integrating of PBL and CAPS, the temptation to show and tell, and the use of groups in PBL.

The teacher interviews were necessary for them to be able to clarify their thinking, as well as for them to be able to ascertain whether the researcher understood their experiences. The interviews were taped, transcribed and analysed with ATLAS.ti, using the open coding technique (see Appendix 13 for an example of a full transcript).

Question 1: *What has been your experience with regard to the integration of PBL demands with the demands of the curriculum (i.e. CAPS)?*

The analysis of the teacher responses to the above question revealed teachers who were confused and under pressure, resulting from having had to implement the CAPS. Figure 6.16 below shows some of the comments that were received, and which indicated that the teachers experienced pressure and confusion.

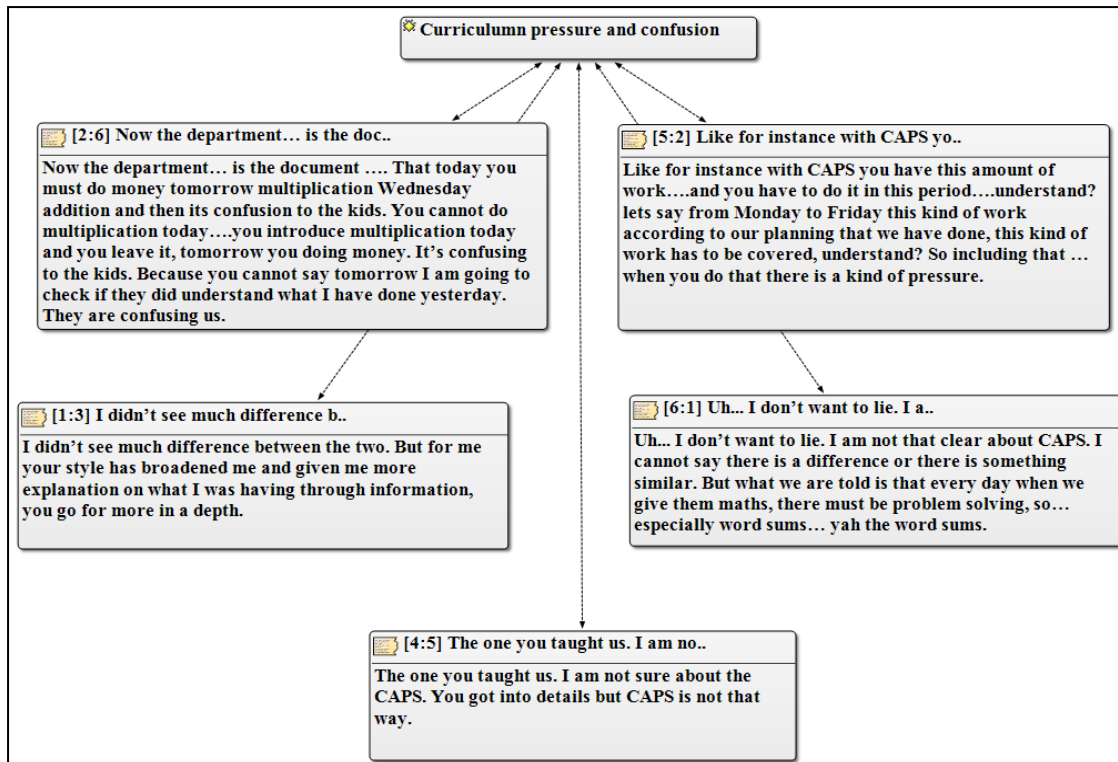


Figure 6.16: Curriculum pressure and confusion

The above comments revealed that the teachers were confused and under pressure to teach, as per the CAPS requirements. The teachers in question found themselves under pressure, because they had their daily work cut out ([5:2] and [2:6]). This pressure to cover everything in the syllabus made it difficult to use another teaching model other than the traditional model (Yoshinobu & Jones, 2012, p. 304; Hennings et al., 2012, p. 454). The traditional model features the teacher as a transmitter of knowledge, and the learner as the consumer. In terms of such a model, the teacher shows, and the learners follow. The pressure that this induces tempts the teachers concerned, who are encouraged to resort to the traditional methods of 'show and tell' in an effort to cover the prescribed daily work. (See also the responses to question 2 below.)

Question 2: *What has your experience been with regard to the temptation to show and tell the learners what to do?*

The data analysis in Section 1 showed that the majority of the teachers' views of teaching emphasised their role in transmitting knowledge, and in providing the correct solutions. This was before they had been exposed to PBL as an alternate view of teaching, which emphasises the teacher's role as a facilitator of active learning, with the learners seeking the solutions themselves. The above question sought to find out whether the teachers had done

away with knowledge transmission practice. Figure 6.17 below shows that 5 of the 12 teachers (42%) interviewed admitted that they were occasionally tempted to show and tell.

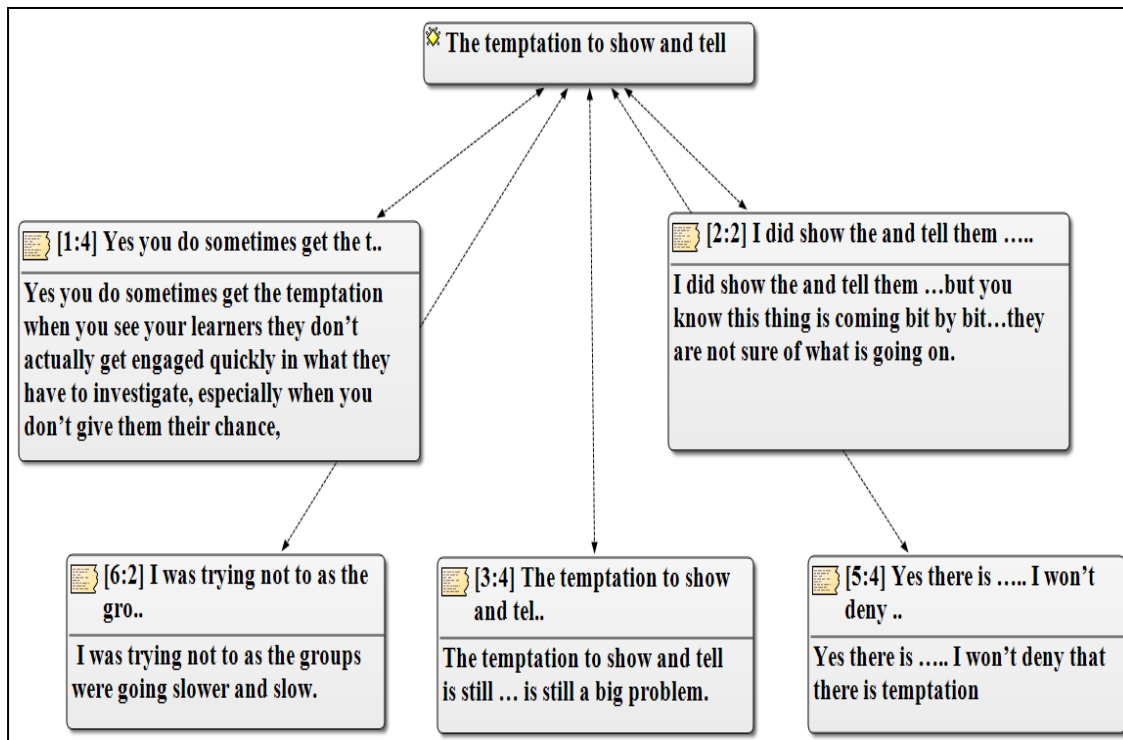


Figure 6.17: Temptation to show and tell

The above extracts show that the teachers were sometimes tempted to use the old traditional methods of showing and telling learners the mathematics involved. The teachers did this when the groups concerned were working too slowly on the task [6:2], and when the learners were slow to engage with the task [1:4]. The extracts in Figure 6.18 below give the teachers' justification for why they sometimes resorted to the traditional method of showing and telling during mathematics lessons.

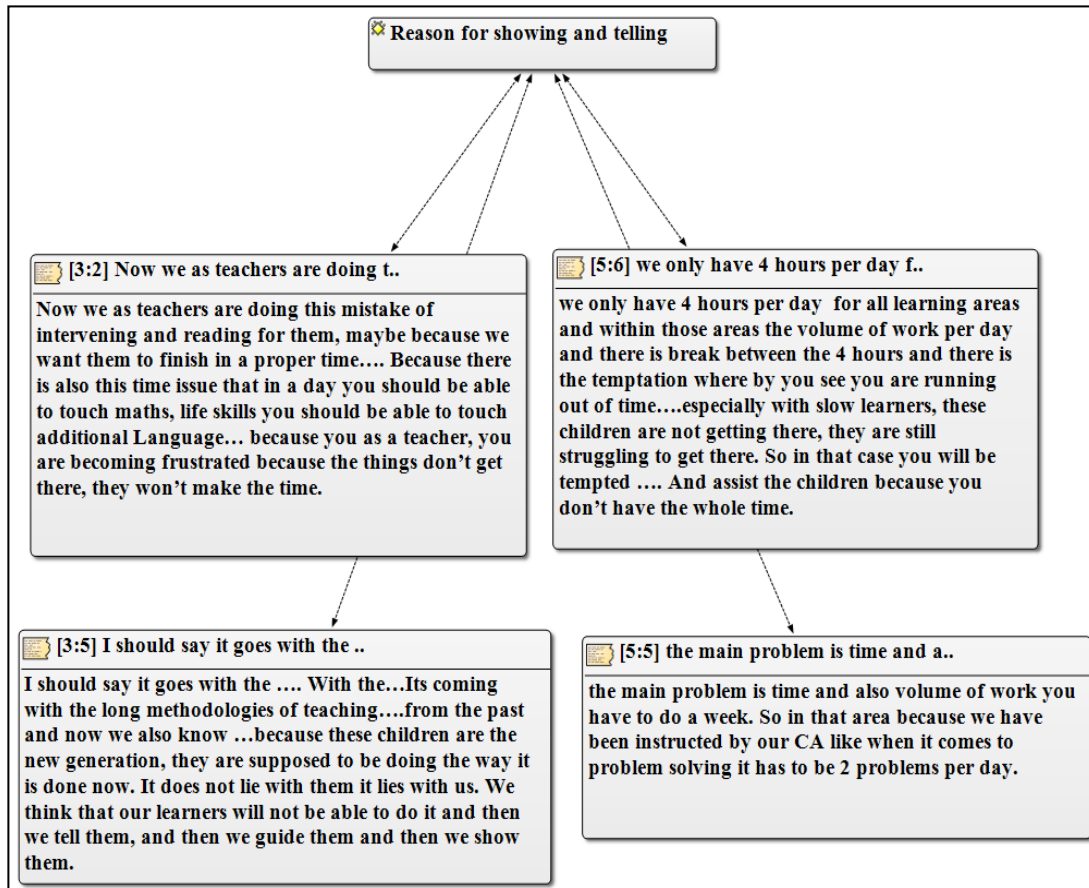


Figure 6.18: Reasons for showing and telling

The teachers' reasons indicate that, whereas they were aware that showing and telling was not an effective teaching and learning strategy, they still did so because:

- of the desire to finish the work of the day in the stipulated amount of time [3:2];
- they were running out of time, especially with the struggling learners [5:6];
- there was a mismatch between the volume of work to be done per week, and the amount of time that was allocated to mathematics [5:5];
- the teachers thought that the learners were not capable of solving the problem on their own, hence they told them how to reach the solution, guided them towards working it out, and then showed them it [3:5],
- they were frustrated when the learners were not achieving the targeted objective within the allocated amount of time [3:2].

Due to the above reasons, the teachers were tempted to show and tell, although their experiences with PBL had shown them that doing so was not an effective approach to the teaching and learning of mathematics. Extracts 49 and 50 below support this point.

Extract 49: "...when they do these things for themselves it's better ...to understand instead of telling, that's the difference, they don't forget. But when you tell[,] sometimes they forget. That's the difference."

Extract 50: "...so in that way its more effective than the old way, it is effective. If you start...I did try it ever since...even after you were here the other day. I can see it is working[,] because I believe before we have undermined the thinking or maybe understanding of our learners. We always assume or take it as if they are to be spoon-fed. Only to find out with problem-solving...that this time you can see...they are...gives [i.e. given] time to think and [to show] interest[,] because they always want to explore and find out. So[,] if you fail to give them that opportunity[,] you will end up struggling all the way."

The teachers resorted to showing and telling as a means of covering the daily workload prescribed by CAPS. Doing so was actually a manifestation of the teachers' belief that the memorisation of facts and formulas, and the practising procedures is sufficient for the learning of mathematics.

Extracts 49 and 50 above show that the teachers had observed that PBL was more effective than were their old ways of teaching ('showing and telling', or 'spoon-feeding'). They also acknowledged that the use PBL enhanced the understanding of the learners more than did merely telling them the solution, with the use of the former method resulting in them remembering what they were learning. The above extracts point to the fact that, in those situations where the teachers are frustrated and pressed for time, they tend readily to fall back on their traditional roles of power, and of being the knowledgeable prime talker, irrespective of whether the use of such a strategy leads to an understanding of the content. They do this solely so as to be able to cover the content within the stipulated timeframe.

Question 3: *What has your experience been regarding how your learners worked in their groups?*

The literature in Subsection 3.5.3 emphasised the importance of using groups in PBL. Learners do not just learn by being put together in groups. When group work is carefully constructed, and when the teachers help the learners deal with the group dynamic issues that comprise group effectiveness, cooperative learning takes place. Question 3, thus,

sought to find out how functional group work was done in the interviewed teachers' problem-based lessons.

The ATLAS.ti network view given in Figure 6.19 below reveals that the teachers had observed the following when allowing their learners to work in groups during PBL lessons:

- Their learners had improved in performance, as a result of the weak ones having worked together with the capable ones [1:6].
- The learners had been willing to work in groups [3:6].
- The group work had turned the lessons into exciting experiences [6:3].
- The learners had not understood the concept of working in groups [4:4].

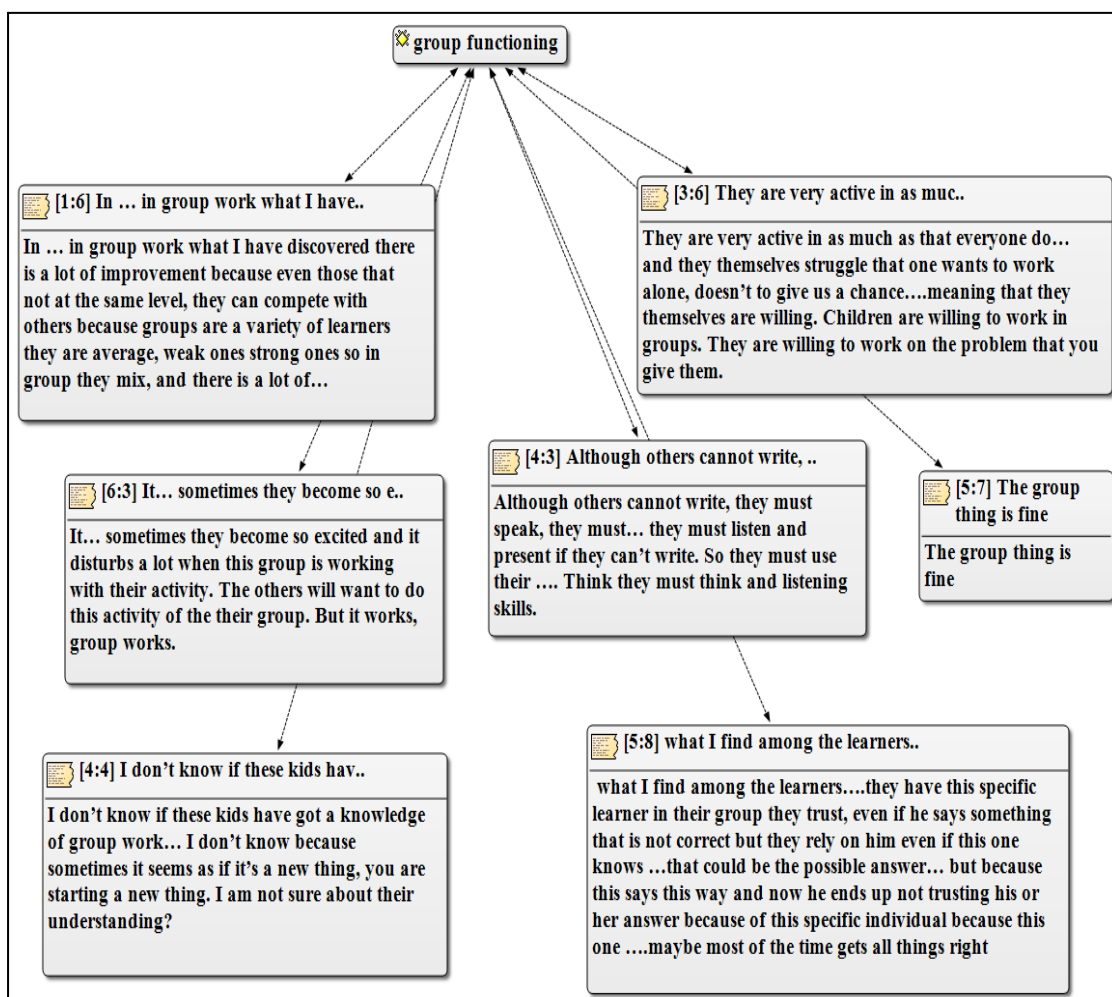


Figure 6.19: Group functioning

The network view that is given in Figure 6.19 above also reveals that the teachers had experienced the following problems related to group functioning:

- In terms of group dynamics, the groups functioned neither efficiently nor effectively [4:4].

- The group members relied on a single group member [5.8].
- Not all learners had the necessary skills, such as reading, that were required to function in a group [4:3].
- Some learners dominated their groups, not giving others a chance to contribute to the group work [3.6].

The quotes that are given in Figure 6.19 above reveal that, whereas the groups were not functioning satisfactorily in some classes, the teachers did not readily fall back on their traditional roles of authority, expert, and prime talker; which they tended to do when they were running out of time.

6.7.1 Dealing with dominating group members

The quotes that are given in Figure 6.20 below show how the teachers dealt with the dominating learners, which was one of the causes of group malfunctioning.

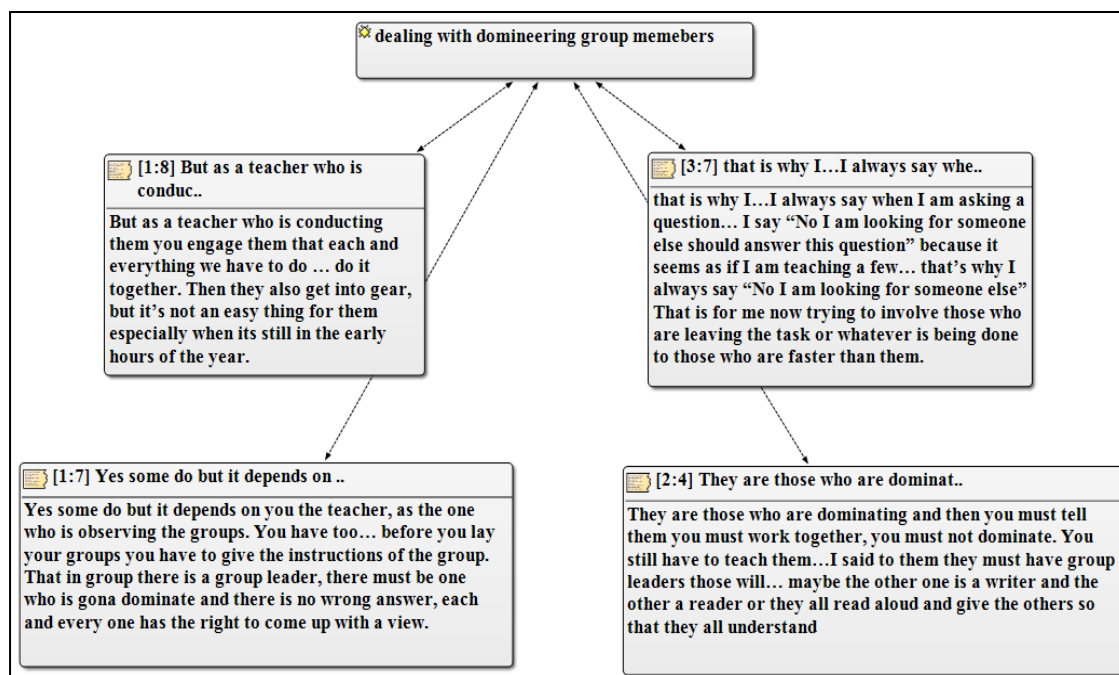


Figure 6.20: Dealing with dominating group members

The quotes that are given in Figure 6.20 above show that the teachers were aware that the dominating learners were detrimental to the holding of effective group discussions. They took the following actions to control the flow of group discussion without dampening the dominating group members' sense of inner self-worth and enthusiasm:

- They gave instructions to the group, stating that there was no wrong answer, and that each and every one had the right to come up with a view [1:7].
- They told the dominating learners not to dominate, but rather to work together, and to allocate functions to the various group members [2.4].
- They looked for someone else to give the answer, other than the dominating learner [3:7].

6.7.2 Discussion, and factual and interpretative conclusions

CAPS, which is a very prescriptive curriculum, allocates the content and topics to specific lessons. The teachers' comments reveal that, when the pressure to complete the curriculum within the CAPS timeframe mounts, the teachers revert to the old traditional methods of showing and telling, in an effort to cover the daily work that is prescribed. The teachers, too, readily fell back, in frustration, on their traditional roles of dispensing knowledge, irrespective of whether or not the use of such an approach has led to the understanding of the content, and thereby compromising learning through the lack of relational understanding.

The researcher interprets this fall back to the old traditional methods to be partly a manifestation of the teachers' beliefs that the memorising of facts and formulas, and the practising of procedures, is sufficient for the learning of mathematics. This is indicated in Subsection 6 of the data presentation and analysis.

In PBL, the functioning of the groups plays a crucial role in stimulating students' learning. The teachers' quotes on the functioning of learners' groups suggest that a viable approach to staff development for these teachers would be to offer them training workshops on the basic strategies that foster well-functioning PBL groups.

6.8 SECTION 7: DATA FROM THE 2ND TEACHERS' INTERVIEWS

A total of 14 out of the 15 teachers were interviewed six months after the end of the four-year intervention project to find out whether, and how, they had continued implementing PBL in their teaching of mathematics. Their responses were transcribed and analysed using ATLAS.ti.

Question 1: *Have you continued using PBL in your mathematics lessons this year?*

The ATLAS.ti network view produced for question 1 is summarised in Table 6.30. It shows that 71% of the interviewed teachers had continued implementing PBL in their teaching of mathematics.

Percentage of teachers who had, or had not, continued with the use of PBL		
	Had continued with PBL	Had not continued with PBL
Question 1: Have you continued using PBL in your mathematics lessons this year?	71%	29%
N = 14		

Table 6.32: Continuation of the use of PBL in the teachers' mathematics lessons

Extracts 51 and 52 below are examples of comments by some of the teachers who had continued with PBL in their teaching of mathematics.

Extract 51: "Yes[,] I continued, and then told other teachers about this and the new teachers that we are using the method."

Extract 52: "Yes, I have. I am implementing that and then I have also shared, we have got quite a few new teachers in Grade 1 and I did mention to why and why it is important... that lessons should be learner-centred."

The two teachers concerned had not only continued implementing PBL, but they had also shared the approach with the new teachers at their schools.

6.8.1 Reasons for continuing with the use of PBL

An analysis of the responses of the teachers who had continued with PBL reveals that the teachers had done so for two reasons:

The learners' enhanced understanding of mathematics: Of the teachers, the 60% who had continued with PBL had done so because they had observed that their learners understood mathematics better when they used PBL than they otherwise did. Their reasons for continuing with the use of PBL are shown in Figure 6.21 below.

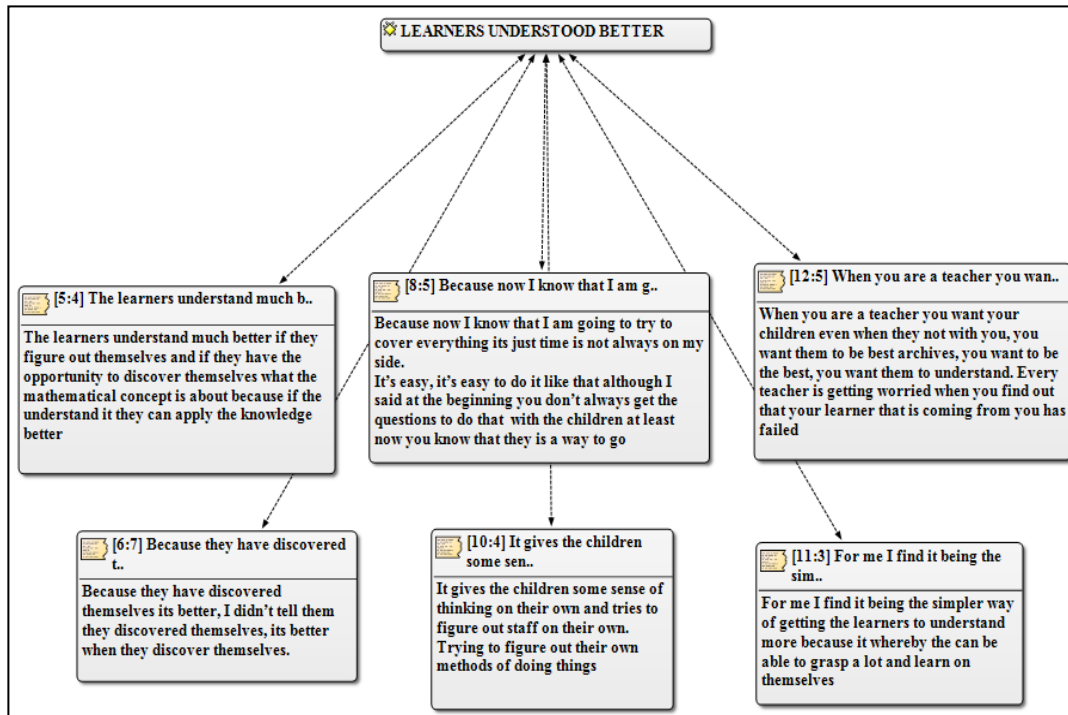


Figure 6.21: Learners' enhanced understanding of mathematics through PBL

The comments that are contained in Figure 6.21 above reveal that the teachers had continued to use PBL because of its effects on the learners: "*learners understood better*" [5:4] or "*more*" [11:3]; and "*it's better when they discover themselves*" [6:7].

6.8.2 An effective teaching/learning method

The other group (40%) of teachers who had continued implementing PBL had done so because they had seen its effectiveness as a method for the teaching and learning of mathematics. Their comments to this effect are shown in Figure 6.22 below.

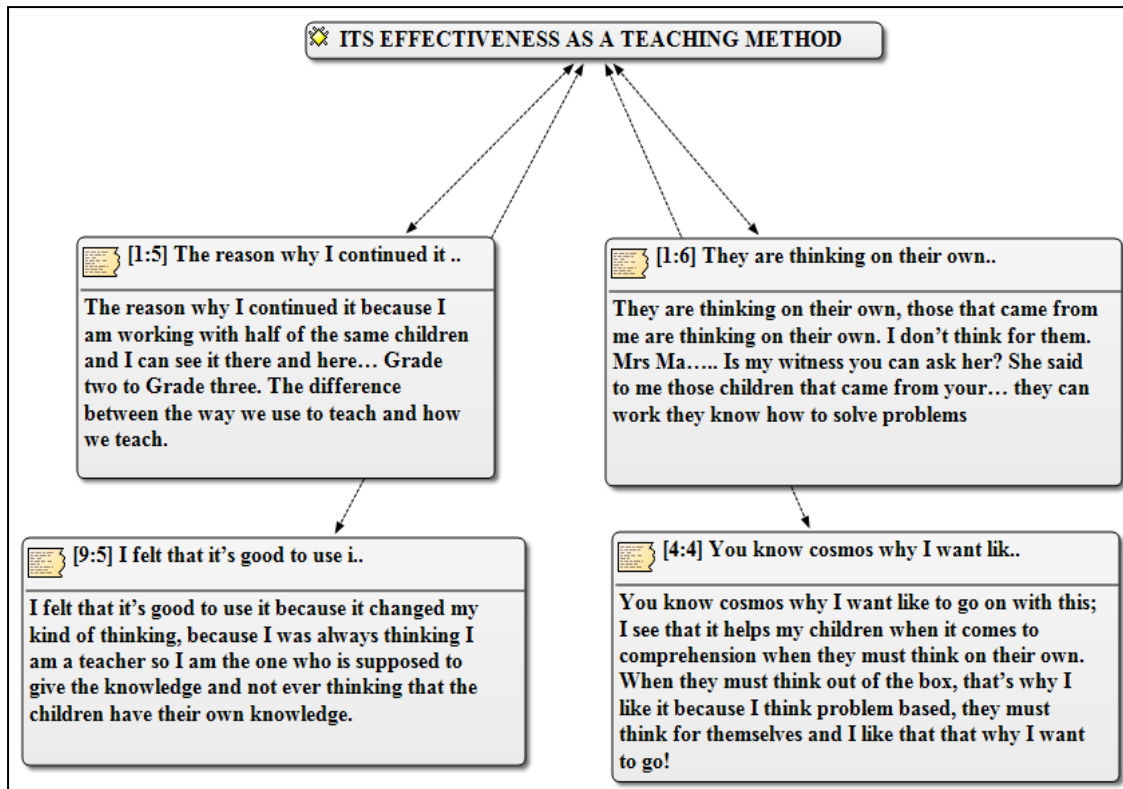


Figure 6.22: The use of PBL as an effective teaching/learning method

Teacher [1:5] had continued with half of her previous class to the following grade, and was seeing a great difference between the performance of her learners, who had been exposed to PBL, and that of the other half of the learners, who had come from the other grade class, where the teacher had not used PBL. The other half of her learners, who had come to be taught by another teacher, were also reported to be showing a marked difference compared to the other half, who had not been exposed to PBL [1:6].

6.8.3 Reasons for not continuing with the use of PBL

The four teachers who had stopped using PBL had done so for the reasons shown in the ATLAS.ti network view that is given in Figure 6.23 below.

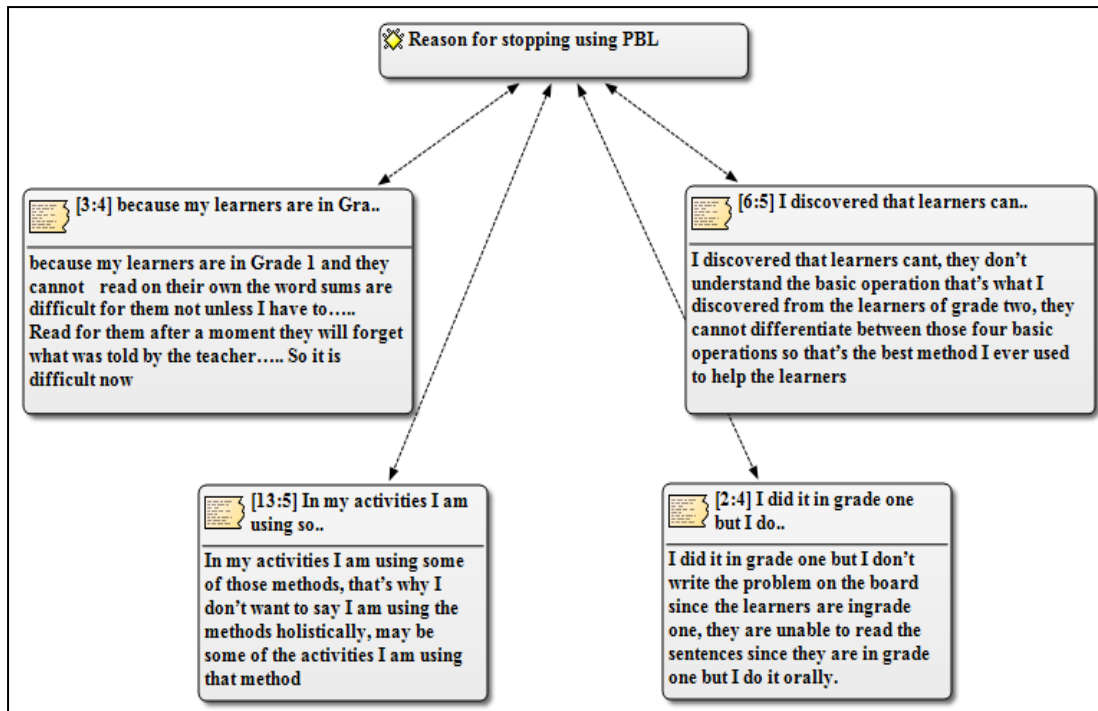


Figure 6.23: Reasons for stopping the use of PBL

According to Figure 6.23 above, two of the teachers, [3:4] and [2:4], had stopped using the PBL approach because they were now teaching Grade 1, and they felt that they could not use PBL in the first term, as their learners had not yet learned how to read. The teachers concerned associated PBL with the ability to read. Although respondent [13:5] did not use PBL holistically, she used it in “some of the activities”. Teacher [6:5] was in the teaching *for* problem-solving mode, hence she had not continued using the method, because she had discovered that her learners “can’t, they don’t understand the basic operations”. Consequently, for her, learners without an understanding of basic operations could not be involved in PBL. Savery (2006, p. 11) describes such an outlook as arising from “confusing PBL as an approach to curriculum design with the teaching of problem-solving”.

Question 2: *Based on the experience that you are having of PBL, what is your assessment of PBL as an approach to the teaching of mathematics?*

The teachers’ responses reveal that 79% of the interviewed teachers assessed PBL very positively, based on the experiences that they had had, or were having, with PBL. The positive comments on PBL from the teachers included the following:

Extract 53: “I think it’s a very successful method to use and I think that the children will...have [mastery of] more of [a] thinking concept than [merely] listening...just of... getting a concept, [they will learn] more about the

thinking concept than [they would if merely] listening to you.”

Extract 54: “I would say [that] it [is] the most effective way which everybody could say, the grouping is very important[,] because with grouping the learners share the knowledge among themselves without you...telling them what to do. They discover, [and] explore things for themselves. That is very good, by the time you intervene at least you know they have the idea rather than when you tell [them] something.”

The following extracts reveal that the teachers concerned had tried to use PBL, and that they had found it to be an effective method that worked well for them. The effectiveness of PBL, as it was described by the teachers, was based on the quality of learning that it generated.

Extract 55: “I think it’s good...I feel that the learners were able to apply the knowledge better[,] as opposed to when I tell the learners...”

Extract 55: “I think it’s [a] very good idea...I think everyone must use it, not only one or two teachers. If we start it in Grade 2 to 7[,] imagine what [i.e. how that] child will come out in Grade 7...”

Question 3: *Can you briefly describe what you understand about problem-based learning?*

Of the teachers, 58% described PBL as being child-centred. This is evident in the following extracts:

Extract 56: “You must not feed the kids, they must think on their own. They must be creative when answering the question. We don’t restrict them...”

Extract 57: “...try to solve problem on their own without me telling them...”

Of the teachers, 17% revealed their confusion regarding the use of PBL as an approach to the curriculum design of material relating to the teaching of problem-solving. Extracts 58 and 59 below are examples of teacher responses that indicated this confusion.

Extract 58: “First[,] the child must know his basics to [the] approach. They must have the knowledge of calculation[,] and I mean you give the child the basics ...you ask them questions.”

Extract 59: “You give learners a problem[,] and then from there[,] if it's addition, then you follow up with the addition sums.”

Of the teachers' descriptions, 25% showed a better conceptual understanding of PBL as a vehicle for learning, rather than for teaching for, problem-solving. Extracts 60 and 61 below are examples of such descriptions.

Extract 60: “Well[,] I understand that it is posing a problem to introduce new mathematics concept, so that the learner...you know has the opportunity to figure out the concept themselves, to come to an understanding [of] what the concept really means[,] as opposed to someone explaining [to] them how they must do it.”

Extract 61: “When we talk about problem-based learning[,] we talk about learning that comes from the learners... something that is more learner-centred.”

Question 4: *Have you discussed PBL with other teachers? What did you discuss?*

Of the 14 teachers interviewed, 10 teachers (71%) had shared the information on PBL with other teachers, as is shown in Table 6.33 below.

Number of teachers told about PBL, and the nature of information on PBL shared with others				
<u>Capacity of others with whom PBL information was shared</u>	<u>Nature of information shared</u>			
	Learner involvement, and the function of groups	Application of PBL and the PBL process	The capacity to allow for different solutions to a problem	The addressing of CAPS through PBL
Colleagues	–	4	–	–
New teachers in the school	2	–	–	–
Colleagues teaching the same grade	–	2	–	–
Teacher from another school	–	–	1	–
Another teacher who asked about PBL	–	–	–	1
N = 14 No. of responses = 4				

Table 6.33: Sharing of PBL with others

Table 6.33 reveals that the teachers shared a variety of information on PBL with their colleagues. The data also reveal the formation of informal communities of learning with teachers from other schools, with colleagues within the same grade, and with new teachers coming into the school.

Question 5: *As your learners work in groups, how do you guide them and ensure that meaningful learning is taking place?*

An analysis of the teachers' responses in relation to ensuring that learning takes place during group work reveals that the teachers had assumed the role of a facilitator to ensure that meaningful learning was, indeed, taking place. They had not restricted themselves to the traditional role of information transmitter.

<p><u>Question 5:</u> As your learners work in groups, how do you guide them and ensure that meaningful learning is taking place?</p>	Strategies used by the teachers to ensure that meaningful learning is taking place as their learners work in groups.			
	Ensure that the learners understand the problem to be solved	Walk around the groups, asking questions, and initiating discussion	Create a familiar problem context to be in line with the learning issues	Lead and guide the learners, so as to get more out of them than would otherwise be possible.
	37%	37%	13%	13%
N = 12				

Table 6.34: Strategies used to ensure that meaningful learning is taking place in groups

The temptation to revert to being a knowledge transmitter was still present among the teachers (see the detailed transcription from the interview with [4:6] below).

Teacher:	If you give a problem, you don't explain, you group your learners. Then let them work in groups[,] and I go from group to group[,] and see how they work. Sometimes they struggle and I want to help, then I just walk away[,] because I feel I want to help.
Interviewer:	You don't have to walk away. You give them a hint.
Teacher:	But sometimes I want to take over instead of just giving a hint. That's why I walk away, I get frustrated[,] then I want to solve for them, then I want to give too much.

The strategies used above and the teachers' responses given above show that the teachers were adapting to their new role of facilitator in relation to PBL.

Question 6: *What support would you need as you continue to use PBL?*

An analysis of the responses to question 5 reveals that the teachers needed the type of support that was associated with material resources and with the PBL process.

Nature of the support, and the percentage of teachers indicating the need for such support		
	Support associated with material resources	Support associated with the PBL process
Question 5: What support will you need as you continue to use PBL?	42%	58%
N = 12		

Table 6.35: Nature of support needed by the teachers who continued with PBL

6.8.4 Support associated with material resources

Of the teachers, 42% indicated that they needed support that was related to material resources and to such learning materials as play boards, concrete objects, and previous examination papers. Extracts 62 and 63 below give examples of the type of support required:

Extract 62: “If I can get more support[,] like[,] for instance[,] having paper[s] of the previous years [–] all that stuff will help me see how they test.”

Extract 63: “Yes, in terms of apparatus because when they did the shop[,] we only have [i.e. had] the money. If we can have[,] say for instances, playing boards.”

6.8.5 Support associated with the PBL process

Of the teachers, 58% indicated that they needed support to improve their implementation of PBL, focused on how effectively to involve the weaker learners and the nonreaders in PBL. Extracts 64, 65 and 66 are examples of the utterances of such teachers.

Extract 64: “I think the support I would like to have is to do in those different groups, I want to apply it. You find with [a] weaker group[,] sometimes they can’t even move.”

Extract 65: “Well[,] since I am not there yet I cannot say now...Because I can only say they cannot read if they cannot read and understand what...the problem...wants...”

Extract 66: “Sometimes it is difficult[,] especially for the weaker learners[,] to...you know[,] give them enough time for them to discover and solve problems for themselves. I think it [is] something easy for the stronger learners to do.”

The second form of support needed was improved knowledge and skills, which could become available through more training and peer demonstration, in order that PBL might effectively be implemented. Extracts 67 and 68 below are examples of specific requests of this nature:

Extract 67: “...attending more training on that in order to ... be[come] more skilled on [i.e. in] that and to gain more knowledge as well.”

Extract 68: “It is always good to see demonstrations and people working[, and] even other teachers who are applying the problem-solving method. Just to see the different ways. How they approach it[,] because I feel that you can always learn from somebody else.”

The third form of support that the teachers stated they needed was in constructing good PBL questions. Extract 69 below is an example of such a request:

Extract 69: “...in the formulation of the questions[,] because sometimes I give them just straight...questions there is no thinking. One of the questions I gave one day was *“I got 15 eggs, some broke and I have got 3 left.”* I really need help in that area.”

Question 7: From whom the support is required

The teachers expected to receive the support they required from the WCED, their colleagues, the university, and other users of PBL. Extracts 70, 71 and 72 below consist of examples of such expectations:

Extract 70: “From the WCED[, and from] whoever can give us the support, whoever can give us the resources.”

Extract 71: “Other people who are applying it[,] because I don’t think that somebody would be able to support me with it that doesn’t know the problem-solving method...that doesn’t use it.”

Extract 72: “Whom? As I have gained training from you guys[,] I am looking forward to gain more from you guys.”

6.8.6 Discussion, and factual and interpretative conclusions

The second set of interviews revealed that the majority of the teachers (72%) had continued to use PBL during their mathematics lessons. The teachers had continued using PBL primarily because it had worked for them. The second reason was because their learners seemed to understand mathematics better through PBL than they did through other methods of learning. One of the teachers who had proceeded with half of her class to the following grade had noted that her half of the class, which had done mathematics through PBL, performed much better than did the other half, which had come from another class that had not used PBL, especially where mathematical reasoning was involved. The teacher who had the other half of the same class to teach noted the same difference between the two groups of learners, too. The teachers needed continued and ongoing support to be able to improve their implementation of PBL, particularly in the case of the slow learners. The teachers also needed help in constructing good PBL problems.

The teachers reported relatively infrequent collaboration with other teachers at the schools at which they taught, and in other schools. Such collaboration, which was beyond being a mere exchange of information, shows that, generally, there is much greater scope for teachers to learn from other teachers. At the same time, the data showed much variation in the support needs of the individual teachers. Such variation underlines the need for individualised and targeted follow-up support as part of any professional development for teachers, rather than the once-off whole-group interventions that tend to dominate current teacher professional development.

6.9 CONCEPTUALISATION OF PBL WITHIN THE FOUNDATION PHASE

As was discussed in Chapter 3, PBL, as the constructivist answer to traditional learning theories, is based on the following three main preconditions for a successful and comprehensive learning process: collaborativeness and learner-centredness, following an active process of knowledge construction (Maurer & Neuhold, 2012). The research showed that PBL, based on the above preconditions, can be meaningfully implemented in the classroom in the Foundation Phase. To illustrate this, an example of one of the Grade 1 teachers who was observed implementing PBL is discussed below. The teacher concerned created a learning environment in which the learners constructed their own mathematical knowledge, and in which they developed their own reasoning and critical thinking skills. She was able to organise the subject matter (Figure 6.24) around a specific problem.

Subject Matter: Grade 1

Topic 1.7: Addition and subtraction

Solve word problems in context, and explain own solution to problems involving addition and subtraction with answers up to 20.

Figure 6.24: Subject matter (CAPS, p. 22)

The above-mentioned teacher, who dealt with the topic given in Figure 6.24 was able to organise the CAPS subject matter around a problem that she then gave to her learners to solve in groups. Their involvement in this process allowed the learners to construct their own understanding of subtraction, and to develop their own subtraction procedures.

The Grade 1 class teacher's focus was on subtraction, starting with the provision of a problem. The subtraction concepts were developed through hands-on experiences with countable objects. Subtraction tells 'how many are left', or 'how many more or less'. In this case, the problem focused on *how many more* there were of an item than before. In this PBL lesson, the teacher created an opportunity for the learners to construct their own understanding of subtraction, and to develop their own subtraction procedures.

6.9.1 Reading the PBL problem in the Foundation Phase

The fact that the Grade 1 learners did not have reading skills was not a barrier to the use of PBL. Three Grade 1 teachers observed were able to use PBL in their classes. They read out the problem that they wrote up on the board to the whole class before asking the groups to

discuss and work out the solution to it. Doing so provided an opportunity for integrating reading into mathematics; making reading meaningful within the mathematical context.

6.9.2 Research in the Foundation Phase PBL environment

If there are issues that require researching, Grade 1s can do oral research, in terms of which they can find out the information that they require by means of asking their fellow learners, siblings, their teacher, and parents. In their groups, the learners become small academic researchers, as they engage with the problem, present their individual viewpoints, look for evidence, formulate their arguments, and present their group solution to the rest of the class. This can be done even in the most under-resourced schools. As learners gain reading skills, they can extend their seeking out of information by engaging with the relevant literature.

6.9.2.1 *Drilling in the Foundation Phase PBL environment*

Though drill and practice may be a popular practice in the teaching and learning of mathematics in the Foundation Phase, it can only be beneficial if it is implemented after learners have been taught meaningfully (Brownell 1935). Brownell challenged the drill-and-practice approach to mathematics. He pointed out that if one was to be successful in quantitative thinking, one needed a repertoire of meanings, not a myriad of automatic responses. He argued that drill does not develop meaning and that repetition does not lead to understanding. In fact, he argued that if mathematics becomes meaningful, it becomes so in spite of drill. Repetitive procedural work is only of value if accompanied by a process of dealing with numbers in a way that leads to a flexible understanding and use of the number concepts, number facts and procedures which are critical if relational understanding (discussed in chapter 2 section 2.3) is the objective of the learning process. Drill and practice should not be used to teach skill, but to increase efficiency. Therefore, the teaching and learning of mathematics in Foundation Phase should be less about drill and more about learning with understanding.

6.9.3 Major issues of concern relating to the implementation of PBL in the Foundation Phase

A major concern about applying PBL in the Foundation Phase is that the learners, at this level, cannot really know what might be important for them to learn. The teacher's role is, therefore, pivotal in addressing this concern. The teacher, as a facilitator, must be able to carefully assess and account for prior knowledge that the learners bring into the classroom, and they must be able to aid the learners toward obtaining the curriculum-desired learning outcomes.

Another concern is that the teachers adopting PBL might not be able to cover as much material as they could have done through the traditional knowledge transmission method. This matter appeared as an issue in this research. During the interviews, the teachers admitted (Section 6.7) that they sometimes resorted to showing and telling, when they felt that they were lagging behind in terms of the CAPS timeframe. PBL can be very challenging to implement, as it calls upon the teacher to plan and work hard. Hence, it became hard for some of the teachers in the research to let go of their control, and to become facilitators who were capable of creating a learning environment in which the learners constructed their own mathematical knowledge, rather than handing knowledge over through showing and telling them the mathematical procedures to follow. In reality, the Foundation Phase should be less about covering all the CAPS material, and more about the learners coming to understand the CAPS material.

6.10 CONCLUSION

This chapter has presented, analysed and interpreted the data in a number of different ways. Open and focused codes of the emerging themes were generated from the theoretical chapters 2, 3 and 4. The original data were assigned to the codes to facilitate their interpretation in a more conceptual way. It was then possible to evolve findings from the data. The main finding of the study emerged as a substantive body of knowledge that is grounded in the underlying theory of the teaching and learning of mathematics. The findings are, thus, theoretically grounded, and they provide answers to the research questions. The next chapter discusses the implications of the current study for future research and practice.

CHAPTER 7: REFLECTIONS AND RECOMMENDATIONS FOR IMPLEMENTING PROBLEM-BASED LEARNING IN THE FOUNDATION PHASE

“Students can learn new skills and concepts while they are working out solutions to problems.”

(Grouws & Cebulla, 2000, p. 15)

7.1 INTRODUCTION

The current study presented and incorporated the following theoretical perspectives:

- the three main theories of learning, behaviourism, and maturationism and constructivism (Section 2.6);
- MKT, incorporating significant ideas from Hill and Ball (2009) (Chapter 2, sections 2.7 to 2.9); and
- mixed research method design (Chapter 3), which was aimed at establishing sound practices and data collection methods.

The above-mentioned theoretical perspectives made it possible for the aims and objectives of the study to be realised, as is outlined below. The concept of PBL was explored in Chapter 3, incorporating significant ideas from Barrett (2005); Camp (2010); Goodnough and Hung (2008); Jonassen and Hung (2008); Norman and Schmidt (1992); Sahin (2010); Savery (2006); Schroeder and Lester (1989); Sockalingam and Schmidt (2011); and Uden (2006). The study of professional development in Chapter 4 guided the planning, and the professional development, of the teachers involved in regard to the implementation of PBL. The theoretical perspectives provided the researcher with the necessary theoretical framework for the analysis and discussion of five data sources, namely: the questionnaire; the workshop evaluations; the lesson observations; the 1st and 2nd interviews; and the theoretical framework for the interpretative and conceptual conclusions presented in Chapter 6.

This chapter commences with a discussion and interpretation of the findings, highlighting the relationships between the findings and the theory. The findings of the study are used to construct a synthesis of the study, and, ultimately, to respond to the research questions. Subsequently, the recommendations, both for future practice and for future research, are discussed, and then the limitations of the study are followed by the concluding remarks.

7.2 BUILDING A REFLECTION FRAMEWORK

The aim of the study was to unpack the teachers' pedagogical knowledge and beliefs about, and practices in, the teaching/learning of mathematics of Foundation Phase teachers, with specific focus on problem-solving, and its use as a vehicle for teaching and learning through the implementation of PBL. In support of this aim, the following research questions were posed:

- 1) What PCK do teachers have in general, and about problem-solving and its use as a vehicle for learning?
- 2) What beliefs do teachers have about problem-solving in general, and in relation to its use as a vehicle for learning?
- 3) What are the support needs of teachers in their use of problem-solving as a vehicle for learning?

The reflection framework shown in Figure 7.1 below reflects the chronological nature of the study, and the sources of data that were used during the course of the study in the quest to find answers to the above research questions.

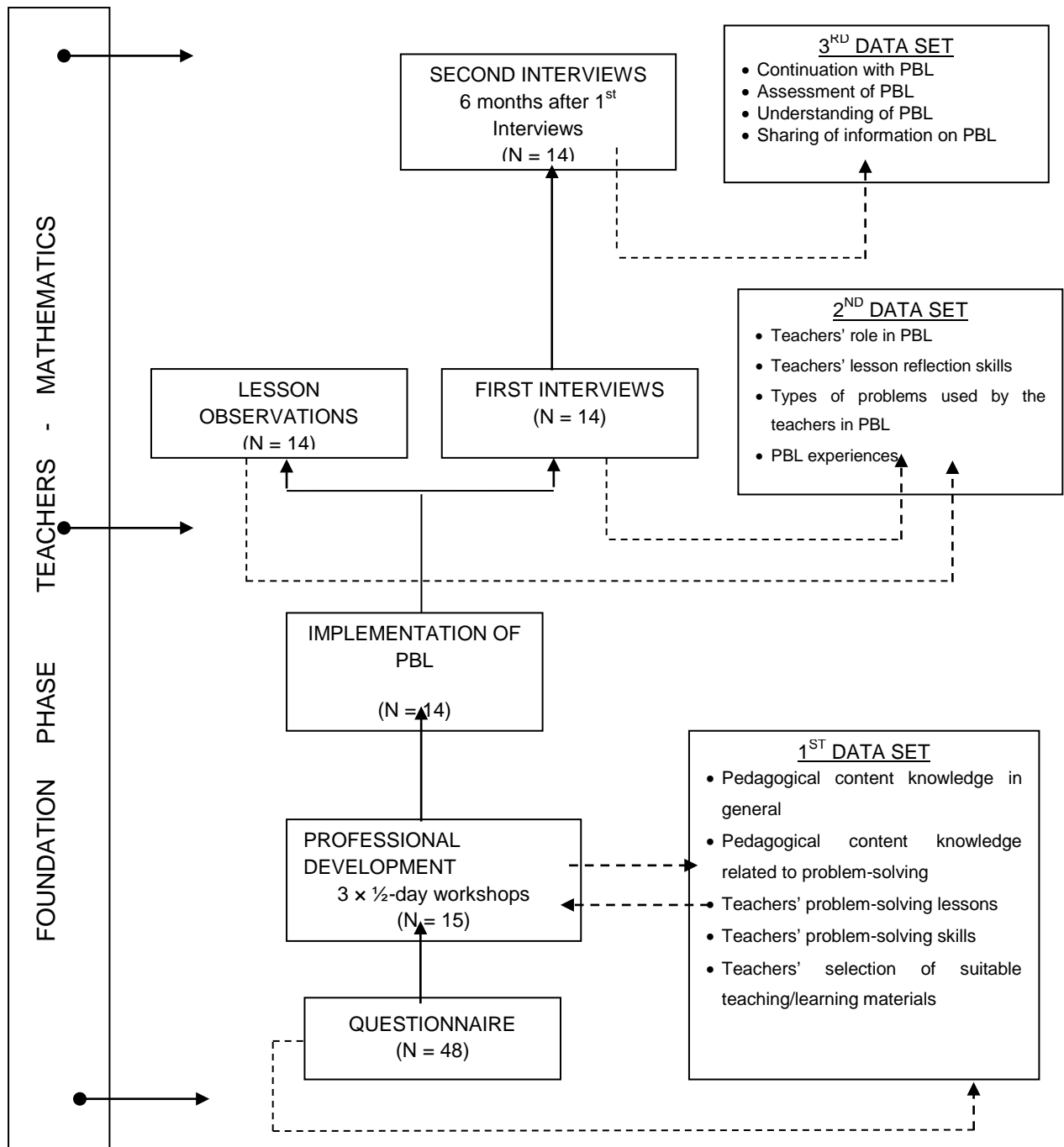


Figure 7.1: Reflection framework for the study

7.3 ADDRESSING RESEARCH QUESTIONS, AIMS AND OBJECTIVES

In seeking answers to the first research question, “*What PCK do teachers have in general and about problem-solving and its use as a vehicle for learning?*”, the first data set from the questionnaire that was distributed to the 48 Foundation Phase teachers in Project X was analysed to find out what knowledge the teachers had of how the learners learned mathematics, of how best to teach mathematics, and of which materials are suitable for teaching mathematics. The data revealed the following:

- The teacher knew that learners learn mathematics by means of memorisation and the mastery of basic facts and procedures (6.2.2.4). As a result of this understanding, learning was demonstrated more through expert memorisation, and through the performances of mathematical procedures, rather than through the ability to show understanding (see 6.2.3.4). This approach has been found to have far-reaching consequences, considering the observation that “[s]tudents who memorize facts or procedures without understanding often are not sure when and how to use what they know and such learning is often quite fragile.” (Bransford & Cocking, 1999, p. 20).
- The teachers knew that the best way of teaching mathematics was through demonstrating the mathematical procedures involved on the chalkboard, and then by letting the learners practise the procedures (6.2.2.3). The understanding that mathematics is best taught through the demonstration of the procedures on the chalkboard is manifested in the teachers’ view of their role as that of knowledge transmitters (see 6.2.2.4). This research finding contradicts one of the findings of the OECD’s 2009 Teaching and Learning International Survey (TALIS), which focused on teacher beliefs, attitudes and practices; on teacher appraisal and feedback; and on school leadership in the 23 participating countries as being the factors that lie behind the differences in learning outcomes. The TALIS finding, as it was related to teacher beliefs and practices, was that the majority of the teachers who were involved in the TALIS survey tended to be more inclined to see their role as supporting active learning, rather than as directly transmitting information.

Cobb et al. (1991) conducted a study in second grade classes in a yearlong project, in which instruction was generally compatible with a socio-constructivist theory of knowledge. At the beginning of the study, before professional development was given in PCL, the beliefs of the teachers concerned were generally not compatible with those of a socio-constructivist perspective. The researchers involved, however, found that the project teachers’ beliefs after one year were significantly more compatible with those of a constructivist perspective than were those of their non-project colleagues. The significance of findings in both studies in this regard confirms the widespread nonconstructivist perspectives that were held by many of the teachers, with the need for professional development being aimed at exposing teachers to the alternative approach of PBL. Both studies show that it is possible for teachers to change from their behaviourist perspective to a constructivist perspective in order to achieve the desired goal of teaching mathematics with understanding.

- The teachers' knowledge and choice of suitable materials for teaching mathematics revealed that the teachers tended to prefer materials with which they could show and tell, or explain and demonstrate (6.2.8.1).
- The participating teachers' approach to problem-solving was a manifestation of their knowledge and understanding of how best to teach mathematics. After the teachers had demonstrated the process of problem-solving, they then gave the learners similar problems to solve.

In answering the first research question, the following research aim and objective, which are outlined in the research mind map (Section 5.1, Figure 5.1), were met.

Aim 2: *To analyse the teachers' pedagogical knowledge related to problem-solving as a vehicle of effective learning*

The majority of the teachers knew that, in terms of problem-solving, effective/good teachers demonstrate the correct way to solve a problem, which is an indication of teaching for problem-solving, rather than of teaching via problem-solving (see Subsection 6.2.3.3).

Objective 2: *To determine the teachers' own problem-solving skills*

The study revealed that all the teachers found it difficult to attempt to solve the given problem (see Subsection 6.2.7.2). The majority of the teachers who attempted to solve the given problem completely misinterpreted the problem, or used only partially correct strategies, but with major faults.

7.4 BELIEFS TEACHERS HAVE ABOUT PROBLEM-SOLVING

In answer to the second research question, "*What beliefs do teachers have about problem-solving in general, and about its use as a vehicle for learning?*" the findings of the study established the following:

- The participating teachers viewed problem-solving as an application of known mathematics, and their focus was on applying the learners' acquired mathematical knowledge to solve routine or nonroutine problems (see subsections 6.2.2.4 and 6.8.3). One factor that influenced the adoption of a problem-solving approach by the teachers in this study was their knowledge and beliefs about what the teaching of mathematics is. The findings of the research described in this research assignment point out that, to the majority of teachers, memorising facts and formulas, and practising procedures, were regarded as being sufficient for being able to learn mathematics (see Subsection 6.7.2). This finding is in line with the work of Hiebert and Stigler (2009), who found out that, despite the massive efforts that had been

made to improve teaching in the United States, most lessons had remained devoted to practising mathematics procedures, rather than to developing conceptual understanding.

- The majority of the teachers in the current study expressed a belief that problem-solving is an add-on in the curriculum, and that the learners are only able to solve problems after they have acquired mastery of basic skills and procedures. This explains why the majority of teachers concerned called for a return to the basics of emphasising the mastery and memorisation of facts and skills. As has already been observed above, the adoption of such a position only serves to contribute towards an approach to the teaching of mathematics that produces learners that have been exposed to mathematics through transmission pedagogy, leaving the development of conceptual understanding, mathematical reasoning, and critical thinking to chance. In later years, such learners are then likely to perform poorly in tests of their procedural knowledge, as was observed by Grouws and Cebulla (2000, p. 15) who state that “students who develop conceptual understanding early perform best on procedural knowledge later”.
- The data showed that the teachers were quick to show and tell their learners how to solve a given problem at the earliest moment that the learners exhibited, or experienced, difficulties. This kind of teacher behaviour is rooted in the traditional belief of the nature of mathematics or in a personal philosophy of mathematics, which regards mathematics as a Platonist unified body of knowledge, in terms of which the teacher is the explainer, and learning entails the reception of knowledge (Ernest, 1989). This finding concurs with the research findings of a longitudinal study by Fennema, Carpenter, Franke, Levi, Jacobs & Empson (1996, p. 415), according to which teachers on a lower level of cognitively guided instruction “usually practiced direct instruction by demonstrating the steps in a procedure as clearly as they could and then having the children practice repeating the steps”.

The belief that children need to be explicitly taught how to do mathematics, and then be made to practise it, has had far-reaching negative effects on the teaching and learning of mathematics (OECD, 2009).

During the intervention, the participating teachers in Project X created rich learning opportunities by posing good problems to their learners. However, they were found to snatch away such opportunities, by means of showing and telling the learners how to solve the problem at the earliest signs of any difficulty being exhibited by the learners concerned. The teachers did not allow sufficient time for the learners to grapple with the mathematical ideas

in the problem, and to engage in a critical thinking process, thus depriving their learners of a vital opportunity for true learning to occur.

In doing the above, the teachers concerned were failing to transform mathematics teaching and learning in terms of the rigid transmission model to one in which the learning was learner-centred, self-regulatory, and inquiry-driven (Kennedy, 2006, p. 82). This minimised the learners' opportunities to develop mathematical reasoning and analytic skills, and it left the development of such essential modern-day mathematics skills to chance. The possible outcome of such teaching practices is that of a learner with knowledge of mathematical facts and procedures, but with very low levels of mathematical reasoning and analytic skills.

The findings are even more concerning against the background of the TIMSS assessments. The study in question was the largest international study of student achievement that had been conducted up to that point in time (1994–1995). The study was conducted at five grade levels (the third, fourth, seventh, and eighth grades, and the final year of secondary school) in more than 40 countries, including South Africa. The learners in South Africa performed badly in this assessment, as well as in the subsequent TIMSS assessment. The TIMSS' cognitive domain target percentages (which are presented in Table 7.1 below), form the framework for the levels of difficulty set for the TIMSS test items.

Cognitive Domains	Percentages	
	Fourth Grade	Eighth Grade
Knowing	40%	35%
Applying	40%	40%
Reasoning	20%	25%

Table 7.1: Target percentage of the TIMSS 2011 mathematics cognitive domains in the fourth and eighth grades (Mullis, Martin, Ruddock, O'Sullivan & Preuschoff, 2009, p. 29)

The '*knowing*' cognitive domain covers the facts, concepts, and procedures that learners need to know, while the '*applying*' domain focuses on the ability of learners to apply knowledge and conceptual understanding to solve problems, or to answer questions. The third domain, '*reasoning*', goes beyond the solution of routine problems to encompass unfamiliar situations, complex contexts, and multistep problems (Mullis et al., 2009, p. 40).

Given the exposition of the cognitive domain scenario of the TIMSS test (see Table 7.1 above), and the above-mentioned scenario of learners whose development of mathematical

reasoning and analytic skills is left to chance, one wonders how well learners from the study participants' classrooms would perform on the TIMSS test, which has a 60% bias towards the application and reasoning cognitive domains.

In answering the second research question, the following research aim and objective, which was outlined in the research mind map (see Section 5.1, Figure 5.1) were met.

Aim 1: *To determine what role beliefs about what mathematics is play in teachers' use of problem-solving as a vehicle for learning*

The study was able to establish that, because the teachers expressed a belief that mathematics was about the memorisation of facts and procedures, they did not view problem-solving as a tool for learning, but rather as a platform from which to apply the acquired facts and procedures (see Subsection 6.2.1.5).

Objective 1: *To determine the teachers' beliefs about problem-solving.*

The study was able to establish that the majority of teachers believed that effective teachers demonstrate the correct way of problem-solving, and that the learners should master basic facts before problem-solving (see Subsection 6.2.3.4).

Objective 3: *To identify how the teachers' beliefs about problem-solving manifest themselves in the teaching and learning of mathematics*

The majority of the teachers expressed a belief that problem-solving was about the acquisition of problem-solving strategies; hence, their problem-solving lessons were about presenting problems to solve, and about the practising of problem-solving strategies. Their demonstration of problem-solving routines manifested this belief (see Subsection 6.2.5.4).

7.5 TEACHER SUPPORT IN PROBLEM-BASED TEACHING METHODS

In answer to the third research question, "*What are the support needs of teachers in using problem-solving as a vehicle for learning?*" data from the interviews and the lesson observation revealed that the teachers needed support in the following areas:

- the construction of quality problems for PBL;
- the involvement of the weaker learners in PBL;
- the avoidance of showing and telling; and
- learner-centred lesson reflection (see subsections 6.8.4 and 6.8.5).

Figure 7.2 below provides details of the nature of support that the teachers would continue to require in order that they might eventually implement PBL effectively, as was revealed by the data from interviews and from the lesson observation.

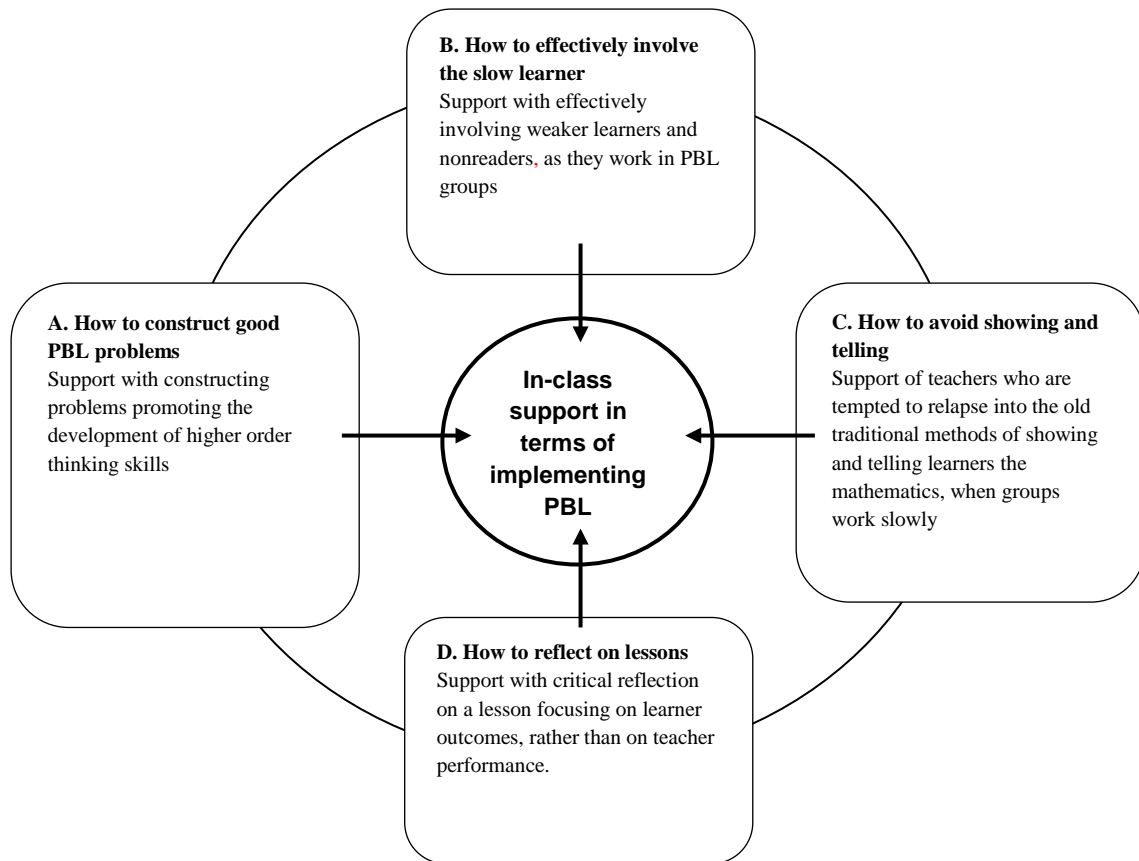


Figure 7.2: Teacher support required for the implementation of PBL

These four aspects will now be elaborated upon.

7.5.1 Constructing quality problems for PBL

The type of problems that a teacher uses in PBL is critical. One of the main teacher challenges observed in this research was their inability to design problems that were able to stimulate critical reasoning (see Subsection 6.6.7). Most of the problems that the teachers used were either problems that were designed for practising step-by-step procedures for calculations (algorithm problems) or story (word) problems. They tended to be a simple translation of equations into mathematical sentences requiring the learners ultimately to focus on the mathematical operation. Such problems neither stimulate, nor build, higher order thinking, critical problem-solving, or reasoning skills. This finding concurs with Franke and Kazemi's (2001) findings, in term of which teachers had difficulty in preparing and posing appropriate mathematics word problems that expanded the learners' current levels of mathematical thinking. Rauscher's (2012) analysis of the situation pertaining to textbooks concluded that none of the typical tasks analysed complied with all of the criteria for tasks considered suitable for use in terms of a problem-based approach. The teachers, therefore, required support in the construction of problems that would meet the critical requirement of stimulating critical reasoning in PBL (see subsections 3.9.2 to 3.9.4).

7.5.2 Involvement of weak learners in PBL

The teachers had admitted to meeting challenges regarding the meaningful involvement of weak learners in PBL. They had observed that the learners had poor reading skills, and, thus, that they took more time to understand, and to engage with, tasks. The teachers, therefore, needed support in terms of how best to involve such learners within their groups. Studies in the project on problem-centred learning (PCL) directed by Piet Human, Hanlie Murray and Alwyn Oliver at Stellenbosch University (see Subsection 3.3.3) found that “students can reason mathematically and construct important understanding without teacher intervention” (Hiebert et al., 1997a, p. 127). The focus of the current study was on how the teachers could, through PBL, facilitate their learners’ mathematical reasoning, and their construction of important understanding, without teacher intervention. The finding of the study was that, although the teachers would be able to facilitate such learning through PBL, they would initially require professional development regarding how to effectively involve the slow learners (see Subsection 6.8.5).

7.5.3 Lesson reflection

Formal or informal lesson reflection is a very important part of a teachers’ professional life. Through self-reflection, teachers assess themselves in terms of whether or not their lessons have been successful. The success of a lesson is ultimately measured by how it has impacted on the learners (i.e. in terms of learner outcomes), and by whether it has fostered optimal learning. The final measure of a lesson is not how well the teacher prepared for, or presented, the lesson. These two measures are but means to an end. Hence, true assessment focuses on learner outcomes. Positive answers to such questions as ‘Did the learners understand the lesson content?’ and ‘Have the lesson objectives (which are learner-centred) been achieved?’ can be regarded as proof of successful lesson preparation and presentation.

Teachers’ lesson reflections in the current study showed a preoccupation with instructional success, in terms of the technical preparation and presentation of the lesson and the logistics, such as the use of groups, involved. The focus in their reflections was mainly on their own teaching performance, and scant attention was paid to how their performance impacted on their learners’ understanding, and on the achievement of learning objectives. The teachers were also not critical of their impact and role in the crafting of learning situations in which the learners had access to quality mathematics learning experiences. Their lesson reflections were not supported by, or based on, evidence drawn from the learners’ performance during the lesson taught (see Section 6.5).

The making of such a finding implies that the quality controller (i.e. the teacher) concerned tends to base their quality control on such secondary measures as lesson preparation and presentation. The primary measure, namely the learning outcome, goes unchecked. However, this does not mean that teachers should not measure their pedagogical approaches, as a substantial amount of research validates that certain approaches promote learner understanding (Learning Mathematics for Teaching Project, 2011). Instead, one argues that the teacher should not evaluate the contributions that the pedagogical approach has made in terms of student learning *without* conducting a reflective appraisal of the learner outcomes achieved.

7.5.4 Showing and telling

In contrast to the desired approach of learner-centred learning and PCL (see Subsection 2.6.3 and Section 3.7), the research data revealed that, in many instances, the teachers were tempted to show and tell learners what to do (see subsections 6.2.5.4 and 6.4.2; Section 6.7; and Subsection 6.8.4). Careful reflection on what emerged from the study emphasises the need to support mathematics teachers in their paradigm shift towards becoming practitioners who can create and sustain learning experiences in which learners are exposed to, and engaged in, mathematical reasoning and higher order thinking, and in which they construct meaning from all such experiences. This finding is similar to that of Murray, Olivier and Human (1998), in their study on learning through problem-solving, in term of which they found that it was necessary to clarify very thoroughly the role of the teacher in employing a problem-centred approach.

Clearly, there is a need for the undertaking of a pre-professional development survey to determine the teachers' current pedagogical beliefs about, and practices in, the teaching and learning of mathematics. The strong presence of traditional pedagogy, or the dispensing of knowledge (see Subsection 6.2.2.4), guided the theoretical focus and emphasis in the professional development that was undertaken in the current study. After the professional development had covered the PBL theoretical framework and its practical implementation, the participants implemented a system of such learning in their classrooms. During the implementation phase, the teachers were provided with classroom support by the researcher in the areas cited in the above framework. After 6 months, an evaluation was undertaken of how PBL was being implemented, so as to determine the problems and successful experiences that had been encountered. After the researcher's observations were then discussed at the second professional development session, the teachers concerned returned to their classrooms to continue their implementation of PBL, with the cycle of support, monitoring and evaluation being repeated accordingly.

In answering the third research question, the following research aims and objectives, which were outlined in the research mind map above (Section 5.1, Figure 5.1) were met.

Aim 3: *To develop the teachers' ability to use problem-solving as a vehicle for learning*

A professional development intervention grounded in PBL was developed and implemented to develop the teachers' ability to teach via problem-solving (see Section 6.3).

Aim 4: *To facilitate the use of problem-solving as a vehicle for learning*

Classroom observations were conducted to gauge the teachers' ability to implement PBL, and individual in-class support was provided to facilitate its implementation of PBL (see Section 6.4). In general, after reflecting on the many interviews and lesson observations conducted, it was apparent that the teachers were attempting to use the PBL approach in their classrooms, as they best understood it in the light of how they had been trained.

Objective 4: *To determine teachers' ability to construct/select problems that are suitable for use in PBL*

The study established that the teachers were able to construct problems that met five of the six function characteristics (see subsections 3.9.3 and 3.9.4), namely problems that: 1) lead to the intended learning issues; 2) promote SDL; 3) stimulate collaboration; 4) promote teamwork; and 5) trigger interest. The majority of the teachers had, however, been unable to construct problems that stimulated critical reasoning (see Subsection 6.6.7).

Objective 5: *To determine the teachers' ability to use problem-solving as a vehicle for learning*

The study established that the majority of the teachers had been able to design lessons that provided the learners with opportunities to learn through problem-solving (see Subsection 6.4.5). Along with their successes with PBL, the teachers also told of their struggles with PBL. The most common challenge that many of the teachers described was the difficulties that were encountered in involving the weaker learners in the learning experience, and with the need to suppress the urge to show and tell struggling learners.

7.6 CONTRIBUTION TO THEORY AND PRACTICE

Research that is undertaken at the level of this study is conducted with a view to addressing a gap in the knowledge about a topic, and it makes an explicit original conceptual contribution to fill the gap in question (Creswell, 2003; Trafford & Leshem, 2008). In the field of education, such a knowledge gap can take the form of the need to identify areas of weakness, the nature of such weaknesses, or faults, and the extent to which the faults affect educational systems. The knowledge gap concerned can also relate to the need to explore possible solutions that are aimed at improving instruction and education in general (Ary, Jacobs, Sorensen & Walker, 2013).

This study has made contributions relating to mathematics teaching and learning. Discussion of the study's contribution to knowledge will focus on the pedagogical knowledge, beliefs and practices of the participating Foundation Phase teachers.

7.6.1 Knowledge of teachers' pedagogical knowledge, beliefs and practices that shape the effective teaching of mathematics

The challenges that poorly performing schools encounter in terms of mathematics teaching and learning continue to impede the development of the South African education system. Addressing them will require the creation of knowledge-rich, evidence-based intervention systems. The findings of this study present rich knowledge of the teachers' pedagogical knowledge, beliefs and practices that shape the effective teaching of mathematics. Those who accept the responsibility of bringing about the desired change in learner performance in Foundation Phase mathematics can draw upon the findings presented in this study to design future teacher professional development.

The data obtained in this study reveal that the majority of teachers in the study had behaviourist pedagogical beliefs, knowledge and practices about mathematics as a Platonist unified body of knowledge, in terms of which the teacher is the explainer, and learning is the reception of knowledge. Such a conclusion is important when one considers that the desired view is a constructivist belief that mathematics is learned through problem-solving, with the teacher serving as facilitator, and with learning consisting of the active construction of understanding. The current study, therefore, contributes towards the understanding of the pedagogical beliefs, knowledge and practices of this group of Foundation Phase teachers in the poorly performing schools involved in Project X.

7.6.2 Professional development framework for introducing PBL in Foundation Phase mathematics

Little evidence exists of the implementation of PBL in the Foundation Phase (Hiebert et al., 1996). Studies on PBL in South Africa have mainly focused on teachers, or on high school, or university, students (Malan, Ndlovu & Engelbrecht, 2014; Rauscher, 2012; Van Loggerenberg-Hattingh, 2003). The framework for introducing PBL in the Foundation Phase (see Figure 7.2 below) was generated from the observations made in this study, which form the conceptual contributions of the study.

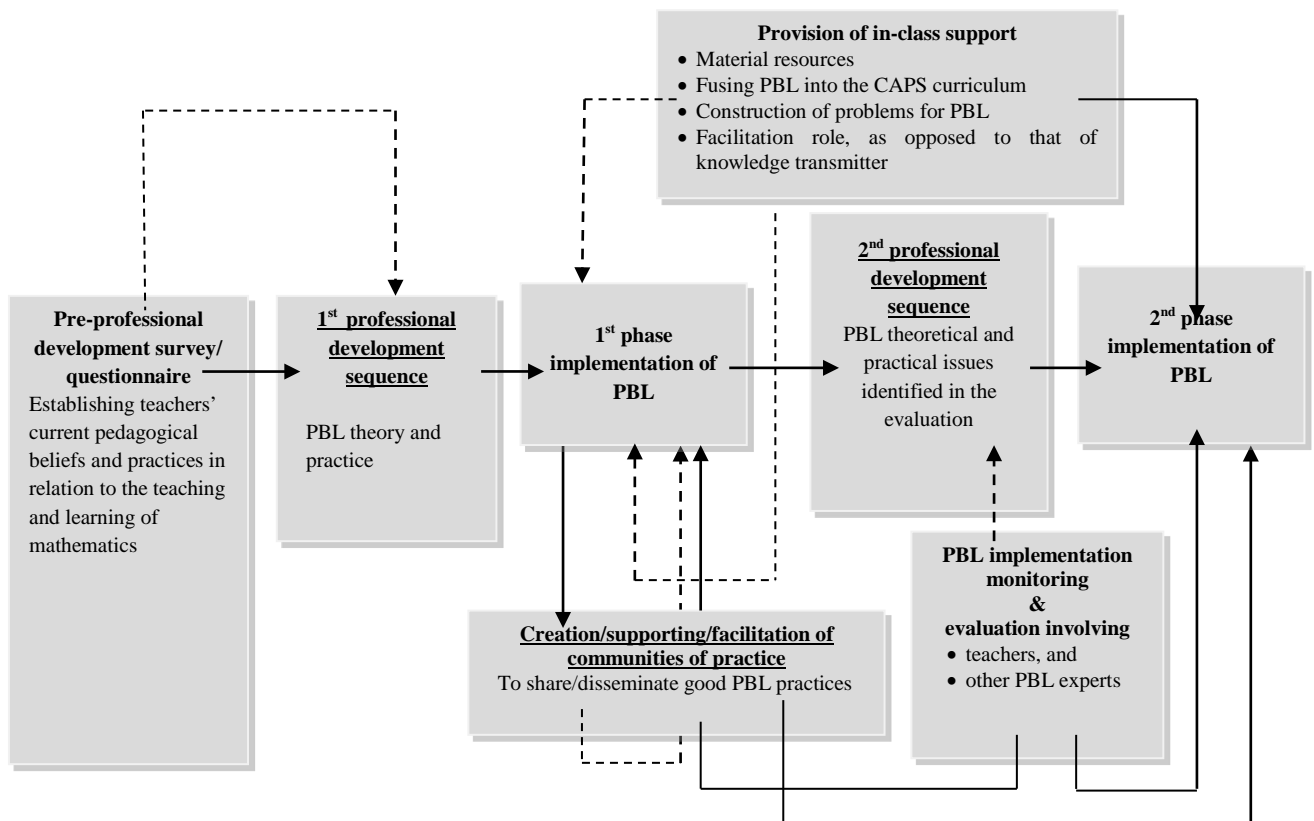


Figure 7.3: The framework for introducing PBL in the Foundation Phase

7.6.3 The instruments developed and used in the study

The study developed three instruments, namely the preprofessional development questionnaire, the lesson observation, and the interview schedules. These instruments might prove to be useful if they are included in other PBL teacher development programmes.

7.6.4 Barriers to the intervention efforts that are aimed at improving the teaching and learning of mathematics

The research revealed the traditional pedagogical knowledge, beliefs and practices that are present in teachers are, in fact, barriers to the changes in classroom practices that are demanded by intervention programmes that are designed to improve the teaching and learning of mathematics. The traditional pedagogical knowledge, beliefs and practices embraced by the teachers made it difficult for them to readily make the desired paradigm shift to the constructivist approaches that are needed to promote learners' conceptual understanding, and to foster their ability to reason and communicate mathematically. Such a shift entails transforming the learning of mathematics from the traditional passive memorisation of facts, skills and procedures, to the desired active construction of own knowledge.

7.7 RECOMMENDATIONS

By means of the current research study, the researcher sought to contribute to the growing body of knowledge in the field of mathematics education, specifically in terms of the use of problem-solving as a tool for the teaching and learning of mathematics in the Foundation Phase. As South Africa continues with the battle to improve the teaching and learning of mathematics, it is hoped that the findings of this study will provide a modest contribution to the efforts of transforming the Foundation Phase teacher from being a transmitter of knowledge to being an engineer of learning environments in which the learners can actively grapple with mathematical problems, and construct their own understandings.

The study expounds on findings that evidence some of the barriers to the intervention efforts that are exerted to improving the teaching and learning of mathematics. It contains elements that are crucial to transforming the teaching and learning of mathematics, with it being based on this body of evidence. The following recommendations are made in this regard:

- A strong campaign is needed for a paradigm shift to be made towards the creation of learning environments in which learners can actively grapple with mathematical problems, and in which they can construct their own understandings. This is in opposition to the dispensing of knowledge that was observed in the case of the majority of teachers in this study.
- Instruction in PBL needs to be part of the content of the professional development of teachers, in order to improve the current low learner performance, as the current pedagogical approaches that were observed to have been adopted in the case of

the participants in this study left the development of learners' mathematical reasoning and critical thinking to chance.

- The teachers involved needed to be assisted by means of the provision of a wide range of material supporting the giving of examples of problems, and of rich tasks, for each of the learning areas concerned.

7.8 LIMITATIONS OF THE STUDY

The use of a purposive sample of 48 Foundation Phase teachers in the intervention project weighs heavily against the generalisability of the findings of this research to schools outside the geographical area of the research (see Section 5.3). The study focused on the implementation of PBL by 48 Foundation Phase teachers from five schools who were involved in an intervention Project X. As the study was set in the context of the schools participating in an intervention project, they cannot be regarded as being representative of other schools outside the project. However, the sample used was made up of all the Foundation Phase teachers from five poorly performing schools within a wide geographical area, hence schools with a similar performance background might find the study findings informative.

Although there is evidence in the literature that PBL improves the retention and recall of important units of information, and that it helps learners to retain knowledge longer than traditional instruction tends to do (see Section 3.8), similar evidence was not collected in the study. This, inevitably, limits the argument to be made for the effectiveness of the participating teachers' implementation of PBL.

The inclusion of the mind map on problem-solving in the questionnaire (in Subsection 1) did not allow for follow-up questions or interviews to seek clarification. As a result, a significant amount of the data collected could not be conclusively categorised into any of the three categories of this section's data analysis, because it would have been too brief to be comprehensible (see subsections 6.2.1.3 and 6.2.1.3).

The research design did not allow for teacher discussions of their own reactions to PBL. However, doing so would have allowed them to interrogate their attitudes and beliefs about teaching using PBL, and about the learning of mathematics by primary school children in general. This factor is significant, particularly in terms of an environment where the opportunities for teachers to share and discuss practice within and across schools are rare, and where the textbooks used fail to refer to PBL.

In this study, the researcher strived to create a reliable and valid questionnaire so as to enhance the accuracy of its assessment and evaluation. Reliability, which is concerned with

the ability of an instrument to measure consistently, is one fundamental element that is present in the evaluation of a measurement instrument. Cronbach's alpha is the most widely used objective measure of reliability. A major limitation of the study in this regard was the reported low Cronbach's alpha in three out of the nine subsections of the measurement instrument (i.e. the questionnaire). The questionnaire had, however, been validated as an instrument that measured what it was meant to measure by the experts in mathematics education who had been consulted in this regard.

The fact that the research was carried out in government schools that are often plagued by bureaucratic delays in the granting of permission was not as much of a limitation as had, at first, been anticipated. Permission to carry out the research in such schools was promptly granted by the relevant office of the WCED within one week of application. (See Appendix 3.)

7.9 RECOMMENDATIONS FOR FUTURE RESEARCH

The need for research into the teaching and learning of mathematics is ongoing. One observation that was made in this study was how teachers concerned held stereotyped beliefs about what mathematics is, and how it should be taught, which made it difficult for the majority of the participants in the study to implement PBL. This observation points to the need for research in the following areas:

- How best to address the teachers' stereotyped beliefs about what mathematics is, and how it should be taught, in order to enable the implementation of changes in the teaching of mathematics, requires research.
- The design of learning environments conducive to the teaching of mathematics through PBL in the Foundation Phase needs to be investigated. Such research should help teachers to understand how, when and why an innovative teaching approach like PBL fosters the teaching of mathematics for understanding.
- Joint projects should be conducted between researchers and educators in regard to such issues as incorporating PBL within the CAPS curriculum, and the accommodating of slow learners and nonreaders (i.e. learners with inadequate reading skills) in PBL.
- The development of supplementary learning support materials for implementing PBL in Foundation Phase should be looked into
- As the construction and supply of suitable problems plays a crucial role in PBL, research into rich problems for use in the Foundation Phase could contribute to making suitable problems available for PBL in the Foundation Phase.

7.10 CONCLUDING COMMENTS

The focus of this study was on the unpacking of Foundation Phase teachers' pedagogical knowledge, beliefs and practices about the teaching and learning of mathematics, with a specific focus on problem-solving, and on its use as a vehicle for teaching and learning through the implementation of PBL. The findings of this study have important implications for the improvement of the teaching and learning of mathematics in the Foundation Phase, and for the use of PBL within this phase. The research findings provide crucial information for teacher educators, for the Education Department concerned, and for those who are involved with teacher professional development, and who desire to change the current traditional approach to the teaching of mathematics to an approach that is entrenched in constructivism. As has previously been discussed, constructivism involves teaching that empowers and promotes critical thinking and reasoning in learners during their foundation years in learning mathematics.

Those who are involved with teacher professional development, and who tend to believe that teaching is influenced by one's pedagogical knowledge, practices and beliefs, should begin improving the teaching and learning process by becoming knowledgeable about the pedagogical knowledge, beliefs and practices that are affecting teachers' effectiveness. Becoming more aware of, and knowledgeable about, how teacher beliefs influence their pedagogical practices should lead to an in-depth understanding of existing instructional practices, and of the factors that can potentially re-engineer them, in order that they might address instructional reform (Fleisch & Schoer, 2014).

Modifying teacher beliefs, as well as teaching and learning practices, so as to foster the teaching of mathematics for understanding is one of the major challenges affecting mathematics education in South Africa. Such modification is, however, what is called for if teaching approaches are to be modified and focused on teaching for understanding. To improve the teaching of mathematics in the Foundation Phase in South Africa, teacher beliefs about what mathematics is, and about how it is learned, will need to be modified. In future, teachers will, furthermore, have to be exposed to professional development in relation to such alternative approaches as PBL, and to examples of the successful implementation of PBL, so as to be able to bring about change in their pedagogical practices.

The literature review showed how strongly teachers' beliefs and practices affect teacher change, as well as teaching and learning itself (Ernest, 1989; Grootenboer, 2008; Li, 2005; Pohan & Aguilar, 2001; Raymond, 1997; Rimm-Kaufman & Sawyer, 2004; Wilkins & Ma, 2003). While the current study did not measure teacher effectiveness, its focus was on

important pedagogical beliefs and practices that shape effective learning. It was shown how these were associated with some of the conditions that are prerequisites for the effective teaching and learning of mathematics. The study showed that, while it was possible, although challenging, to implement PBL in the Foundation Phase, the teachers needed ongoing support to change their set beliefs about the teaching and learning of mathematics, and so that they might be able to construct sound PBL problems. Reflecting on the study, one notes that the teachers who were involved in this study, by means of personally experiencing PBL, and by means of observing the changed mathematical performance of their learners, became convinced that PBL provided an effective alternative to their traditional teaching methods.

The current researcher believes that the focus should not be on whether or not to introduce PBL in the Foundation Phase, but rather how to provide support, and what support to provide to teachers so that they can come to successfully integrate PBL into the existing CAPS curriculum.

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APPENDICES

APPENDIX 1: TEACHER QUESTIONNAIRE

TEACHER PROFILING

FOR PROFESSIONAL DEVELOPMENT

FOUNDATION PHASE

NAME: _____

SCHOOL: _____

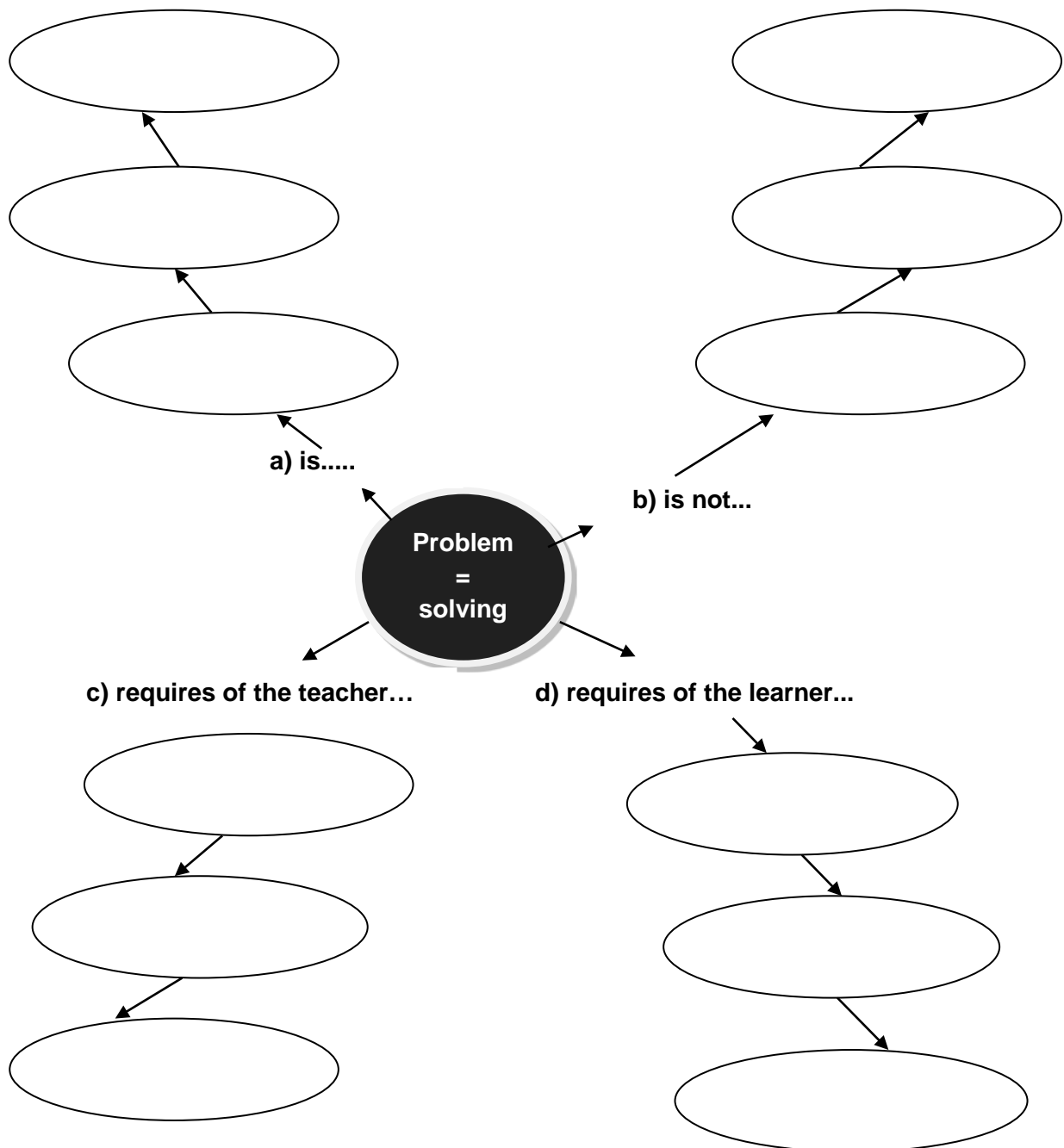
All responses to this document shall remain confidential.

Please complete all the questions.

Q1

Complete this mind map about problem-solving by writing out your own views in each bubble, about what you think...

- a) Problem-solving is.
- b) Problem-solving is not.
- c) Problem-solving requires of the teacher.
- d) Problem-solving requires of the learner.



This year in your mathematics lessons, how often did you do the following? Mark X for each item.

		Never	Less than once a month	1–3 times per month	1–3 times per week	3–4 times per week	Every day
Q2	Ask the learners to do pen-and-paper calculations, and to practise.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q3	Demonstrate to the class a procedure on the chalkboard, and then let the learners practise.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q4	Teach the learners mathematics by focusing on rules and procedures.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

How do you rate yourself in the following areas? Mark X for each item.

		Very poor	Poor	Fair	Very good
Q5	Knowledge and the ability to use problem-solving as a vehicle for learning mathematics	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q6	Knowledge of the teaching of mathematics at Foundation Phase level.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q7	The ability to understand learners and their learning needs	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q8	Motivation to teach mathematics at Foundation Phase level	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

To what extent do you agree or disagree with the following statements?

Mark X to show whether you strongly agree, agree, disagree or strongly disagree.

		Strongly agree	Agree	Disagree	Strongly disagree
Q9	Problem-solving is about word sums.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q10	Teaching should be built up around problems with clear, correct answers, and around ideas that most students can quickly grasp.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q11	A quiet classroom is generally required for effective learning to take place.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>

Q12	Learners should learn and master basic number facts before they do problem-solving.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q13	How much students learn depends on how much background knowledge they have: that is why the teaching of basic facts is necessary.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q14	Effective/Good teachers demonstrate the correct way of solving a problem.	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>	<input type="checkbox"/>
Q15	To what extent do you, as a teacher, believe that “Many learners are just not able to learn mathematics.” Mark X to show the extent of your belief.				
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	
Q16	To what extent do you, as a teacher, believe that “Teachers must return to the basics of emphasising the mastery and memorisation of facts and skills.” Mark X to show the extent of your belief.				
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	
Q17	To what extent do you, as a teacher, believe that “Learners learn mathematics best when they sit, listening carefully, and watching you, the teacher, demonstrate and explain the mathematics, and then practise what they have seen and heard.” Mark X to show the extent of your belief.				
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	
Strongly do not believe		Do not believe		Believe	
<input type="checkbox"/>		<input type="checkbox"/>		<input type="checkbox"/>	

Scenario Q18: Problem-solving with Mr Adams

Mr Adams is a Grade 3 teacher. Here he explains to his colleague how he handles problem-solving with his class.

Q18

Study the above scenario. How often do you handle problem-solving in the same way in which Mr Adams handles problem-solving with his class?

Always

Most of the time

Sometimes

Rarely

Never

☐
☐
☐
☐
☐


Problem-solving with Mr Adams

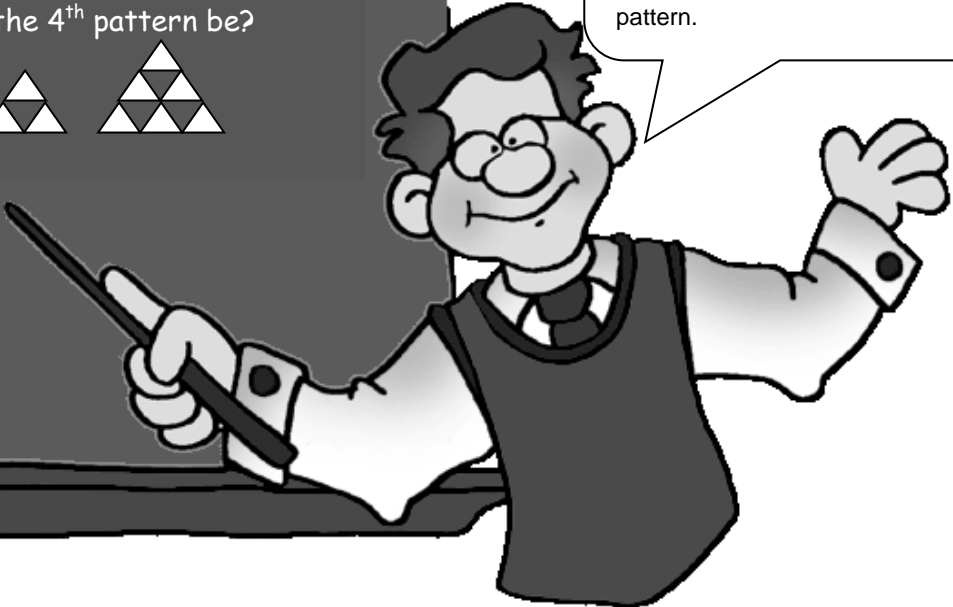
Mr Adams' Grade 3 class is working on the problem on the chalkboard. He moves around, visiting the different groups in turn, and, on finding that most of the groups are struggling with the problem, he then says the following.

Monday 18 June 2012

Problem for today

What will the 4th pattern be?





This is how to solve the problem. Look at how many triangles are in the 1st, 2nd and 3rd patterns to find the pattern. Use this pattern to work out how many triangles will be in the 4th pattern.

Q19

How often do you, like Mr. Adams, have to show or tell your learners how to solve a problem when they find it difficult to do so?

Always

Most of the time

Sometimes

Rarely

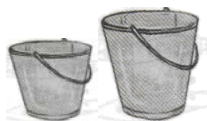
Never

☐
☐
☐
☐
☐

Solve the following problem in the space provided in any way that you see fit, and show how you reached the answer.

Q20

You are standing on the bank of a river with two pails.





One pail holds exactly 3 litres of water, and the other holds exactly 5 litres. The pails are not marked for measurement in any other way. How can you carry exactly 4 litres of water away from the river?


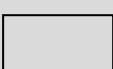
Q21

Ms Miller wants her students to be able to recognise and name triangles, as well as to discuss whether their sides are straight or curved. To help them, she wants to give them some shapes that they can use to test their ability. She goes to the store to look for a visual aid to help with the lesson. Which of the following aids is most likely to help the students improve their ability to recognise and name triangles? (Circle ONE answer.)

Shapes

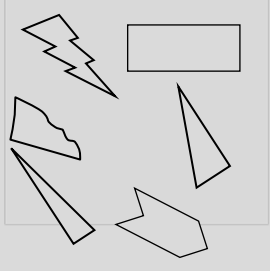



Square triangle

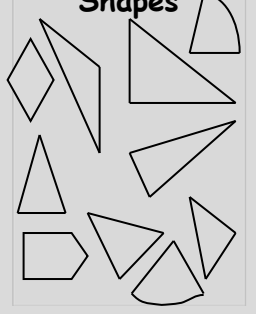



Circle Rectangle


Shapes



Shapes



Shapes



Look at these shapes.
They are called
triangles. A triangle has
3 sides and 3 corners.

Q22

Please explain why you chose this activity.

Q23

Have you seen the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document at your school?

YES

NO

Q24

Have you read the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document for mathematics?

Q25

Have you implemented the requirements of the Government Gazette no. 30880: Foundations for Learning Campaign 2008–2011 document for mathematics in your planning?

Q26	Have you seen the CAPS document at your school?		
Q27	Are you familiar with the requirements of these documents with regard to problem-solving?		
Q28	Have you participated in a professional development session that focused on using problem-solving as a vehicle for teaching mathematics at the Foundation Phase?		
		School	WCED
		University	Other (specify)
Q29	If YES . Who was the organiser of the session?		
Q30 How long ago did this professional development take place?			
		1–6 months ago	½–1 year ago
			+1 year ago

Biographical information

	Q31 What is your gender? <input type="checkbox"/> Male <input type="checkbox"/> Female	Q32 Please indicate your age below. _____ years
	Q33 Please indicate your professional qualification. <input type="checkbox"/> Junior Primary Teachers' Diploma <input type="checkbox"/> B Ed Foundation Phase <input type="checkbox"/> Other (specify)_____	Q34 Please indicate how many years of teaching you have altogether, including this year. _____ years
	Q35 Indicate how many years of teaching you have had in the Foundation Phase, including this year. _____ years	Q36 Grade currently teaching <input type="checkbox"/> 1 <input type="checkbox"/> 2 <input type="checkbox"/> 3

APPENDIX 2: INFORMATION SHEET AND CONSENT FORM

STELLENBOSCH UNIVERSITY

CONSENT TO PARTICIPATE IN RESEARCH

Unpacking the teachers' pedagogical content knowledge and skills to develop the learners' problem-solving skills

You are asked to participate in a research study conducted by Cosmas Toga Tambara (B Phil, M Ed) from the Department of Curriculum at Stellenbosch University. You were selected as a possible participant in this study because you are a mathematics teacher in the Foundation Phase and are involved in the Curriculum, Literacy, Mathematics and Management Buzz (CLIMMB) project.

1. PURPOSE OF THE STUDY

The purpose of this research is primarily to examine the knowledge and skills teachers have to facilitate the development of the learners' problem-solving skills.

2. PROCEDURES

If you volunteer to participate in this study, we would ask you to do the following things:

- Complete the teacher profiling instrument, in your spare time.
- Hand over the completed questionnaire when the researcher comes to collect it in a week's time.
- Attend the professional development workshops.
- Allow the researcher to interview you.
- Allow the researcher to observe some of your mathematics lessons.

3. POTENTIAL RISKS AND DISCOMFORTS

Participating in this research will not put either your physical self, nor your professional person at any risk, or in any discomfort, pain, or possible complications. It will also not put you at any risk of persecution, stigmatisation or negative labelling.

4. POTENTIAL BENEFITS TO SUBJECTS AND/OR TO SOCIETY

Participation in this research will contribute towards your professional growth as a mathematics teacher in the Foundation Phase. It will entail that you attend a needs-based professional development programme, whose aim is to assist, motivate and equip the Foundation Phase teachers with the necessary subject knowledge, didactical skills and sound classroom management skills. Such skills will enable you to effectively improve the numeracy skills and the knowledge of your learners, and to create a stable and organised environment for learning to take place.

5. PAYMENT FOR PARTICIPATION

There will be no financial, or other forms of benefit, other than those that are stated in Section 4 above.

6. CONFIDENTIALITY

Any information that is obtained in connection with this study and that can be identified with you will remain confidential, and it will be disclosed only with your permission, or as required by law.

Confidentiality will be maintained by means of strictly adhering to the University's research ethics guidelines, and to general research ethics. Confidentiality and anonymity shall be upheld throughout the research, and in any publication thereafter.

7. PARTICIPATION AND WITHDRAWAL

You can choose whether or not to take part in this study. If you volunteer to be included in this study, you may withdraw from it at any time, without consequences of any kind. You may also refuse to answer any questions that you do not want to answer, and still remain in the study. The investigator may withdraw you from this research if circumstances arise that warrant doing so.

8. IDENTIFICATION OF INVESTIGATORS

If you have any questions or concerns about the research, please feel free to contact any of the following:

Cosmas T. Tambara Stellenbosch University Tel: 021 808 3483

Dr H. Wessels (Promoter) Stellenbosch University Office: +27 21 808 2286
--

Dr J.H. Smit (Copromoter) Stellenbosch University Tel: 021 808 3483

9. RIGHTS OF RESEARCH SUBJECTS

You may withdraw your consent at any time, and discontinue participation without penalty. You are not waiving any legal claims, rights or remedies because of your participation in this research study. If you have questions regarding your rights as a research subject, contact Stellenbosch University's Unit for Research Development.

SIGNATURE OF RESEARCH SUBJECT OR LEGAL REPRESENTATIVE

The information above was described to _____ by _____ in [Afrikaans/English/Xhosa/other] and (I am / the subject is / the participant is) in command of this language, or it was satisfactorily translated to [me/him/her]. [I/ The participant / The subject] was given the opportunity to ask questions and these questions were answered to [my/his/her] satisfaction.

[I hereby consent voluntarily to participate in this study / I hereby consent that the subject/participant may participate in this study.] I have been given a copy of this form.

Name of Subject/Participant

Name of Legal Representative (if applicable)

Signature of Subject/Participant

or Legal Representative

Date

SIGNATURE OF INVESTIGATOR

I declare that I explained the information given in this document to _____ [name of the subject/participant] and/or [his/her] representative _____ [name of the representative]. [He/She] was encouraged and given ample time to ask me any questions. This conversation was conducted in [Afrikaans/*English/*Xhosa/*Other] and [no translator was used / this conversation was translated into _____ by _____].

Signature of Investigator

Date

APPENDIX 3: LETTER OF PERMISSION TO CARRY OUT RESEARCH IN SCHOOLS



WESTERN CAPE
Education Department

Provincial Government of the Western Cape

RESEARCH

research@westerncape.gov.za

Tel: +27 021 476 9272

Fax: 0865902282

Private Bag x9114, Cape Town, 8000

wced.wcape.gov.za

REFERENCE: 20110919-0085

ENQUIRIES: Dr A T Wyngaard

Mr Cosmas Tambara
Institute for Mathematics and Science Teaching
Curriculum Studies
Stellenbosch University

Dear Mr Cosmas Tambara

RESEARCH PROPOSAL: UNPACKING TEACHERS' PEDAGOGICAL CONTENT KNOWLEDGE AND SKILLS TO DEVELOP LEARNERS' PROBLEM SOLVING SKILLS IN MATHEMATICS

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from 11 January 2012 till 30 September 2012
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A.T Wyngaard at the contact numbers above quoting the reference number.
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

The Director: Research Services
Western Cape Education Department
Private Bag X9114
CAPE TOWN
8000

We wish you success in your research.

Kind regards,

Signed: Audrey T Wyngaard
for: HEAD: EDUCATION
DATE: 20 September 2011

APPENDIX 4: PBL WORKSHOP PROGRAMME

Problematising mathematics: Introducing problem-based learning in the Foundation Phase:

3 ½-day workshops

Day 1

TOPIC
Introduction
Motivation for PBL
What is problem-based learning?
PBL: The process
The steps of PBL

DAY 2

Cooperative learning: What the research shows
Suggestions for using groups
Group roles to be rotated
Using groups in larger classes, with inexperienced learners
The role of the teacher in PBL

DAY 3

Important considerations in writing problems
Drafting the problem
Characteristics of a good problem in PBL
The three-part lesson format
Conclusion and evaluation

1. How has the workshop been helpful to you?
2. Will you be able to apply PBL in your class?
3. What further assistance will you require to be able to apply PBL?
4. Do you wish to make any other comments?

APPENDIX 6: EXAMPLE OF WORKSHOP EVALUATION RESPONSE

Feedback on the Workshop.

1. The workshop has been helpful to me because I learnt a new approach to problem solving that I have never been exposed to before. A brilliant approach!
2. I will begin to implement it to my class as this approach stimulates the high thinking level to the learners. (Creative thinking)
3. As I will start to implement I might get stuck along the way then I will call for help.
4. Thank you Cosmas Jambasa for all the sacrifice, the passion and love for teaching, you are indeed a motivation. God bless you!

APPENDIX 7: LESSON OBSERVATION SCHEDULE**LESSON OBSERVATION SHEET****MATHEMATICS LESSON OBSERVATION SHEET**

School: _____ Educator: _____ Date: _____

The role of the teacher

Mark (✓) if teacher behaved as described, or (X) if the teacher did not behave as described.

Students treated as listeners ☐Pursued in-depth from among the ideas that students brought up during a discussion ☐Decided when and how to attach mathematical notation and language to students' ideas ☐Posed questions and tasks that elicited, engaged, and challenged each student's thinking ☐Teacher demonstrated and showed the learners the correct way to solve the problem. ☐**Lesson Structure**

Mark (✓) if the practice is not observed and (X) if the practice is observed.

Preparation**Students' work****Class discussion**The teacher made certain that the problem was understood. ☐The teacher let go, and avoided stepping in front of a struggle. ☐The teacher encouraged a community of learners. ☐The teacher activated useful prior knowledge. ☐The teacher listened carefully. ☐The teacher listened and accepted learners' solutions without evaluation. ☐The teacher established clear expectations. ☐The teacher provided appropriate hints. ☐The teacher summarised main ideas and identified future problems. ☐**Nature of the problem.**

Mark (✓) if the problem is as described and (X) if the problem is not as described.

The problem cast students in a realistic situation. ☐The problem presented was a complex situation that had no single, clear-cut solution." ☐The problem presented was authentic, and the students stayed motivated and worked together to solve the problem. ☐The problem presented encouraged the learners to develop higher level thinking and group collaboration skills. ☐**REFLECTION EDUCATOR:****REFLECTION FACILITATOR:****SIGNATURE** _____**SIGNATURE** _____

APPENDIX 8: EXAMPLE OF COMPLETED LESSON OBSERVATION SCHEDULE

LESSON OBSERVATION SHEET				
MATHEMATICS LESSON OBSERVATION SHEET				
School: <u> </u>		Educator: <u> </u>		Date: <u>17 Oct (Gr 2)</u>
The role of the teacher				
Mark (✓) if teacher behaved as described or (X) if the teacher did not behave as described.				
Students were treated as listeners.				<input checked="" type="checkbox"/>
Pursued in depth from among the ideas that students brought up during a discussion.				<input checked="" type="checkbox"/>
Decided when and how to attach mathematical notation and language to students' ideas.				<input checked="" type="checkbox"/>
Posed questions and tasks that elicit, engage, and challenge each student's thinking.				<input checked="" type="checkbox"/>
Teacher demonstrated and showed learners the correct way to solve the problem.				<input checked="" type="checkbox"/>
Lesson Structure				
Mark (✓) if the practice is not observed and (X) if the practice is observed.				
Getting Ready		Students' Work		Class Discussion
The teacher made certain the problem was understood.	<input checked="" type="checkbox"/>	The teacher let go, and avoided stepping in front of struggle.	<input checked="" type="checkbox"/>	The teacher encouraged a community of learners.
The teacher activated useful prior knowledge.	<input checked="" type="checkbox"/>	The teacher listened carefully.	<input checked="" type="checkbox"/>	The teacher listened and accepted learners' solutions without evaluation.
The teacher established clear expectations.	<input checked="" type="checkbox"/>	The teacher provided appropriate hints.	<input checked="" type="checkbox"/>	The teacher summarized main ideas and identified future problems.
Nature of the problem				
Mark (✓) if the problem is as described and (X) if the problem is not as described.				
AC1	The problem cast students in a realistic situation.			<input checked="" type="checkbox"/>
AC2	The problem presented was a complex situation that has no single, clear-cut solution."			<input checked="" type="checkbox"/>
AC3	The problem presented was authentic students stayed motivated and worked together to solve the problem.			<input checked="" type="checkbox"/>
AC4	The problem presented encouraged the learners to develop higher level thinking and group collaboration skills.			<input checked="" type="checkbox"/>
REFLECTION EDUCATOR:		REFLECTION FACILITATOR:		
Learners tried their level best but as a teacher I still have to create more challenging problems to enhance their thinking.		Learners were actively involved in their group and the teacher moved from group to group providing and facilitating discuss well prepared lesson!		
SIGNATURE <u> </u>		SIGNATURE <u> </u>		

APPENDIX 9: EXAMPLE OF A LESSON OBSERVATION SUMMARY**Teacher 03****Lesson:** Money**Grade:** 2**1.The role of the teacher**

The teacher gave each group a written copy of the problem to read and solve. She explained how they had to work on the problem as a group. The teacher, as a facilitator, moved from one group to another (See figures A9.1 and A9.2).

The teacher did not demonstrate the correct way to solve the problem, but, rather, she provided assistance in helping the learners understand the mathematics surrounding the problem. She provided the groups with real notes and coins to identify and to use in solving the problem (Figure A9.3).

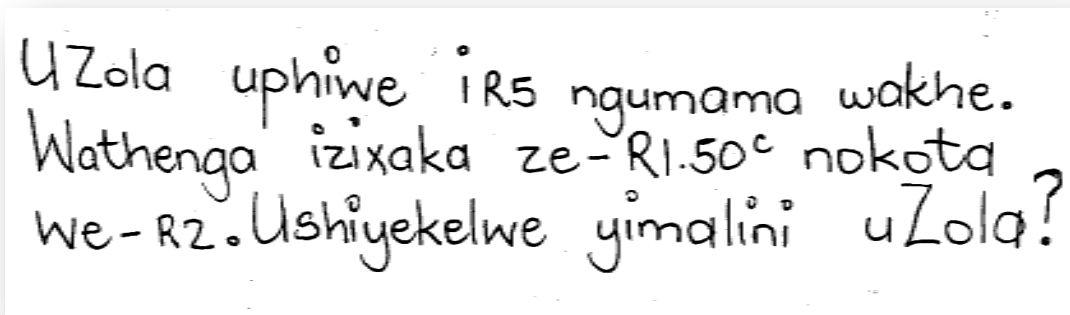
**Figure A9.1****Figure A9.2****Figure A9.3**

2. Lesson structure

The lesson partially followed the three-part PBL lesson format:

- In relation to **getting it right**, the teacher made certain that the problem was understood. She did not, however, activate useful prior knowledge. For example, she could have identified the South African coins and notes.
- The **students worked** in groups, with the teacher letting go, and allowing the learners to work on the problem, this avoiding 'stepping in front of the struggle'.
- No **class discussion** was held after the learners had solved the problem in their groups. Hence, a very important part of the PBL lesson was omitted.

3. Nature of the problem



The teacher had, in developing this problem, considered the level of her class and CAPS demands, which are:

- *Recognise and identify the South African coins (5c, 10c, 20c, 50c, R1, R2, R5, and bank notes R10, R20, R50);*
- *Solve money problems involving totals and change in cents up to 99c or Rands to R99.*

(CAPS Grade 2, term 4. 1.11)

The problem was appropriate for the grade level, and it could be used as a base for the teaching and learning of the above CAPS concepts and skills requirement. The problem met the following criteria of a good problem for PBL:

- casting the learners in a realistic situation; and
- authenticity, with the learners staying motivated and working on the problem.

The problem did not, however, present a complex situation that had no single clear-cut situation. The problem, being not so complex, did not, therefore, develop high-order skills.

APPENDIX 10: PBL PROBLEMS DEVELOPED AND USED BY THE TEACHERS

1. Lizo has two 5c pieces. His sister has three 5c pieces. Write down how much money this is altogether.
2. Zola was given R5 by her mother. She bought sweets for R1,50 and $\frac{1}{4}$ of a loaf of bread for R1,50. How much money does Zola have left?
3. A farmer had to pack eggs in boxes to take to the store. He takes 15 eggs, which he gives to his child to pack in 3 boxes. How many eggs will be in each box?
4. A grandmother wants to bake 4 cakes. Each cake will require 4 eggs. How many eggs must she buy from the shop?
5. Father has 12 apples. He gave these to 3 children. How many apples did each child get?
6. Tina walks 15 footsteps, and his brother 24 footsteps. How many footsteps do they walk altogether?
7. Mr Priza fills his car with petrol once a week. His tank holds 43 litres. How much petrol does he use in a month?
8. Carla earns R14,00 an hour from cleaning houses. She works from 8:00 a.m. to 2:00 p.m. How much money does she earn?
9. A taxi had 6 learners on board. It stopped to pick up learners. When the driver counted them, there were now 10 learners in the taxi. How many learners did the taxi pick up?
10. Tina saves 35 cents in a week. What are his savings over 4 weeks?
11. Jese has 342 stamps. She gave out 149 stamps. How many stamps does Jese now have?
12. Andile has R70,00. Which items will she be able to buy for breakfast?

NB: 2 teachers did not use PBL.

APPENDIX 11: EXAMPLES OF THE TEACHERS' LESSON REFLECTION COMMENTS

REFLECTION EDUCATOR:

The interesting part in the lesson is to see how the learners helped each other when one is stuck in answering a certain question or how to solve the problem.

SIGNATURE _____

REFLECTION EDUCATOR:

Learners must be able to analyse the problem and be able to solve it from their own way and style.

SIGNATURE _____

REFLECTION EDUCATOR:

Show different methods on how to solve a problem.

SIGNATURE _____

REFLECTION EDUCATOR:

Learners tried their level best but as a teacher I still have to create more challenging problems to enforce their thinking.

SIGNATURE _____

REFLECTION EDUCATOR:

Well presented, however drawings were used instead of real coins and that was to help them think and imagine and be able to compare and apply.

SIGNATURE _____

REFLECTION EDUCATOR:

I wasn't so well prepared for the lesson, due to the fact that I forgot, but I carried on on what we have learned. Next time it will be different.

SIGNATURE _____

REFLECTION EDUCATOR:

I worked in groups with a problem surrounding open sums. More practise on the concept is needed. Learners are still unsure.

SIGNATURE _____

REFLECTION EDUCATOR:

I wanted the learners to see how long "the metre" was. And I got through to everyone. I can go on now to next step. Everyone understood well.

SIGNATURE _____

APPENDIX 12: INTERVIEW 1 SCHEDULE**GUIDED INTERVIEW SCHEDULE**

School: _____

Educator: _____

Grade: _____

Date: _____

Q1

What has been your experience with regard to integrating the problem-based learning demands with, and the demands of, the curriculum (CAPS)?

Q2

What has your experience been with regard to the temptation to show and tell the learners what to do?

Q3

What has your experience been with regard to how your learners have worked in their groups?

APPENDIX 13: EXAMPLE OF A TRANSCRIBED INTERVIEW FOR INTERVIEW 1

INTERVIEWER: Cosmas Toga Tambara

RESPONDENT: 03

GRADE TAUGHT: 1

Interviewer: What has been your experience with regard to your planning for, and you implementing of, problem-based learning?

Teacher: Is it regarding problem-solving only, or the entire planning?

Interviewer: When you compare the two, what you have been doing in planning in general, and in planning for problem-based learning.

Teacher: *Ja...*especially after that day even after the workshops there is...ah...if we are...so focused definitely there is a difference and because if you plan in that way, it means children will be more involved, so you don't have to do everything on your selves so now...ah, so definitely in your planning...there is that space you gonna leave it for the learners, it's not your part...you have to give everything to the learners like you spoon-feed them...

So, there is a difference, there is less time spending, it's easier now. It more clear than before ... children have come to understand better when they do for themselves than ... because if I do it myself, you have to do it over and over before they can understand.

Interviewer: What has your experience been with regard to integrating PBL demands with the demands for CAPS?

Teacher: No ... with ... CAPS has everything. Everything is there for CAPS. It's just that sometimes you are required to do lots of things in a short space of time. Like, for instance, with CAPS you have this amount of work...and you have to do it in this period ... understand? Let's say from Monday to Friday, this kind of work according to our planning that we have done, this kind of work has to be covered, understand? So including that ... when you do that, there is a kind of pressure.

Interviewer: Do you see that when we say problem-based learning...in other words what you are supposed to do in the 5 days within CAPS eh...instead of teaching it ...showing it. You start each of the days with a problem. You identify a problem to facilitate the teaching and learning of what it is that you are supposed to do for each of the 5 days with CAPS.

So, in that way its more effective than the old way, it is effective. If you start...I did try it...ever since...even after you were here the other day. I can see it is working, because I believe before we have undermined the thinking, or maybe understanding, of our learners. We always assume, or take it as if, they are there to be spoon-fed only to find out that with problem-solving...that is the time you can see...that also gives them time to think and interest because they always want to explore and find out. So, if you fail to give them that opportunity you will end up struggling all the way through. Understand?

Interviewer: What has been your experience with regard to the temptation to show and tell the learners what to do?

Teacher: Yes there is ... I won't deny that there is temptation, especially because we also have...the main problem is time, and also volume of work, you have to do a week. So in that area, because we have been instructed by our CA like when it comes to problem-solving, it has to be 2 problems per day.

So remember that if the children are not used...like with the timeframe...first of all you must look at ... if they are not trained when it comes to speed... if the pace is not there...you end up doing only that...however, that is the effective way, however, because of the timeframe and time, we only have 4 hours per day for all learning areas and within those areas the volume of work per day and there is break between the 4 hours and there is the temptation whereby you see you are running out of time...especially with slow learners, these children are not getting there, they are still struggling to get there. So in that case you will be tempted ... and assist the children because you don't have the whole time. Yes, with the clever ones you will...like that but with the slow learners, that is where the problem lies. Understand? Because they are still trying to get there, up until you come and give a clue. Some of them you give a clue, still they don't see. So you have to...teach! I put that way...not to teach but to spoon-feed.

Interviewer: What has been your experience with regard to how your learners worked in their groups?

Teacher: Have observed that when they are working in groups...the problem I had in that area...The group thing is fine, but what I find among the learners....they have this specific learner in their group they trust, even if he says something that is not correct, but they rely on him even if this one knows ...that could be the possible answer... but because this says this way, and now he ends up not trusting his or her answer because of this specific individual because this one ... maybe most of the time gets all things right. Understand... and then go and do something on the board, it's everybody's interest in class they like it very much so that it is a problem. When you want to control a group they don't... when it comes to answers especially when the solution is eh...the problem is a little difficult it's not everyone who wants to give out answers but when it's time to show it's everyone. The other thing is... let's say you give out a problem ... while the others are thinking, still working...so now I have learnt from that one that it must not be the same problem, it cannot be if you give this group a problem, that one has to be different from that one because if you give them one problem like that, one individual jumps up with the answer...so now that person has killed the whole class, because everybody cannot think any more.

Interviewer: It also depends on the problem, if it's a problem that has one simple straight solution it will quickly end up the discussion...

Teacher: Em...it will abort everything.

Interviewer: It has to be a problem that has many stages to go through when solving. Maybe, in future, we want to look more into how to construct a problem that will sustain discussion leading on to learning issues.

Em...okay. Thank you very much, madam.

APPENDIX 14: INTERVIEW 2 SCHEDULE

Main Question

During the CLIMMB project intervention, you were introduced to problem-based learning (PBL) and were later observed giving a PBL lesson. I would like to find out if you have continued to use PBL in the teaching of mathematics. Have you continued using PBL in your mathematics lessons this year?

Yes <input type="checkbox"/>	No <input type="checkbox"/>
Questions for those who answer YES	Questions for those who answer NO
1. Can you briefly describe what you understand about problem-based learning?	1. Can you briefly describe what you understand about problem-based learning?
2. Have you discussed PBL with other teachers? <u>Yes</u> What did you discuss? <u>No</u> Why not?	2. Have you discussed PBL with other teachers? <u>Yes</u> What did you discuss? <u>No</u> Why not?
3. What are the reasons for your continuation with this approach to teaching mathematics?	3. What are your reasons for your discontinuation of this approach to teaching mathematics?
4. With what part of PBL did you have problems?	4. With which part of PBL did you have problems?
5. As your learners work in groups, how do you guide them and ensure that meaningful learning is taking place?	5. What support would you need to resume using problem-based learning?
6. Based on the experience that you are having of PBL, what is your assessment of PBL as an approach to teaching mathematics?	6. From whom would you need this support?
7. What support would you need as you continue to use problem-based learning?	7. Based on the experience that you had of PBL during the time you used it, what is your assessment of PBL as an approach to teaching mathematics?
8. From whom would you need this support?	

APPENDIX 15: EXAMPLE OF A TRANSCRIBED INTERVIEW FOR INTERVIEW 2

INTERVIEWER: Cosmas Toga Tambara

RESPONDENT: 03

GRADE TAUGHT: 1

Interviewer: I would like to find out if you have continued using PBL in teaching mathematics. Have you continued using PBL in your mathematics lessons this year?

Teacher: Yes, I have. I am implementing that, and then I have also shared [that] we have got quite [a] few teachers in Grade 1. I have shared that information, and I did mention to why, and why it is important, because we have been to the workshop, and we were taught that the lessons should be learner-centred, and will first start with the learner to find [i.e. finding] out, I mean they is [i.e. have] a knowledge that is there already... so, in order for us to retrieve the knowledge we start with problem-solving, so we start from there. I am still doing that. You can even see my books. We are still implementing that and on [a] daily basis. I share with the new teachers.

Interviewer: Can you briefly describe what you understand about problem-based learning?

Teacher: What I have just mentioned. I have [i.e. It has] to be your lesson. It's not about spoon-feeding a child. It has to involve the child. By involving a child, first get the knowledge from the child. See how much the child knows, you take it from there. Let the child work out of, I mean work out on his own or her own, and assist where possible.

Interviewer: Have you discussed PBL with other teachers?

Teacher: Yes, I did.

Interviewer: Do you remember what you talked about?

Teacher: The importance, ok not despite the knowledge in learners, it's not about the teacher leading the lesson. It's about everybody being involved, mostly the learners.

Interviewer: What did other teachers think about it? What are the reasons why you have continued using implementing PBL?

Teacher: I felt that it's good to use it, because it changed my kind of thinking. Because I was always thinking I am a teacher, so I am the one who is supposed to give the knowledge, and not ever thinking that the children have their own knowledge. I was underestimating their thinking, and being for that I was not giving them the opportunity. I saw it was very important to involve [the learners].

Interviewer: Is it an effective method?

Teacher: It is very. I mean, as far as I am concerned, it's more than being effective to me, to me it has to be.

Interviewer: What have you seen in the learner which makes you conclude that it is effective?

Teacher: It's even worse with . . . other learners, like you will be expecting this kind of answers, but you will get something you never thought of, being at mine [i.e. in terms of my] own understanding, you get surprised the way they put things [that they] understand. I think it is very important to give them the opportunity.

Interviewer: With what part of PBL did you have problems?

Teacher: The only way, they [i.e. there] is nothing wrong with teaching itself. {It is} the only way, the learners they cannot really push themselves to do that. They cannot use their common sense. They don't want to be involved . . . unless you push them, they are still in that method of being pushed to do something. They can't, they don't want to think [or] understand. They always think what they say, it cannot be right, unless the teacher say[s] so.

Interviewer: How do you ensure that the process is meaningful in your groups?

Teacher: It was, *ja*, thank you. They are in groups, and then it's very different because in this same year I had two different classes. With Grade 2, it was more practical, because if you give them this kind of task to do, they do it because they understand it better. But what I have noticed in [a] group, I mean Grade 1 is when I give them different task[s], like I give them A, B, C. You'll find out they [i.e. there] is only one. They don't even come together. You ask them to work together, but they have not seen that. They still want to shine as

individuals. Talk together, this is what I am gonna need from you, but that is still not ringing a bell. That [is] still a challenge, too.

Interviewer: Based on the experience that you are having of PBL, what is your assessment of PBL as an approach to teaching mathematics?

Teacher: Uummh, I would say that it is the most effective ways [i.e. way] which everybody could say. The grouping is very important, because with grouping the learners share the knowledge among themselves. Without you being involved, without you telling them what to do to discover...explore things for themselves. That is very good, by the time you intervene at least you know they have the idea, rather than when you tell [them] something. You know when you tell me something I don't know, I will take time to understand.

Interviewer: What support would you need in order to continue [with the use of] PBL?

Teacher: Especially with the Grade 1s, if I can get a kind of . . . support that can help the children to be more like to kind of understand the method. To get away from being spoon-fed, to get away from always waiting on teacher to say something in order for them to learn, and in a kind of, let them . . . explore things, experience whether they are coming out with the wrong answer. It doesn't matter, as long as they participate. How I can help them to fully participate, to listen. Skills are very poor at this stage.

Interviewer: From whom do you want to get support?

Teacher: Whom? I don't think that's not a very easy question. If I look at that, how can I say whom, because, first of all, I have to maybe if I ask for support. I may have to ask from my HOD, first from her, then maybe from the other teacher's as well from with the same grade, but they might not be experiencing what I am experiencing. Maybe my CA, *ja*, otherwise we all attend the workshop. She is also helping us in this category, but [the] time is very limited. I need someone, you know. When you go to the workshop, the children are not there, so things are just theorised. The only thing I always wish for is it can be practical, and somebody can be in class and can experience what you experience.

Interviewer: Thank you very much for the interview. That will be all.